Chapter 8

Elasticity of Demand

8.1 Motives and objectives

Broadly

To understand the mark-up above marginal cost that arises from market power, we need to understand marginal revenue. Marginal revenue is related to how price-sensitive the demand curve is, when we measure price-sensitivity in percentage terms: the percentage fall in demand from a percentage increase in price. This measure is called the elasticity of demand.

The purpose of this chapter is to revisit and explore the elasticity of demand functions and demand curves.

More specifically

Here are some cases in which sensitivity of demand is important.

1. For a firm with market power, its optimal price depends on the price sensitivity of the demand for its good. For example, the benefit of raising its price depends on how abruptly demand would fall.
2. The impact of a tax on the competitive equilibrium price—and also the tax revenue generated and the deadweight loss—depend on the price sensitivity of demand and supply.
3. If a supply curve shifts, perhaps because of change in technology, the effect on the price depends on the price sensitivity of demand.

In particular, price sensitivity will be an important concept during our study of pricing by imperfectly competitive firms.

You might expect to measure price sensitivity by the slope of a demand curve. But for many applications, a more useful measure of the sensitivity of a dependent variable \( Y \) to an independent variable \( X \) is elasticity: the percentage change in \( Y \) divided by the percentage change in \( X \). This is true for all the applications of price sensitivity just described.

Here is a little teaser from Chapter 9. The following statement is intuitive: “Consider a firm that can segment its market. The firm should charge a higher price in the market segment in which demand is less price sensitive.” We will see that this statement is correct if we measure price sensitivity by elasticity, while it is false if we measure it by slope.
8.2 Measuring elasticity

Loosely

Thus, we measure the responsiveness of demand to changes in a good’s own price by the *(own-price) elasticity of demand*, which we denote by $E$:

$$ E = -\frac{\% \text{ change in } Q}{\% \text{ change in } P}. $$ (8.1)

Because the change in demand goes in the opposite direction of the change in price, we have inserted a negative sign to make sure that elasticity is a positive number. This convention, which is common practice within the discourses of economists, makes it much easier to discuss elasticity because a higher value means more responsive demand. However, the formal definition of elasticity does not have this sign change, so you should not be surprised to see own-price elasticity as a negative number in some articles. (Then, if elasticity has gone *up* from $-3$ to $-2$, demand has become *less* elastic!)

Elasticity is “unit free”. A percentage change is the same whether we measure quantity in liters or gallons or whether we measure price in euros or bahts, whereas any such variation in units changes the numerical value of a demand curve’s slope.

**Discrete changes: Arc elasticity**

Elasticity varies along a demand curve; that is, the responsiveness of demand to a change in price depends on the initial price that is being charged. Furthermore, quantitatively it measures responses to small changes in price. We therefore say that elasticity is a *local* property of demand. This introduces a few technical issues when translating the intuitive equation (8.1) into specific formulas.

Consider first the elasticity that we measure based on a discrete change in the price level. Suppose that the price changes from $P_1$ to $P_2$ and that, as a result, demand changes from $Q_1$ to $Q_2$ as a result. We end up with different numbers for the elasticity depending on whether we measure changes as percentages of the initial or of the final values. Which price level should we treat as the status quo?

We give the two points equal status by measuring changes as percentages of the averages of the initial and final values. This yields the formula

$$ E = -\frac{\frac{1}{2}(Q_2 - Q_1)}{\frac{1}{2}(P_2 - P_1)} \frac{P_2 - P_1}{\frac{1}{2}(P_1 + Q_2)}, $$

which is called the *arc elasticity* between the points $(P_1, Q_1)$ and $(P_2, Q_2)$ on the demand curve.

**Example 8.1** Let the demand function be

$$ Q = 16 - 0.6P. $$
Now suppose the price is initially $20K and hence demand is 4. Suppose the price increases to $20.2K and hence demand falls to 3.88. Then the arc elasticity is

$$\frac{0.12}{\frac{1}{4}(4 + 3.88)} \div \frac{0.2}{\frac{1}{4}(20 + 20.2)} = 3.06.$$ 

**Smooth changes: Point elasticity**

If the demand function is smooth, then we have a nice definition of the elasticity at a point on the demand curve:

$$E = -\frac{dQ}{dP} \frac{P}{Q}.$$ 

This is called the point elasticity. The slope $dQ/dP$ is “normalized” by multiplying by $P/Q$ (and by $-1$ so that it is positive). The point elasticity is approximately equal to the arc elasticity for small price changes.

Point elasticity is a local measure: When you use it to predict a change in demand due to a discrete change in price, your answer will only be approximate—but with the percentage error going to zero as the size of the price change decreases.

**Example 8.2** In Example 8.1, the demand curve is $Q = 16 - 0.6P$. Let’s calculate the point elasticity at $P = 20$ and $Q = 4$. The slope $dQ/dP$ of the demand curve is $-0.6$. Hence, the point elasticity is

$$E = 0.6 \frac{20}{4} = 3.$$ 

**Why 1 is a special value**

Own-price elasticity can range from 0 to $\infty$ (the symbol for infinity). We have the following terminology for the different values of the elasticity.

<table>
<thead>
<tr>
<th>We say demand is …</th>
<th>if …</th>
</tr>
</thead>
<tbody>
<tr>
<td>perfectly inelastic</td>
<td>$E = 0$</td>
</tr>
<tr>
<td>inelastic</td>
<td>$E &lt; 1$</td>
</tr>
<tr>
<td>unit-elastic</td>
<td>$E = 1$</td>
</tr>
<tr>
<td>elastic</td>
<td>$E &gt; 1$</td>
</tr>
<tr>
<td>perfectly elastic</td>
<td>$E = \infty$</td>
</tr>
</tbody>
</table>

Why is $E = 1$ the right division between elastic and inelastic demand? Here is one reason. Suppose the price rises. What happens to the consumers’ expenditure (and hence the firms’ revenue)? Does it go up, go down, or stay the same?

1. Consumers pay more per unit. This causes expenditure to go up—in percentage terms, by the percentage increase in price.
2. Consumers buy less. This causes expenditure to go down—in percentage terms, by the
percentage decrease in demand.

Which effect dominates? It depends on whether the percentage decrease in demand is larger than the percentage increase in price.

- **Inelastic demand:** $E < 1$. The percentage decrease in demand is *smaller* than the percentage increase in price. Expenditure rises.
- **Elastic demand:** $E > 1$. The percentage decrease in demand is *larger* than the percentage increase in price. Expenditure falls.
- **Unit-elastic demand:** $E = 1$. The percentage decrease in demand just offsets the percentage increase in price. Expenditure stays roughly the same.

In Example 8.1, the elasticity is 3.06, meaning that demand is elastic. When the price rose from $20K to $20.2K, expenditure fell (from $20 \times 4 = 80$ to $20.2 \times 3.88 = 78.38$), as predicted.

Worldwide demand for coffee is inelastic (as it is for many commodities). The inflation-adjusted price of coffee fell by half from 1998 to 2002 owing to increases in supply from Vietnam (and some other new producers) and Brazil (which expanded production in frost-free zones). (Vietnam’s exports grew from 78 million kilos in 1991 to 418 million kilos in 1998 and then 896 million kilos in 2000; since 1999, it has displaced Colombia as the world’s second-largest exporter of coffee.) The Association of Coffee Producing Countries tried to get producing countries to retain 20% of their production. The UN Food and Agricultural Organization estimated that the elasticity of demand for coffee was about 0.6 (inelastic), so that such a program would have raised the price of coffee by about 32% and resulted in an increase in revenue of 5.5%.

(However, as is common with such cartels, getting countries to participate is subject to a “free rider” problem. Only Brazil, Colombia, Costa Rica, and Vietnam pledged to participate. Their reductions would have raised the price by only 17%. Although total revenue would still have risen, the non-participating countries would have profited at the expense of the participating countries. Brazil, Colombia, Costa Rica and Vietnam would have seen their revenues fall by 6.5%, whereas the revenues of the other countries would have risen by 17%. Eventually, the program was abandoned.)

### 8.3 Elasticity with respect to other parameters

It is possible to measure how demand responds to changes in other parameters, such as income and prices of other goods. For example, the responsiveness to changes in the price $P_s$ of another good is measured by

$$\frac{\% \text{ change in } Q}{\% \text{ change in } P_s}$$
This is called the cross-price elasticity of demand. Cross-price elasticity is positive if the two goods are substitutes and is negative if the two goods are complements.

The responsiveness to changes in income is measured by

\[
\frac{\% \text{ change in } Q}{\% \text{ change in } I}
\]

and is called the income elasticity of demand. It is positive if the good is normal and it is negative if the good is inferior.

The elasticity with respect to changes in the good’s own price is called the own-price elasticity of demand (when it is necessary to distinguish it from these other elasticities). In this text, we make use mainly of own-price elasticity of demand and so refer to it simply as the elasticity of demand.

Recall that a good is a luxury good if the fraction of income spent on the good rises when income rises. This happens if, following a rise in income, the percentage increase in demand is larger than the percentage increase in income—that is, if the income elasticity is greater than 1.

In European countries, where bicycles are often used for leisure, the income elasticity of the demand for bicycles is greater than 1. In China, on the other hand, bicycles are an inferior good (negative income elasticity) because wealthier people swap their bicycles for cars.

### 8.4 Elasticity of special demand curves

#### Elasticity of linear demand curves

The slope of a linear demand curve \( Q = A - BP \) is \( dQ/dP = -B \). Thus, point elasticity when the price is \( P \) equals

\[
E = \frac{dQ/P}{Q} = \frac{P}{A - BP} = \frac{P}{A/B - P} = \frac{P}{\bar{P} - P}.
\]

Hence, elasticity at a given price depends on the parameters \( A \) and \( B \) of the demand function only through their ratio \( A/B \), which is the choke price \( \bar{P} \).

Observe that demand becomes less elastic as the price falls. In fact, if the price is close to zero then the elasticity is close to zero, whereas elasticity of demand increases without bound as \( P \) approaches the choke price \( \bar{P} \). Demand has unit elasticity when \( P/(\bar{P} - P) = 1 \), that is, when \( P = \bar{P}/2 \).

**Example 8.3** Recall our linear demand curve for minivans:

\[
Q = 16 - 0.6P
\]

The choke price is \( \bar{P} = 16/0.6 = 26.67 \), so demand has unit elasticity when \( P = 13.33 \). Figure 8.1 shows the demand curve and labels the regions of elastic, unit-elastic, and inelastic demand.
Exercise 8.1. Market research revealed that the market demand function for home exercise equipment is

\[ Q = 2400 - 2P - 15P_v, \]

where \( P \) is the price of exercise equipment and \( P_v \) is the price of exercise videos. The current price of exercise equipment is 300 and the current price of exercise videos is 20.

a. Given these prices, calculate the own-price elasticity of demand for exercise equipment.

b. Are exercise videos and exercise equipment complements or substitutes?

c. Suppose the price of exercise videos increases to 40. Does the own-price elasticity of demand increase or decrease?

Constant-elasticity demand curves

For an log-linear demand curve \( Q = AP^{-B} \), elasticity equals \( B \) everywhere on the demand curve. Hence, another name for log-linear (or power) demand curves is constant-elasticity demand curves.

Perfectly inelastic and elastic demand

Sometimes demand is extremely inelastic. We can approximate such inelastic demand by the limiting case of a perfectly inelastic demand curve: demand is the same no matter what
price is charged, so the demand curve is a vertical line. Figure 8.2 shows an example.

Figure 8.2

A 1996 study of the demand for outpatient services in Japan measured elasticities for different categories of service, with demand measured as a function of the patients’ out-of-pocket expenses.\footnote{J. Bhattacharya, W.B. Vogt, A. Yoshikawa, and T. Nakahara, 1996, “The Utilization of Outpatient Medical Services in Japan.” *Journal of Human Resources*, 31:450–476.} Elasticities ranged from 0.12 to 0.54 for most categories. Demand for outpatient services is typically not perfectly inelastic because many of the services are elective and are not required for survival, or a person can postpone a visit hoping that an illness subsides on its own. However, for genitourinary disorders the elasticity was not statistically distinguishable from zero. For this category, the main service provided was kidney dialysis, which patients need at fixed intervals in order to survive.

At the other extreme, demand may be very elastic. There is a price $P$ such that (a) when the price is a little higher than $P$, demand drops off quickly; and (b) when it is little below $P$, demand rises quickly. It can be useful to approximate such elastic demand by the limiting case in which demand is zero when the price is higher than $P$, is infinite when the price is below $P$, and is any amount when the price equals $P$. The demand curve is a horizontal line at $P$. Such a demand curve is *perfectly elastic*; Figure 8.3 shows an example.

Figure 8.3
8.5 The reality of estimation of demand

Linear demand versus constant-elasticity demand

Linear demand functions and constant-elasticity (log-linear) demand functions are the simplest classes of demand functions. Neither is a true representation of real-world demand functions (which are too complicated to work with or measure exactly), but each is a useful approximation. One might say that linear functions are too straight and constant-elasticity functions are too curved, with the curvature of real-world demand functions lying between the two.

We can now see for what purposes each form is useful.

Linear demand: for simple graphical and algebraic illustrations. A linear demand curve is easy to draw and work with. Furthermore, it has the property that demand becomes more elastic moving up the demand curve, which holds in the real world and is important for certain qualitative conclusions pursued in this book.

Constant-elasticity demand: for empirical estimation. One cannot accurately estimate an entire demand curve—instead, the goal is to obtain a good local estimate in the region of the data used for the estimation. Furthermore, one is typically interested in estimating the elasticity of demand (you will see why in subsequent applications of elasticity). This is best done using a power demand function in its log-linear form. When you estimate the linear regression equation

$$\log(Q) = \log(A) - B \log(P),$$

the coefficient $B$ is simply the elasticity.

Any such estimation is an approximation, because (a) there are other variables that affect demand and (b) the relationship between the included variables and demand is not perfectly linear or log-linear (or whatever functional form is used). These effects are picked up in the “error term” of the regression. Adding more variables or more parameters to the functional form might seem to give a more accurate description of the market, but it will weaken our empirical estimates of the coefficients for the other variables.\(^2\)

Interpreting the results of demand estimation

In fact, nearly all estimates of demand functions use the log-linear form or some variant thereof. Researchers typically report only the elasticities. It can be surprising to see no description of the units by which quantities and prices are measured, but these are not reported because elasticities are unit-free.

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2. In regression theory, we say that there is a loss of “degrees of freedom”. The goodness of fit ($R^2$) goes up as we add more explanatory variables, but the accuracy of the estimates of the coefficients for the explanatory variables eventually goes down.
For example, a 2005 econometric study of the demand for tobacco and other addictive goods in India summarized its results for rural India as shown in Table 8.2.

Table 8.2

<table>
<thead>
<tr>
<th>Item</th>
<th>Bidi</th>
<th>Cig</th>
<th>Tleaf</th>
<th>Pan</th>
<th>Alcohol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bidi</td>
<td>−0.997</td>
<td>−0.100</td>
<td>−0.010</td>
<td>−0.026</td>
<td>0.023</td>
</tr>
<tr>
<td>Cigarette</td>
<td>−0.187</td>
<td>−0.626</td>
<td>−0.018</td>
<td>0.010</td>
<td>0.150</td>
</tr>
<tr>
<td>Tleaf</td>
<td>−0.093</td>
<td>0.212</td>
<td>−0.848</td>
<td>−0.129</td>
<td>−0.030</td>
</tr>
<tr>
<td>Pan</td>
<td>−0.075</td>
<td>−0.021</td>
<td>−0.010</td>
<td>−0.600</td>
<td>−0.023</td>
</tr>
<tr>
<td>Alcohol</td>
<td>−0.258</td>
<td>0.114</td>
<td>−0.022</td>
<td>0.084</td>
<td>−1.032</td>
</tr>
</tbody>
</table>

Bidi is made by rolling a piece of temburini leaf around flaked tobacco into a cone shape. Pan is a composite of betel leaf, areca nut, slaked lime, catechu, and tobacco. Tleaf stands for unprocessed leaf tobacco.

The own-price elasticities are all negative. This is the way own-price elasticities are reported in written documents. We have adopted a convention in this textbook to give own-price elasticities as magnitudes (positive numbers), which is common in discourse. Hence, we would say that the own-price elasticity of demand for Bidi is 0.997; it is close to unit-elastic. If a tax on bidi is imposed, consumption will fall but expenditure on bidi will remain roughly constant. The demand for cigarettes is less elastic, only 0.626. If the tax on cigarettes is raised, consumption will fall but people will still spend more on cigarettes.

The cross-price elasticities of the demand for bidi with respect to the price of cigarettes is negative: −0.100. Thus, these two goods are complements. This is common for some pairs of addictive goods. Unusually, however, these data show that cigarettes and alcohol are substitutes in India.

8.6 Wrap-up

We introduced elasticity, a measure of the sensitivity of demand that is percentage terms: percentage change in demand over percentage change in price. Elasticity has many applications, including within the model of perfect competition. However, all our applications will be to understanding the mark-up over marginal cost that arises due to market power, i.e., when there is imperfect competition.

Additional exercises

Exercise 8.2. Calculate the price elasticity at current prices in the following examples. If you do not have enough information, say so.

a. The firm’s demand curve is $Q = 2000 - 5P$, and the firm’s output is 500.

b. The firm’s demand curve is $Q = 5P^{-1.55}$; the firm’s price and output are unobserved.

Exercise 8.3. Table E8.1 shows actual data about the prices of Model T touring cars in different years and the sales volumes at those prices.

Table E8.1

<table>
<thead>
<tr>
<th>Year</th>
<th>Retail price</th>
<th>Sales volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1908</td>
<td>850</td>
<td>5,986</td>
</tr>
<tr>
<td>1909</td>
<td>950</td>
<td>12,292</td>
</tr>
<tr>
<td>1910</td>
<td>780</td>
<td>19,293</td>
</tr>
<tr>
<td>1911</td>
<td>690</td>
<td>40,402</td>
</tr>
<tr>
<td>1912</td>
<td>600</td>
<td>78,611</td>
</tr>
<tr>
<td>1913</td>
<td>550</td>
<td>182,809</td>
</tr>
<tr>
<td>1914</td>
<td>490</td>
<td>260,720</td>
</tr>
<tr>
<td>1915</td>
<td>440</td>
<td>355,276</td>
</tr>
<tr>
<td>1916</td>
<td>360</td>
<td>577,036</td>
</tr>
</tbody>
</table>

a. Assuming that these data represent points on a fixed demand curve, calculate the arc elasticity of demand by comparing the data (i) for the years 1910 and 1911 and (ii) for the years 1915 and 1916.

b. Give two reasons why we might not want to consider these data to be points on a fixed demand curve.