Wealth, Information Acquisition, and Portfolio Choice

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I solve (with an approximation) a Grossman-Stiglitz economy under general prefer-
ences, thus allowing for wealth effects. Because information generates increasing
returns, decreasing absolute risk aversion, in conjunction with the availability of
costly information, is sufficient to explain why wealthier households invest a larger
fraction of their wealth in risky assets. One no longer needs to resort to decreasing
relative risk aversion, an empirically questionable assumption. Furthermore, I show
how to distinguish empirically between these two explanations. Finally, I find that the
availability of costly information exacerbates wealth inequalities.

The effect of wealth on households' demand for risky assets has long been
studied, starting with the works of Cohn et al. (1975) and Friend and
Blume (1975). They document that the fraction of wealth households
invest in stocks increases with their wealth. Several recent studies using
different datasets and estimation techniques confirm their observation.[1]
One common explanation for the observed pattern of portfolio shares is
that relative risk aversion decreases with wealth [e.g., Cohn et al. (1975)].
Moreover, some authors [e.g., Morin and Suarez (1983)] use portfolio
data to elicit households' preferences and conclude from the observation
of shares that relative risk aversion is decreasing. However, abstracting
from portfolio data, there is not much evidence in favor of decreasing
relative risk aversion. Several studies reject this hypothesis using data that
contains information about attitudes toward risk such as farm data, survey
data, or experimental data.[2] Here, I suggest an alternative explanation for

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[1] These studies estimate the elasticity of portfolio shares with respect to wealth to be around 0.1, where
portfolio shares refer to the fraction of financial wealth invested in risky assets, both directly and
indirectly, conditional on holding some risky assets. I review the evidence in detail in Section 1.

[2] In addition, Arrow (1971) makes a theoretical argument in favor of increasing relative risk aversion. The
empirical studies are reviewed in Section 1. Section 6 also rules out alternative explanations for portfolio
shares based on fixed entry costs and psychological biases.

the observed pattern of portfolio shares and wealth. This explanation only requires *absolute* risk aversion to be decreasing with wealth, an assumption that is supported by all empirical studies. (In particular, the model reconciles the common assumption that relative risk aversion is constant with the observed pattern of portfolio shares.)

In addition to decreasing absolute risk aversion, the explanation offered in this article relies on the possibility to acquire, at a cost, information about stocks. Though they are not directly observable, there is evidence that differences in information do matter to investors' decisions and that these differences are related to households' measurable characteristics such as wealth. Several surveys in Europe and the United States document the importance of information for stock ownership.[3] For example, Alessie, Hochguertel, and Van Soest (2002) use data from a Dutch survey that includes a measure of interest in financial matters and find that this variable has a significant and positive effect on portfolio shares. Moreover, Donkers and Van Soest (1999) show that this financial interest variable is strongly positively correlated to income. In the same spirit, Lewellen, Lease, and Schlarbaum (1977) report that the money spent by investors on financial periodicals, investment research services, and professional counseling increases with both income and education. On another front, research in accounting shows that small trades react less to earnings news than large trades do, suggesting that wealthier investors (i.e., investors who place large orders) process the news and adjust their orders faster than poorer investors.[4]

This article explains the cross-sectional pattern of stockholdings and wealth by endogenous differences in information. For that purpose, I model explicitly how investors acquire information. I show that though they do not have lower relative risk aversion, wealthier investors hold a larger fraction of their wealth in stocks.[5] The reason is that the value of information increases with the amount to be invested, whereas its cost does not. This implies that agents with more to invest acquire more information. Consequently they purchase even more stocks and hold a larger portfolio share. Thus they do so not because they are relatively less

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[3] For Europe, see Alessie, Hochguertel and Van Soest (2002), Borsch-Supan and Eymann (2002), Guiso and Jappelli (2002). For the United States, see King and Leape (1987).

[4] See Cready (1988) and Lee (1992). In addition, some articles argue that costly information processing explains some puzzling phenomena in finance such as the "home equity bias" [French and Poterba (1991), Kang and Stulz (1994), Coval and Moskowitz (1999)] and the "weekend effect" [Miller (1988) and Lakonishok and Maberly (1990)] and others provide evidence on the role of financial education and social interactions for stock ownership [Bernheim and Garret (1996), Chiteji and Stafford (1999), Weisbenner (1999), Bernheim, Garret, and Maki (2001), Huberman (2001), Duflo and Saez (2002)].

[5] Therefore one should be cautious when infering the determinants of relative risk aversion from portfolio shares. What looks like decreasing relative risk aversion (increasing portfolio shares) may in fact be the result of decreasing absolute risk aversion combined with information purchase. This applies not only to wealth, as the article shows, but also to other determinants of risk aversion such as age or education.

risk averse, but because the stock is less risky to them. Importantly, this result does not rely on any form of increasing returns to scale embedded in technology or preferences: it is obtained in spite of a strictly convex information acquisition cost and prevails when relative risk aversion is increasing.

The model builds on Grossman and Stiglitz (1980) and Verrecchia (1982). In Grossman and Stiglitz (1980), traders may purchase private information about the payoff of a stock, which they use to trade competitively in the market. Their information gets revealed by the equilibrium price, but only partially because there is some noise in the system. In Verrecchia (1982), traders are allowed to choose continuously the precision of their private signal. A key assumption of these rational expectations models with asymmetric information is that agents have constant absolute risk aversion utility (CARA or exponential). Hence these models ignore the role of wealth, though it is an important determinant of stockholdings. To capture wealth effects, I solve the model under general preferences.[6] A closed-form solution is derived by making a small risk approximation. The point of the article is that, as long as absolute risk aversion decreases with wealth, there will be increasing returns to acquiring private information even though it gets revealed by public signals.[7]

Finally, I study the link between wealth inequality and stock prices. Because information generates increasing returns, the demand for stocks is a convex function of wealth. Hence the more unequal the distribution of wealth, the higher the stock price. Conversely, wealthier investors achieve a higher expected return, a higher variance, and a higher Sharpe ratio on their portfolio. Consequently, the distribution of final wealth as measured by expected wealth or by certainty equivalent is more unequal than the distribution of initial wealth. This fact also suggests a simple way of discriminating the information model from the decreasing relative risk aversion model: in the former, the Sharpe ratio on an agent's portfolio increases with her wealth, whereas in the latter, it is constant. Using a comprehensive dataset on Swedish households, Massa and Simonov (2003) report that Sharpe ratios increase with financial wealth, in accordance with the information model. More research is needed to confirm these results.

The remainder of the article is organized as follows. Section 1 reviews the evidence on the relations between wealth, portfolio shares, relative risk aversion, and information acquisition. Section 2 describes the economy. Section 3 defines the equilibrium concept. Section 4 solves the model: the

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[6] However, it should be noted that the model presented here is static and hence does not capture hedging demands. This important feature of portfolio choice is considered in dynamic models with CARA preferences.

[7] The idea of increasing returns to information is not new, but to my knowledge, it has not been modeled in a setup where private information gets partially revealed by public signals, as in the stock market [Wilson (1975) and Arrow (1987)].

equilibrium is characterized and the relation between wealth and portfolio shares is described. Section 5 studies the effect of information acquisition on wealth and return inequality. Finally, Section 6 addresses some empirical issues: I calibrate the model to U.S. data, show how to discriminate the information acquisition model from the decreasing risk aversion model using micro data, and finally, discuss alternative explanations based on fixed entry costs and psychological biases. Section 7 concludes and suggests some applications. Proofs and robustness checks are in the appendix.

1. Evidence

In this section I review the evidence on the relations between wealth, portfolio shares, relative risk aversion, and information acquisition.

1.1 Wealth and portfolio shares
This article is motivated by the observation that the share of wealth households invest in stocks increases with their wealth, so let me now be more precise about how portfolio shares are measured. First, stocks refer to equity that is held both directly and indirectly through mutual funds. Second, depending on how housing is treated (whether it is excluded, included as a riskless asset, included as a risky asset, priced at market value, or priced at owner's equity value), different studies reach different conclusions about the effect of wealth on portfolio shares of risky assets. However, virtually all agree that the fraction of *financial* wealth invested in stocks (i.e., total wealth excluding housing, capitalized labor, private businesses, social security, and pension incomes) increases with financial wealth. Third, portfolio shares of stocks are computed conditional on owning some stocks. Accordingly, the purpose of this article is to explain the fraction of financial wealth households invest in risky assets, both directly and indirectly, conditional on being a stockholder.

Several recent articles estimate the elasticity of portfolio shares with respect to wealth to be around 0.1. The ones mentioned below use different datasets and econometric techniques, but all conform with the three points made above and, in particular, separate the share choice from the participation decision. Vissing-Jørgensen (2002) uses the Panel Study of Income Dynamics and finds estimates of 0.09, 0.12, and 0.10, depending on the specification of the model.[8] Bertaut and Starr-McCluer (2002) use several waves of the Survey of Consumer Finance and find estimates of 0.17, 0.04, and 0.06. Finally, Perraudin and Sørensen (2000) use the 1983 Survey of Consumer Finance and find an estimate of 0.09. Other articles

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[8] For example, in Table 2, Vissing-Jørgensen (2002) reports regression coefficients on wealth and wealth squared equal to 0.0011 and -0.00000149, which imply an elasticity of 0.12 using the average wealth of $74,810.

report elasticities but differ either in their measures of wealth or do not condition on participation.[9] An often-cited explanation for the observed positive elasticity is that relative risk aversion is decreasing with wealth. As the next section shows, this hypothesis does not hold in the data.

1.2 Wealth and relative risk aversion

In contrast to decreasing absolute risk aversion, there is not much support for decreasing relative risk aversion outside portfolio data. The evidence instead points to increasing or constant relative risk aversion *in environments where information cannot be acquired*.[10] First, studies in agricultural economics use data on farmers who allocate their land across crops of different risks, the same way an investor allocates her wealth across securities. Saha, Shumway, and Talpaz (1994) and Bar-Shira, Just, and Zilberman (1997) find a clear pattern of decreasing absolute risk aversion and increasing relative risk aversion using different estimation techniques and datasets.

Second, surveys have been designed to elicit the respondents' risk aversion by asking questions about hypothetical lotteries. Barsky et al. (1997) offered the respondents of the Health and Retirement Study gambles involving new jobs and found that relative risk aversion rises and then falls with wealth. Similarly, Guiso and Paiella (2001) asked the respondents of the Bank of Italy Survey of Household Income and Wealth for the maximum price they would be willing to pay to participate in a lottery. The answers show that absolute risk aversion is a decreasing function of wealth, while relative risk aversion is an increasing function. Furthermore, when portfolio shares of risky assets are regressed on the measure of risk aversion, wealth, and other demographic variables, the coefficient on risk aversion is significantly negative and the coefficient on wealth is significantly positive, suggesting that wealth plays a role not captured by risk aversion.[11]

Finally, experimental studies provide some interesting insights on risk aversion. Gordon, Paradis, and Rorke (1972), Binswanger (1981), and

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[9] For example, King and Leape (1998) use net worth as their measure of wealth and Heaton and Lucas (2000) do not condition on stock ownership in their regressions of portfolio shares on financial wealth (Table IX).

[10] An exception is Ogaki and Zhang (2001), but this study focuses on households close to their subsistence level.

[11] Studies in other fields strengthen the case against decreasing relative risk aversion. Szpiro (1986) uses aggregate data on property and liability insurance in the United States from 1951 to 1975 and finds that relative risk-aversion is constant. Wolf and Pohlman (1983) examine the bids of a U.S. bond dealer who gets most of his income from a fixed share of the profits he generates. Combining this information with the dealer's returns forecasts, they find that absolute risk aversion is decreasing and that relative risk aversion is constant or slightly increasing. Aït-Sahalia and Lo (2000) and Jackwerth (2000) use options prices to estimate the risk-neutral and subjective distributions of the S&P 500 index (a measure of aggregate wealth) from which they infer a representative investor's risk aversion. They find that relative risk aversion is a nonmonotonic function of wealth.

Quizon, Binswanger, and Machina (1984) offered subjects (MBA students or Indian villagers) gambles with real prizes. The result is that the fraction of wealth they play declines as their wealth increases, pointing to increasing relative risk aversion. This pattern is, however, in sharp contrast with U.S. households' portfolio data. The model presented here provides a way to reconcile these conflicting observations. Indeed, in these experimental studies, subjects have to choose among gambles with known odds and information cannot be acquired, in contrast to the real world. Hence an interpretation is that relative risk aversion is really increasing, but that the returns to scale generated by information acquisition are so powerful that they overturn the tendency for portfolio shares to decrease with wealth into a tendency to increase. Next I review the relation between wealth and information.

1.3 Wealth and information acquisition

The evidence on the effect of wealth on information relies mainly on surveys. Lewellen, Lease, and Schlarbaum (1977) asked a sample of customers of a large U.S. retail broker how much they spent on financial periodicals, investment research services, and professional counseling. They find that information expenditures increase very significantly with income. In the same spirit, Donkers and Van Soest (1999) use data from a Dutch survey which contains information on interest in financial matters and show that it is strongly positively correlated to income. I now turn to the model.

2. The Economy

The model is in the spirit of Grossman and Stiglitz (1980) and Verrecchia (1982). There are three periods, a planning period ($t = 0$), a trading period ($t = 1$), and a consumption period ($t = 2$). Agents receive public information and may purchase private information about the payoff of a stock, which they use to trade competitively in the market. Some noise prevents the equilibrium price from fully revealing agents' private information.

2.1 Investment opportunities

Two assets are traded competitively in the market, a riskless asset (the bond) and a risky asset (the stock). The stock represents the equity market as a whole, which investors attempt to time.[12] Unfortunately there exists in general no closed-form solution for the equilibrium in this economy when absolute risk aversion is not constant because the demand for risky

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[12] For concreteness, the stock may be viewed as a share of a mutual fund. Most mutual funds are specialized in equity or bonds. In 1998 there were more than 7,000 mutual funds in the United States; hybrid funds accounted for only 7% of all funds and managed only 9% of the industry's assets according to the Investment Company Institute.

assets is no longer a linear function of the expected payoff. For this reason, I resort to a local approximation to compute the equilibrium when the stock has small risk. Specifically, I create a continuum of economies, each with a different set of fundamentals and hence a different portfolio problem. Each economy is indexed by a parameter, z, that scales the variables representing risks, payoffs, and trading costs. In particular, the stock's expected payoff and variance are both proportional to z so that the mean to variance ratio is constant across the continuum of economies. The model will then be solved in closed form by driving z toward zero.[13] The riskless asset is in perfectly elastic supply and has a net rate of return of $r^f z$. The risky asset has a price P and a random payoff Π that is log-normally distributed. Let πz be the "growth rate" of Π:

$$\pi z \equiv \ln \Pi.$$

With the stock price acting as a public signal, one more source of risk is needed to preserve the incentives to purchase private information. This role is played by the supply of stocks emanating from noise traders.[14] Let θ represent the net supply of stocks (i.e., the total number of shares plus the supply from noise traders). By assumption, θ and π are jointly normally distributed and independent and the mean and variance of θz and πz are linear in z:

$$\begin{pmatrix} \ln \Pi \\ \theta \end{pmatrix} \sim N \left\{ \begin{pmatrix} E(\pi)z \\ E(\theta) \end{pmatrix}, \begin{pmatrix} \sigma_\pi^2 z & 0 \\ 0 & \sigma_\theta^2/z \end{pmatrix} \right\}.$$

2.2 Information structure

Agents may spend time and resources gathering information about the stock market, i.e., about the stock's payoff Π. For example, they may read newspapers, listen to radio and TV reports, surf the Web, participate in seminars, subscribe to newsletters, join investment clubs, or hire a financial advisor. Agent j may purchase a signal S_j about the payoff of the stock Π,

$$S_j = \ln \Pi + \varepsilon_j, \tag{1}$$

where $\{\varepsilon_j\}$ is independent of Π, θ, and across agents. Let x_j denote the precision of agent j's signal. I assume that ε_j is normally distributed:

$$\varepsilon_j \sim N\left(0, \frac{z}{x_j}\right).$$

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[13] The scaling factor z has the flavor of the time increment dt in a continuous-time model.

[14] Noise or liquidity traders are a group of agents who trade for reasons not explicitly modeled. For example, these agents may have access to a private investment opportunity such as human capital, durables, or nontraded assets. Alternatively, they could make common random errors in their forecasts of the stock's payoff.

The signal costs $C(x_j)z$ dollars, where C is increasing and strictly convex in the precision level. Specifically, I assume that

$$C(0) = 0, \quad C'(\cdot) \geq 0, \quad C''(\cdot) > 0 \text{ on } [0, \infty] \quad \text{and} \quad \lim_{x \to \infty} C'(x) = +\infty.$$

These assumptions ensure the existence of an interior solution. They capture the idea that each extra piece of information is more costly than the previous one; for example, because they are correlated. Allowing for a nonconvex cost function would only strengthen the point of the article, that wealthier investors acquire more information. For example, the specification $C(x) = x^c$ for $c > 1$ satisfies the assumptions. Agency problems (not modeled here) preclude investors from sharing or selling their private information.

Finally, in a rational expectations equilibrium, agents know that the equilibrium price P contains some information about the risky payoff Π and they will use it as an informative signal. $\mathcal{F}_j$ denotes investor j's information set: $\mathcal{F}_j = \{S_j, P\}$ if investor j acquires a private signal and $\mathcal{F}_j = \{P\}$ if she does not. $E_j(\cdot \mid \mathcal{F}_j)$ and $E_j(\cdot)$ refer respectively to period 1 and period 0 expectations, by investor j, where the private signal S_j is distributed with precision x_j.

2.3 Investors

There is a continuum of heterogeneous agents in number normalized to one. Their objective is to maximize expected utility from final wealth, W_2, where their preferences are represented by the utility function U. I assume that absolute risk aversion is decreasing with wealth, or equivalently, that its inverse, absolute risk tolerance is increasing:

$$\tau(W_2) \equiv -\frac{U'(W_2)}{U''(W_2)} \quad \text{is increasing with } W_2.$$

For convenience, I assume further that $\lim_{W_2 \to 0} \tau(W_2) = 0$ and $\lim_{W_2 \to \infty} \tau(W_2) = \infty$, but these limit conditions are not necessary to the results. Importantly, there is no assumption about relative risk aversion: it may be increasing, decreasing, or constant. For example, preferences could display constant relative risk aversion [under CRRA, $U(W_2) = \frac{W_2^{1-a}}{1-a}$ and $\tau(W_2) = \frac{W_2}{a}$].

In general, agents may differ in their risk aversion, initial endowments, and cost of information. Here, I assume the only source of heterogeneity across agents is their initial endowments in stocks and bonds. Let W_{0j} be agent j's total endowment in stocks and bonds (i.e., the number of stocks plus the number of bonds she initially owns) and let α_{0j} be the fraction of that endowment held in the form of stocks. From these definitions, it follows that the number of stocks and bonds initially owned are $\alpha_{0j}W_{0j}$
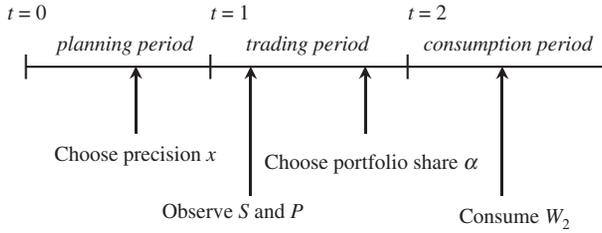
Figure 1
Timing

and $(1 - \alpha_{0j})W_{0j}$.[15] Let G be the cumulative joint distribution function of W_{0j} and α_{0j} on a compact set $[\mathcal{W}_0, \overline{\mathcal{W}_0}] \otimes [\alpha_0, \overline{\alpha_0}]$.

A measure of agents' *aggregate risk tolerance*, n, will help characterize the equilibrium. Let

$$n \equiv \int_j \tau(W_{0j}) dG(W_{0j}, \alpha_{0j}).$$

The choice variables of an agent are the precision of her private signal, x_j, and the fraction of wealth she allocates to the risky asset, α_j (i.e., the value of her stockholdings divided by the value of her endowment).

2.4 Timing
The timing is depicted in Figure 1. There are three periods. Period 0 is the planning period: the agent chooses how much information to acquire, if any [she chooses x_j and pays $C(x_j)z$]. The second period ($t = 1$) is the trading period. The investor observes her private S_j with the precision x_j she chose in the previous period. At the same time, markets open and she observes the equilibrium price. She uses the public and private signals to compute $E_j(\ln \Pi \mid \mathcal{F}_j)$ and $V_j(\ln \Pi \mid \mathcal{F}_j)$ and then chooses her portfolio share of stocks, α_j. In the third period ($t = 2$), the agent consumes the proceeds from her investments, W_{2j}.

3. Equilibrium Concept

3.1 Individual maximization
The investor's problem must be solved in two stages, working from the trading period to the planning period. In the trading period ($t = 1$), she observes P and S_j (where x_j, the precision of S_j, is inherited from the first

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[15] The exogenous variables W_0 and α_0 approximate at the order zero in z an agent's initial wealth and initial portfolio share, which are endogenous variables. Indeed, as shown in Theorem 1, the stock price is $P = \exp(pz) \approx 1 + pz$ in equilibrium.

period) and then forms her portfolio taking P, r^f and $C(x_j)$ as given:

$$\max_{\alpha_j} E_j[U(W_{2j})\,|\,\mathcal{F}_j] \quad \text{subject to} \begin{cases} W_{1j} = (P\alpha_{0j} + 1 - \alpha_{0j})\,W_{0j} \\ W_{2j} = W_{1j}(1 + r_j^p z) - C(x_j)z \\ r_j^p z = \alpha_j\left(\dfrac{\Pi - P}{P} - r^f z\right) + r^f z. \end{cases} \tag{2}$$

Note that agents may borrow at rate $r^f z$ and short stocks if they wish. W_{1j} is the investor's wealth in period 1, i.e., her endowed portfolio valued at the observed equilibrium price. $r_j^p z$ is the net return on investor j's portfolio (excluding the cost of information). Call $v(S_j, x_j, W_{1j}; P)$ the value function for this problem.

In the planning period ($t = 0$), the agent chooses the precision of her private signal in order to maximize her expected utility averaging over all the possible realizations of S_j and P and taking $C(\cdot)$ as given:

$$\max_{x_j \geq 0} E_j[v(S_j, x_j, W_{1j}; P)]. \tag{3}$$

3.2 Market aggregation
The gains from private information depend on how much gets revealed by the public signal P. Call i the aggregate precision or *informativeness* of the price implied by aggregating individual precision choice:

$$i \equiv \int_j x_j \tau_j\, dG(W_{0j}, \alpha_{0j}). \tag{4}$$

Equivalently its inverse is a measure of the noisiness of the price. Private precisions are weighed by risk tolerance because investors transmit their information through their demand for stocks, which is proportional to their risk tolerance. Individual decisions both depend on and determine the aggregate variable i. We are now ready for the formal definition of an equilibrium.

3.3 Definition of an equilibrium
A rational expectations equilibrium is given by two demand functions α_j and x_j, a price function P of Π and θ, and a scalar i such that

1. $x_j = x(W_{0j}, \alpha_{0j}; i)$ and $\alpha_j = \alpha(S_j, x_j, W_{0j}, \alpha_{0j}; P, i)$ solve the maximization problem of an investor taking P and i as given [Equations (2) and (3)].
2. P clears the market for the risky asset:

$$\int_j \alpha(S_j, x_j, W_{0j}, \alpha_{0j}; P, i)\frac{W_{1j}}{P}\, dG(W_{0j}, \alpha_{0j}) = \theta.$$

3. The informativeness of the price i implied by aggregating individual precision choices equals the level assumed in the investor's maximization problem:

$$i = \int_j x(W_{0j}, \alpha_{0j}; i)\tau(W_{0j})dG(W_{0j}, \alpha_{0j}).$$

4. Description of the Equilibrium

For clarity, I will break the presentation of the equilibrium into two parts, but the equilibrium is completely characterized by both parts. Theorem 1 describes the equilibrium in the trading period (i.e., gives the price and demand for stocks for a given level of aggregate information) and Theorem 2 describes the equilibrium in the planning period (i.e., the information acquisition decision). Theorem 3 characterizes the level of information and states the unicity of the equilibrium. Lemma 4 shows the implications for portfolio shares.

4.1 Existence and characterization of the equilibrium
Theorem 1 (price and demand for stocks). *Assume the scaling factor z is small. Assume information decision have been made (i.e., i and x_j are given). There exists a log-linear rational expectations equilibrium.*

*   *The equilibrium price is given by*

$$\ln P = pz, \quad \text{where } p + r^f = p_0(i) + p_\pi(i)(\pi - \mu\theta), \tag{5}$$

$$h_0(i) \equiv \frac{1}{\sigma_\pi^2} + \frac{i^2}{\sigma_\theta^2}, \quad h(i, x) \equiv h_0(i) + x, \quad \bar{h} \equiv h\left(i, \frac{i}{n}\right),$$

$$p_0 \equiv \frac{1}{\bar{h}}\left(\frac{E(\pi)}{\sigma_\pi^2} + \frac{iE(\theta)}{\sigma_\theta^2} + \frac{1}{2}\right), \quad p_\pi \equiv \left(1 - \frac{1}{\bar{h}\sigma_\pi^2}\right), \quad \text{and} \quad \mu \equiv \frac{1}{i}. \tag{6}$$

*   *The optimal portfolio share of stocks for an investor j with a signal of precision x_j (possibly equal to zero) is given by*

$$\alpha_j = \frac{\tau(W_{1j})}{W_{1j}} \frac{E_j(\pi z \mid \mathcal{F}_j) - (p + r^f)z + \frac{1}{2}V_j(\pi z \mid \mathcal{F}_j)}{V_j(\pi z \mid \mathcal{F}_j)}$$

$$= \frac{\tau(W_{1j})}{W_{1j}}\left(\frac{E(\pi)}{\sigma_\pi^2} + \frac{iE(\theta)}{\sigma_\theta^2} + \frac{i^2}{\sigma_\theta^2}(\pi - \mu\theta) + x_j\frac{S_j}{z} + \frac{1}{2}\right.$$

$$\left. - (p + r^f)h(i, x_j)\right). \tag{7}$$

The price function calls for a few remarks. First, the equilibrium price depends on the log-payoff π and the net supply of stocks θ. θ enters the

price equation, although it is independent of π because it determines the value of stocks to be held, and hence the total risk investors have to bear in equilibrium. π appears directly in the price function, though it is not known by any agent, because individual signals S_j are aggregated and collapse to their mean $\ln \Pi \equiv \pi z$.

Second, observing the price is equivalent to observing $\pi - \mu\theta$, which acts as a noisy signal for π with noise $-\mu\theta$. For given σ_θ^2, the parameter $\mu \equiv 1/i$ measures the noisiness of the price signal. The smaller the noise μ (the bigger i), the more informative the price. The function $h_0(i)$ is the precision of the public signal. Similarly the function $h(i, x)$ is the total precision of an investor's signal using both private and public signals (the precisions simply add up). i/n is a measure of the average private information, so $\bar{h}$ is the average total precision in the market.

Third, it is insightful to decompose the random part of the price, p, in two components: $p = [p_0 + \frac{i}{\sigma_\theta^2 \bar{h}}(\pi - \mu\theta) + \frac{i}{n\bar{h}}\pi] + [-\frac{\theta}{n\bar{h}}] - r^f$. The first term captures the signal extraction problem. It is a weighted average of the priors (contained in p_0) and of the public and private signals. The second term reflects the discount on the price demanded by risk-averse investors to compensate them for the risk in Π. The discount is increasing in the net supply of stocks θ, the market risk aversion $1/n$, and the amount of risk per stock, $1/\bar{h}$, the average investor has to bear in equilibrium. Two extreme cases are of interest. If μ is equal to zero ($i = \infty$), then there is no noise and the price reveals the true π. There is no risk in this economy and the price function, p, reduces to $(\pi - r^f)$ so that the two assets have the same net return, $r^f z$. On the other hand, if μ is infinite ($i = 0$), then the price contains no information about π. The price function p becomes $E(\pi) + \frac{1}{2}\sigma_\pi^2 - r^f - \sigma_\pi^2 \frac{\theta}{n}$. The price coefficient p_π is increasing in i, while p_0 might be increasing or decreasing in i depending on the range of parameters considered.

Finally, the fraction of her wealth an investor with a signal of precision x_j allocates to stocks can be written as $\alpha_j = \alpha_{x=0} + \frac{\tau(W_{1j})}{W_{1j}} x_j(\frac{S_j}{z} - p - r^f)$. In other words, her portfolio share equals the optimal share had she been uninformed, plus the stock's premium as predicted by her private signal, scaled by precision and relative risk aversion.

The proof of the theorem is presented in the appendix, so I only outline its key steps. First, guess that the price function p is linear in π and θ. Second, solve the portfolio problem for an investor who observes P and S_j (set x_j to zero if the investor did not acquire a signal). Because of the normality assumption, the signal extraction problem yields an estimate of the stock's payoff $E_j(\pi z \mid \mathcal{F}_j)$, which is linear in p and S_j, and a precision $h(i, x_j) = \frac{z}{V_j(\pi z \mid \mathcal{F}_j)}$, which neither depends on p nor S_j. In addition, approximating the Euler equation at the order 1 in z implies that the demand for stocks is simply $\alpha_j = \frac{\tau(W_{1j})}{W_{1j}} \frac{E(\pi z \mid \mathcal{F}_j) + \frac{1}{2}V_j(\pi z \mid \mathcal{F}_j) - (p + r^f)z}{V(\pi z \mid \mathcal{F}_j)}$, which in turn is

linear in p and S_j.[16] Third, when summing up individual demands for stocks, apply the law of large numbers for independent, but not identically distributed random variables, and the individual signals all collapse to their conditional mean, πz. Hence the value of aggregate demand is linear in π and p, and equating it to the supply θ will yield an equilibrium price linear in π and θ as guessed. The information decision will not affect the linearity of the price since it is made ex ante, that is, before P is observed (of course, it will affect the coefficients of the price equation through i). The next theorem describes the information choice.

Theorem 2 (demand for information). *Assume the scaling factor z is small.*

- *There exists a wealth threshold $W_0^*(i)$ such that only agents with initial wealth above $W_0^*(i)$ acquire information.*
- *Their optimal precision level, $x_j = x(W_{0j})$, is characterized by the first-order condition*

$$C'(x_j) = \tfrac{1}{2}\tau(W_{0j})\varphi'(x_j; i) \tag{8}$$

*and by the second-order condition*

$$C''(x_j) - \tfrac{1}{2}\tau(W_{0j})\varphi''(x_j; i) \geq 0, \tag{9}$$

where φ is an increasing and convex function of x as shown in the appendix.

Depending on the value of her initial endowment (but irrespective of how it is split between stocks and bonds), an agent will choose to acquire information or not. In fact, information will only pay off for agents who are wealthy enough. The wealth threshold W_0^* (derived in Appendix B) is defined as the level of wealth that makes an investor indifferent between acquiring information and remaining uninformed. The value of W_0^* determines whether all investors are informed, whether none are, or whether informed and uninformed investors coexist in equilibrium.

The precision informed agents choose is then given by Equation (8). The function φ is defined in equation Equation (13) in Appendix B. φ measures the squared Sharpe ratio an investor expects in the planning period given that she will receive some information in the trading period. For an informed investor, φ is an increasing and convex function of x. Its derivative, φ', is increasing and concave. Equation (8) is illustrated by

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[16] The approximation does not amount to assuming quadratic preferences since the expansion is done around different wealth levels. Instead, preferences are modeled as an envelope of quadratic functions. An alternative specification of the model is to posit up front that the demand for stocks is given by this equation.
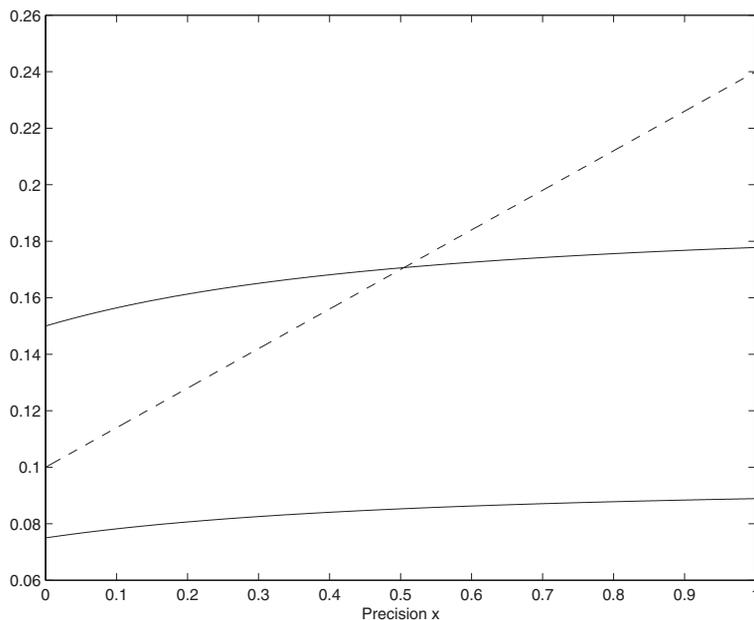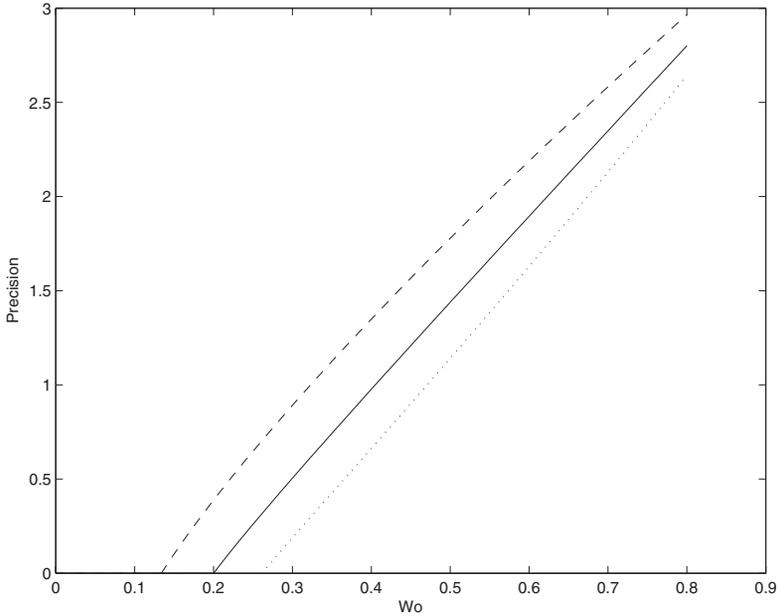
Figure 2
The optimal precision choice
$C'(x)$ (dashed curve) and $\frac{1}{2}\tau(W_0)\varphi'(x)$ (top solid curve for a rich investor and bottom solid curve for a poor investor). Rich investors acquire information of precision at the intersection of $C'(x)$ and $\frac{1}{2}\tau(W_0)\varphi'(x)$. Poor investors do not acquire information. The picture is drawn for $C(x) = 0.07x^2 + 0.01x$, $\tau(W_0) = W_0$, where W_0 equals 0.3 for the rich investor and 0.15 for the poor investor, $E(\pi) = 1$, $\sigma_\pi^2 = 1$, $E(\theta) = 0.01$, $\sigma_\theta^2 = 0.02$, $n = 1$, $\mu = 100$, and $A = 1.25$.

Figure 2 and states that, at the optimum, the gain from a small increase in precision is exactly offset by its extra cost. It shows that the optimal precision level is increasing with absolute risk tolerance, which by assumption is increasing with wealth. Thus wealthier investors acquire more information. The reason is that investors with greater absolute risk tolerance purchase a larger number of stocks and hence find information more valuable. Putting it differently, there are increasing returns to information: the cost of achieving a given precision is independent of the scale of the investment (i.e., of the amount invested), whereas its benefit is increasing with the scale. Note that this increasing returns to scale property is obtained in spite of a strictly convex cost function. Figure 3 depicts the wealth-precision relationship.

The other properties of the optimal precision, x_j, are the following. First, x_j is finite so no investor has an arbitrage opportunity. Second, x_j is decreasing in the marginal cost of information and in risk aversion (a less risk-averse investor will buy more stocks and hence will find information more valuable). Third, x_j is decreasing in the informativeness of the price, i (greater informativeness implies that prices are more revealing and

Figure 3
The optimal precision for different levels of wealth under constant (solid curve) decreasing (dotted curve), and increasing (dashed curve) relative risk aversion
For wealth levels below W_0^*, no information is acquired. The picture is drawn for $C(x) = 0.07x^2 + 0.01x$, $\tau(W_0) = W_0^b$, where $b = 0.8$, 1, and 1.2, $E(\pi) = 1$, $\sigma_\pi^2 = 1$, $E(\theta) = 0.01$, $\sigma_\theta^2 = 0.02$, $n = 1$, and $\mu = 100$.

consequently decreases the incentives to acquire private information).[17] These results correspond to Lemma 2 and Corollaries 1 to 4 in Verrecchia (1982) in the case of CARA preferences. The next theorem characterizes the level of information in equilibrium.

Theorem 3 (equilibrium level of information and unicity). *Assume the scaling factor z is small.*

- *In equilibrium, price informativeness i solves*

$$i = \int_{W_0^*(i)}^{\overline{W_0}} x(W_{0j}; i)\tau(W_{0j})dG(W_{0j}, \alpha_{0j}). \tag{10}$$

- *Assume $\sigma_\pi^2 \leq 2$. There exist a unique log-linear equilibrium.*

Equation (10) characterizes the aggregate level of private information in equilibrium, *i*. It follows directly from the definition that was given in

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[17] Strictly speaking, this statement requires that $\sigma_\pi^2 \leq 2$. The problem is that investors care about the expected instantaneous return which involves the payoff Π, whereas information is about the logarithm of the payoff $\ln \Pi \equiv \pi z$. For that reason, the expected return carries a volatility term that complicates the derivations. The upper bound on σ_π^2 ensures that this term does not become too big.

Equation (4). I have only managed to prove that Equation (10) admits a unique solution under the assumption that $\sigma_\pi^2 \leq 2^{18}$. In this case, the equilibrium is unique within the class of log-linear equilibria. The next section puts the results from Theorems 1 and 2 together to study the effect of wealth on portfolio decisions.

4.2 Wealth and portfolio shares

Let ε_τ be the elasticity of absolute risk tolerance with respect to wealth, ε_c the elasticity of marginal cost with respect to precision, and ε_α the elasticity of portfolio share with respect to wealth:

$$\varepsilon_\tau \equiv \frac{W_0 \tau'(W_0)}{\tau(W_0)}, \quad \varepsilon_c \equiv \frac{x C''(x)}{C'(x)}, \quad \text{and} \quad \varepsilon_\alpha \equiv \frac{W_0 \alpha'(W_0)}{\alpha(W_0)}.$$

By assumption $\varepsilon_c > 0$ and $\varepsilon_\tau \geq 0$. In the definition of ε_α, α is the unconditional portfolio share, that is, the share of her wealth W_0, an investor allocates to stocks, averaging over the possible realizations of all the random variables π, θ, and ε_j. (Alternatively, one could average over the idiosyncratic shocks ε_j only and consider the shares conditional on the economy-wide shocks π and θ.)

Lemma 4 (wealth and portfolio shares).

- *For an uninformed investor,*

$$\varepsilon_\alpha = \varepsilon_\tau - 1$$

- *For a well-informed investor (i.e., an investor with large precision x_j),*

$$\varepsilon_\alpha \approx \varepsilon_\tau - 1 + \frac{\varepsilon_\tau}{\varepsilon_c}.$$

- *Under CRRA preferences, for any informed investor,*

$$\varepsilon_\alpha = \frac{1}{\varepsilon_c}.$$

The lemma shows that the pattern of shares increasing with wealth ($\varepsilon_\alpha > 0$) may hold even if relative risk aversion is not decreasing. This is the case under CRRA ($\varepsilon_\tau = 1$), regardless of the cost function, and under increasing relative risk aversion ($\varepsilon_\tau < 1$), provided the cost function is not too convex ($\varepsilon_c < (\frac{1}{\varepsilon_\tau} - 1)^{-1}$). Figure 4 illustrates the lemma.

The mechanism through which information acquisition operates on the demand for stocks is again the following: under decreasing absolute risk

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[18] See the previous footnote.
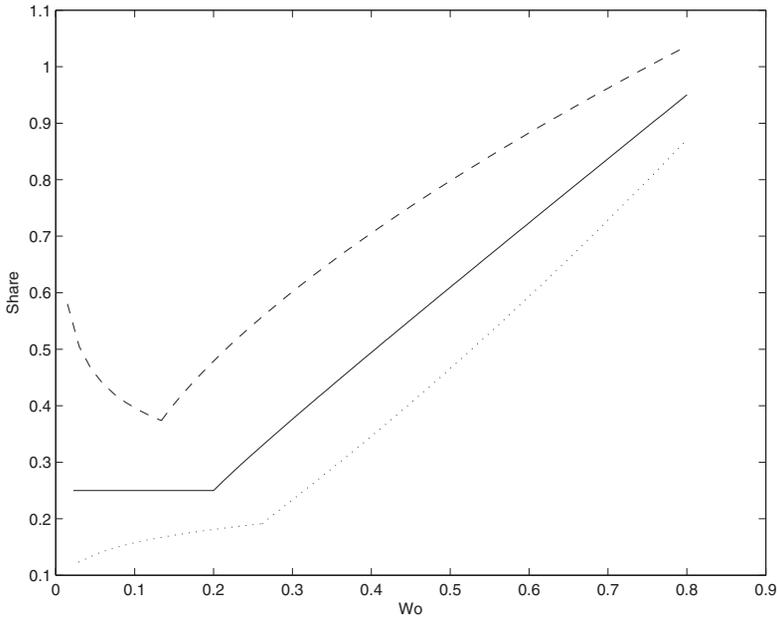
Figure 4
Portfolio share of stocks for different levels of initial wealth under constant (solid curve), decreasing (dotted curve), and increasing (dashed curve) relative risk aversion
Only investors with wealth above W_0^* acquire information. The picture is drawn for $C(x) = 0.07x^2 + 0.01x$, $\tau(W_0) = W_0^b$, where $b = 0.8$, 1, and 1.2, $E(\pi) = 1$, $\sigma_\pi^2 = 1$, $E(\theta) = 0.01$, $\sigma_\theta^2 = 0.02$, $n = 1$ and $\mu = 100$.

aversion, wealthier investors purchase more stocks for a given precision level [Equation (7)]. Having a riskier portfolio makes information more valuable for these investors, so they acquire more private information. Finally, a higher precision induces investors to hold even more stocks. Thus wealth has a double effect on the demand for stocks: a traditional direct effect and an indirect effect through the demand for information. Under decreasing relative risk aversion, both effects work in the same direction, making portfolio shares increasing with wealth. Under increasing relative risk aversion, the direct effect is reversed so the net effect is ambiguous. It depends on the shape of absolute risk aversion relative to that of the cost function. If C is not too convex, then a small increase in wealth will lead to a large increase in private information that will over-turn the increase in relative risk aversion.

The following example illustrates the theorem. Let $C(x) = x^c$ for $c > 1$. Differentiating C yields $\varepsilon_c = c - 1$. Such a function can reconcile the observed pattern of shares with *any* increasing relative risk aversion utility: it suffices to choose $c < \frac{1}{1-\varepsilon_\tau}$. For example, if $\varepsilon_\tau \geq \frac{1}{2}$ and the cost function is quadratic, then portfolio shares increase with wealth in spite of

increasing relative risk aversion. Conversely, suppose preferences are CRRA and all investors are informed, then the observed share elasticity of 0.1 implies a cost elasticity of 10 and hence a cost function in x^{11}. The next section studies the connection between the stock market and wealth inequalities.

5. The Stock Market and Wealth Inequality

In Section 3, I showed that wealthier households acquire more information and hence that the demand for stocks is a convex function of wealth. This means that if a dollar is transferred from a poor to a rich investor, the demand for stocks of the rich will increase by more than the demand of the poor will fall, resulting in a rise in aggregate demand. Consequently the price will increase. In short, the more unequal the distribution of wealth (keeping the average wealth constant), the smaller the equity premium. Interestingly, this is the case regardless of relative risk aversion.[19] The provision of information through prices also increases with wealth inequality (again keeping the average wealth constant). In Section 6, the model is calibrated to U.S. data and the effects on the equity premium are shown to be quantitatively significant for plausible parameter values.

So far I have studied the effect of wealth inequality on the stock price, but I can also look at the reverse causality, that is, at the link from stocks to wealth inequality. It is a well-known fact that wealth is unevenly distributed. In the United States, for example, the top decile of all households own 82.9% of all financial wealth in the nation [Wolff (1998)]. While several factors may explain these differences [see Quadrini and Rios-Rull (1997) for a review], the model focuses on the role played by the availability of costly information about assets. The model shows how information generates increasing returns which magnify wealth inequality: wealthier agents acquire more information and more stocks and achieve a higher expected return, a higher variance, and a higher Sharpe ratio on their portfolio. It follows that the distribution of final wealth as measured by expected wealth or by certainty equivalent is more unequal than the distribution of initial wealth. Arrow (1987) makes this point, albeit in a partial equilibrium setting.

Formally, recall that $r_j^p z$ is the net return on investor j's portfolio (before accounting for the information cost) and let $r_j^{pe} z \equiv r_j^p z - r^f z$ be the associated excess return. Using Equation (7) and integrating over all

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[19] In contrast, in a standard frictionless symmetric information economy with CRRA preferences, the distribution of wealth has no implication on the equity premium. This is no longer the case under different assumptions on preferences [e.g., Gollier (2001)] or if frictions such as entry costs or market incompleteness [e.g., Constantinides and Duffie (1996), Heaton and Lucas (1996)] are introduced.
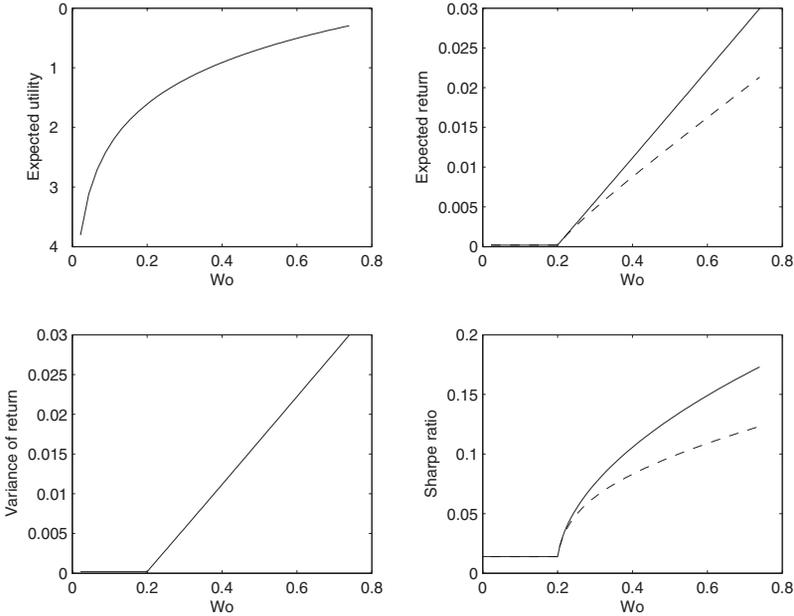
Figure 5
The relation between initial wealth and expected utility (top left panel), expected return (top right panel), variance of return (bottom left panel), and Sharpe ratio (bottom right panel)
In the right panels, solid curves do not include the cost of information, dashed curves do. Rich investors acquire information while poor investors do not. The graphs are drawn for log utility, $C(x) = 0.07x^2 + 0.01x$, $E(\pi) = 1$, $\sigma_\pi^2 = 1$, $E(\theta) = 0.01$, $\sigma_\theta^2 = 0.02$, $n = 1$, $\mu = 100$, $r^f = 0.03$, $z = 0.01$, and $\alpha_0 = 0$.

the random variables yields,

$$E(r_j^{pe}z) = \frac{\tau(W_{0j})}{W_{0j}}\varphi(x_j)z, \quad V(r_j^{pe}z) = \left(\frac{\tau(W_{0j})}{W_{0j}}\right)^2 \varphi(x_j)z, \quad \text{and}$$

$$\frac{E(r_j^{pe}z)}{\sqrt{V(r_j^{pe}z)}} = \sqrt{\varphi(x_j)z}. \tag{11}$$

Recall that investor j's private precision x_j is increasing in her wealth W_{0j} and that the function φ is increasing in x_j for an informed investor. These results are illustrated by Figure 5. The next section addresses some empirical issues raised by the model.

6. Empirical Issues

In this section I calibrate the model. Then I show how to discriminate among different models of portfolio choice.

6.1 Calibration

The model shows that the ability to acquire information explains, both qualitatively and quantitatively, why richer households invest a larger fraction of their wealth in risky assets. A consequence, pointed out in the previous section, is that the distribution of wealth has an impact on the moments of asset returns. To assess whether the effects on returns are quantitatively important for plausible parameter values, I calibrate the model to U.S. stock market data. Starting from a benchmark economy where no information is acquired, I increase wealth inequalities and examine the consequence on the equity premium and its variance.

I begin by describing the benchmark economy. In this economy, no household collects information (the wealth is below the threshold W_0^*). I assume that they have CRRA preferences (with a baseline coefficient of relative risk aversion $a = 5$) so that the distribution of wealth has no effect on asset returns. In 1995, 69.3 million households owned equity in the United States. On average, they had \$74,810 in financial wealth with 55% invested in stocks.[20] It follows that the aggregate level of financial wealth was \$5,184 billion and that aggregate risk tolerance n was \$1,037 billion (for $a = 5$).

I now turn to the assets in the economy. The riskless interest rate is set to 3% per year (the scaling factor z is set to 1). The parameters of the distributions of π and θ are chosen so that, in the benchmark economy, the equity premium, variance, and portfolio shares match their historical values. Over the 1889–1978 period, the average annual equity premium was 6.18%, its standard deviation was 18%, and its variance was 3.24%. In the benchmark economy, the average portfolio share invested in stocks is $E(\alpha) = \frac{q + \frac{1}{2}\sigma_\pi^2}{a\sigma_\pi^2}$, where $q \equiv E(\ln\frac{\Pi}{P}) - r^f z \equiv E(\pi) - E(p) - r^f = (\frac{E(\theta)}{n} - \frac{1}{2})\sigma_\pi^2$ is the equity premium. Therefore $\sigma_\pi^2 = \frac{q}{aE(\alpha) - \frac{1}{2}} = 0.0275$ and $\frac{E(\theta)}{n} = aE(\alpha) = 2.750$. The variance of the equity premium is $v \equiv \text{var}(\ln\frac{\Pi}{P}) - r^f z \equiv \text{var}(\pi - p) = \sigma_\pi^2(\frac{\sigma_\pi^2\sigma_\theta^2}{n^2} + 1)$, implying that $\frac{\sigma_\theta^2}{n^2} = \frac{1}{\sigma_\pi^2}(\frac{v}{\sigma_\pi^2} - 1) = 6.539$. Note that positive σ_θ^2 imposes a lower bound on a: $a > \frac{1}{E(\alpha)}(\frac{q}{v} + \frac{1}{2}) = 4.38$. $E(\pi)$ is irrelevant and normalized to one.

Next, I describe the distribution of wealth. I assume wealth is evenly distributed among households in the benchmark economy (under CRRA preferences and no information acquisition, the distribution of wealth is irrelevant). The goal here is to analyze the economy when wealth becomes unequally distributed. For simplicity, I assume the distribution of wealth is bimodal: the economy is populated by two groups of agents, the rich

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[20] The number of households holding equity is reported by Poterba (1998). Average financial wealth and portfolio shares are based on the 1994 Panel Study of Income Dynamics (PSID) and are measured conditional on having positive financial wealth and positive stockholdings, respectively [Vissing-Jørgensen (2002)]. Wolff (1998) reports a larger number for financial wealth using the 1995 Survey of Consumer Finances (SCF), but his measure includes business equity.

and the poor. The rich, in proportion M, have wealth W_{rich} while the poor, in proportion $1 - M$, have wealth W_{poor}. I simulate the economy for different combinations of the fraction of rich, M, and the fraction of aggregate wealth they own. Importantly, M, W_{rich}, and W_{poor} are varied in such a way that aggregate wealth remains constant, so as to capture only the effects of inequality. In practice, financial wealth is very unevenly distributed. For example, Wolff (1998) reports that the top decile of U.S. households owned 82.9% of all financial wealth in 1995. However, this figure overestimates the relevant number for this calibration because Wolff conditions neither on positive wealth nor on positive stockholdings and includes business equity in his definition of financial wealth (which is mostly concentrated in the hands of the very wealthy).

Finally, I specify the information acquisition technology. I assume $C(x) = x^c + dx$. A large enough d ensures that no information is acquired in the benchmark economy[21] and that only the rich acquire information in the unequal economies, that is, $W_{rich} > W_0^* > W_{poor}$. The parameter c is chosen to match the average share elasticity in the United States. Because the share elasticity equals zero for uninformed investors and $\frac{1}{c-1}$ for (well) informed investors (recall that preferences are CRRA), the average elasticity is $\frac{M}{c-1}$. For example, when $M = 0.2$, an average share elasticity of 0.1 (see Section 1) implies $c = 3$.

I simulate the economy for a relative risk aversion of 5 and 7, an average share elasticity of 0.05 and 0.1, and wealth distributions such that the top 5%, 10%, and 20% of the population own 25%, 50%, or 75% of aggregate wealth. The results are reported in Table 1. As expected, the equity premium and its variance are lower in unequal economies where information is collected. The more unequal the economy, the greater the effect on returns. This can be seen either by move along the lines (e.g., 10% of the population owns 25%, 50%, and 75% of aggregate wealth) or up the columns (e.g., 50% of aggregate wealth is owned by 20%, 10%, or 5% of the population). The effects are quantitatively important, especially close to the benchmark economy. For example, with a relative risk aversion of 5 and an average share elasticity of 0.1, the equity premium decreases by 43% (from 6.18% to 3.53%) and its variance by 39% (from 3.24% to 1.99%) relative to the benchmark economy when 10% of the investors own 25% of financial wealth. Furthermore, the effect of inequalities is enhanced by lower risk aversion and higher share elasticity. Indeed, less risk-averse investors acquire more information because they purchase more shares [Equation (8)]. A greater share elasticity implies a lower coefficient c in the information cost function and therefore that the optimal precision is more sensitive to differences in wealth.

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[21] Formally, $W_0^*(i=0) > 74,810$, which implies that $d > \frac{74,810(\sigma_\epsilon^2)^2}{2a}\left(\frac{1}{\sigma_\epsilon^2} + \frac{E(\theta^2)}{n^2} - \frac{1}{4}\right) = 220$ when $a = 5$. The value of d, beyond this threshold, has little impact on the results reported in Table 1.

Table 1
The equity premium and its variance in unequal economies when relative risk aversion equals 5 and 7, the average share elasticity equals 0.05 and 0.10, the fraction of rich is 5%, 10% and 20%, and they own 25%, 50%, and 75% of aggregate wealth (see text for other parameter values).

| % of aggregate wealth owned by the rich		0	25	50	75
Relative risk aversion = 5					
Average share elasticity = 0.05					
% of rich = 5%	Equity premium	6.18%	2.14%	1.08%	0.72%
	Variance	3.24%	1.18%	0.57%	0.37%
% of rich = 10%	Equity premium	6.18%	5.62%	4.30%	3.21%
	Variance	3.24%	3.07%	2.42%	1.80%
% of rich = 20%	Equity premium	6.18%	6.11%	5.89%	5.64%
	Variance	3.24%	3.23%	3.17%	3.07%
Average share elasticity = 0.10					
% of rich = 5%	Equity premium	6.18%	1.58%	0.83%	0.57%
	Variance	3.24%	0.85%	0.43%	0.29%
% of rich = 10%	Equity premium	6.18%	3.53%	1.87%	1.27%
	Variance	3.24%	1.99%	1.02%	0.67%
% of rich = 20%	Equity premium	6.18%	6.05%	5.06%	4.05%
	Variance	3.24%	3.21%	2.82%	2.28%
Relative risk aversion = 7					
Average share elasticity = 0.05					
% of rich = 5%	Equity premium	6.18%	3.20%	1.77%	1.21%
	Variance	3.24%	1.60%	0.80%	0.52%
% of rich = 10%	Equity premium	6.18%	5.92%	5.35%	4.73%
	Variance	3.24%	3.12%	2.83%	2.48%
% of rich = 20%	Equity premium	6.18%	6.14%	6.03%	5.91%
	Variance	3.24%	3.22%	3.17%	3.11%
Average share elasticity = 0.10					
% of rich = 5%	Equity premium	6.18%	1.89%	1.05%	0.73%
	Variance	3.24%	0.87%	0.44%	0.29%
% of rich = 10%	Equity premium	6.18%	4.48%	2.68%	1.89%
	Variance	3.24%	2.34%	1.30%	0.87%
% of rich = 20%	Equity premium	6.18%	6.11%	5.68%	5.20%
	Variance	3.24%	3.21%	3.00%	2.74%

6.2 Information acquisition versus risk aversion

In this section I show how to distinguish empirically between two models of portfolio choice, the information model and the decreasing relative risk aversion model mentioned in the introduction. In a symmetric information economy, where agents do not acquire information but differ in their relative risk aversion, the expected excess return, variance, and Sharpe ratio on investor j's portfolio in period 0 are

$$E(r_j^{pe}z) = \frac{\tau(W_{0j})}{W_{0j}} \frac{E[(E(\pi) - p - r^f + \frac{1}{2}\sigma_\pi^2)^2]}{\sigma_\pi^2} z,$$

$$V(r_j^{pe}z) = \left(\frac{\tau(W_{0j})}{W_{0j}}\right)^2 \frac{E[(E(\pi) - p - r^f + \frac{1}{2}\sigma_\pi^2)^2]}{\sigma_\pi^2} z, \quad \text{and}$$

$$\frac{E(r_j^{pe}z)}{\sqrt{V(r_j^{pe}z)}} = \frac{\sqrt{E[(E(\pi) - p - r^f + \frac{1}{2}\sigma_\pi^2)^2]}}{\sigma_\pi} \sqrt{z}.$$

Clearly the Sharpe ratio is independent of risk aversion and consequently of wealth. This observation suggests a simple way of testing the information model against the risk aversion model. Indeed, we saw in Section 5 that, in the information model, the Sharpe ratio of an investor's portfolio increases with her wealth as long as absolute risk aversion is decreasing [Equation (11)]. These differences are illustrated in Figure 6, where the optimal portfolio is displayed for a poor and a rich investor in both models. Each investor chooses the portfolio at the tangency of her highest utility curve with her efficient frontier. In the information model, the investors have the same utility functions but face different efficient frontiers (the slope of the frontier is the Sharpe ratio), whereas in the risk aversion model, they have different utility functions (it is determined by risk aversion) but the same efficient frontier. These results are summarized in Table 2.

Yitzhaki (1987) goes part of the way in testing these models. He uses a sample of 58,000 federal income tax returns that reported stock capital gains in the years 1962 and 1973 to examine the relation between capital gains and income. He divides his sample into 5 income groups and divides transactions into 11 holding periods. For each combination of income class and holding period, he computes returns on stocks. He finds that the holding period does not vary, but that returns and their standard
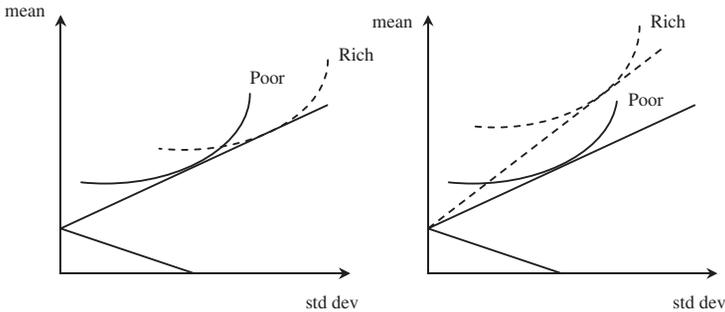
Figure 6
Efficient frontier and investors' portfolio choice in the risk aversion model (left panel) and the information model (right panel)
The figure displays utility curves and mean-variance frontiers for two investors with different levels of initial wealth.

Table 2
Portfolio returns in the information acquisition and decreasing risk aversion models.
The table shows how an increase in the wealth of an investor affects the mean return, the standard deviation of returns and the Sharpe ratio on the portfolio.

	Information acquisition	Decreasing relative risk aversion
Mean return	↗	↗
Standard deviation of return	↗	↗
Sharpe ratio	↗	Constant

deviation increase with income. Unfortunately Yitzhaki does not examine the relation between Sharpe ratio and income. In a recent study, Massa and Simonov (2003) combine several data sets to create a comprehensive sample of Swedish households that include, in particular, information on wealth (financial and non financial), stockholdings (direct and indirect), and capital gains and losses. Their data cover the owners of 98% of the market capitalization of publicly traded Swedish companies over the 1995–1999 period (about 300,000 households for each year). They split their sample into two groups, the wealthy (the top decile in terms of total wealth) and the low-wealth (all others), and then rank the wealthy by financial wealth. They compute the Sharpe ratios over a one-year horizon for the different groups and find that they are larger for the wealthy than for the low-wealth households and that, among the wealthy, they are larger for households with high levels of financial wealth. Therefore the current evidence supports the information model, but more research is needed to confirm these results.[22] Next, I consider briefly an alternative explanation for the relation between wealth and portfolio shares, an explanation based on fixed entry costs.

6.3 Fixed entry costs

To rationalize the pattern of portfolio shares, one can appeal to an entry cost, that is, a fixed cost investors have to pay to be allowed to trade stocks. This assumption is often made to explain why so many households do not hold any stocks. As an alternative explanation for portfolio shares, this cost must be distinct from the cost of acquiring information, or else one would fall back to the model presented here. Furthermore, for this explanation to work, the cost must be unrelated to financial wealth and hence to any variable linked to wealth such as trade size. This rules out all proportional costs such as proportional broker commissions and mutual fund sales loads, bid-ask spreads, and price impact. Therefore, what the entry cost is left summarizing are fixed fees, the time spent setting up and maintaining an account and the time and money spent filing tax forms for dividend income and capital gains.

Such costs do not get close to matching the observed share elasticity of 0.1. This can be seen easily in the setup of this article by assuming CRRA utility (with relative risk aversion a) and by dropping information acquisition. Let F be the entry cost. In this economy, the demand for stocks by a stockholder is proportional to her net

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[22] Barber and Odean (2000) report different results. They find no significant difference in gross and net returns, risks, and risk-adjusted returns across portfolios. However, their data are subject to important limitations. First, they analyze a particular class of households, namely households that hold their investments at a discount brokerage rather than a retail brokerage firm. Discount brokers charge lower fees but do not provide customers with advice, so the sample may be subject to a selection bias. Second, they do not observe the entire portfolio of households who own stocks at other brokers or mutual funds (for this reason, they do not report the Sharpe ratio on these portfolios).

wealth: $PX_j = (W_{0j} - F)\frac{E(\pi) - p - r^f + \frac{1}{2}\sigma_\pi^2}{a\sigma_\pi^2}$. It follows that her portfolio share, that is, the fraction of her gross wealth invested in stocks, is $\alpha_j = \frac{W_{0j} - F}{W_{0j}} \frac{E(\pi) - p - r^f + \frac{1}{2}\sigma_\pi^2}{a\sigma_\pi^2}$ (what is measured empirically is wealth before costs). The share elasticity with respect to gross wealth is $\varepsilon_\alpha = (\frac{W_0}{F} - 1)^{-1} \approx \frac{F}{W_0}$. In words, agents choose a dollar amount of stocks proportional to their net wealth which, as a percentage of gross wealth, increases with wealth. To match the empirical estimates mentioned above, one would need a ratio of entry cost to financial wealth on the order of 0.1 per period, an unrealistic number.[23] Therefore, though the fixed entry cost assumption can explain the pattern of shares qualitatively, it fails quantitatively. In the next section I examine whether psychological biases succeed.

6.4 Psychological biases

Departing from the rational paradigm, one may appeal to investor psychology to explain portfolio decisions. Indeed, behavioral scientists have pointed out a number of biases that affect agents' beliefs and preferences. For example, loss aversion, optimism, overconfidence, familiarity, narrow framing, and mental accounting have all been shown to influence investment decisions [for detailed surveys, see Hirshleifer (2001) and Barberis and Thaler (2002)].

Importantly, for a psychological bias to explain why wealthier households invest a larger fraction of their wealth in stocks requires that it varies systematically with wealth. The behavioral literature does not report such evidence. Yet one could argue, perhaps from casual observation, that some biases vary with wealth. For example, wealthier people may be more overconfident (possibly the result of a self-attribution bias; i.e., the tendency to attribute one's successes to one's own ability) and may overestimate the return-risk ratio. Alternatively, they may apply naive diversification rules such as the "$1/n$ heuristic" to a larger set of stocks or stock funds. Otherwise they may simply be more familiar with the stock market (this may happen if they interact closely with other wealthy people, who are themselves stockholders). Any of these assumptions implies that portfolio shares rise with wealth.

Nevertheless, a fully successful bias should also predict that wealthier investors achieve greater risk-adjusted returns, as the evidence suggests [Massa and Simonov (2003)]. Overconfidence leads to lower risk-adjusted returns [Barber and Odean (2001)]. The "$1/n$ heuristic" generates portfolios that are not on the efficient frontier [Benartzi and Thaler (2001)]. As for familiarity, it is not clear how it can help achieve greater risk-adjusted returns without the help of better

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[23] To make matters worse, the dramatic decrease in costs due to the 1975 deregulation and the intense competition that followed should imply a large increase in shares, for which there is no evidence.

information.[24] Overall, further research is needed to discover which psychological biases, if any, can explain the observed pattern of wealth, portfolio share, and performance.

7. Conclusion

The goal of this article is to explain differences in households' portfolios by differences in private information that derive from differences in wealth. In a rational expectations equilibrium, the price partially reveals private signals and hence dampens the incentives to spend resources on information. The article assumes that absolute risk aversion is decreasing with wealth, but does not take a stand on relative risk aversion. From this assumption and the availability of costly information (and without relying on any form of increasing returns to scale in preferences or technology), the model shows that the demand for information increases with wealth. It follows that the share of their wealth agents invest in stocks increases with wealth. This result matches and rationalizes the data without appealing to decreasing relative risk aversion, a hypothesis that does not seem to hold empirically. Also, from a more technical standpoint, the article solves (with an approximate closed form) a Grossman-Stiglitz economy with general preferences instead of the usual CARA, thus allowing for wealth to matter.

In addition, I show that the availability of costly information about the stock's payoff exacerbates wealth inequality: because there are increasing returns to information, wealthier agents acquire more information, more stocks, and achieve a higher Sharpe ratio on their portfolio. Finally, I study how the information and the decreasing risk aversion models of portfolio choice have different implications for the relation between initial wealth and mean return, standard deviation of return, and Sharpe ratio that can be exploited to tell them apart empirically.

While the current model is essentially static, its emphasis being on the cross section of stock ownership, it would be interesting to extend it to a dynamic setup. This is difficult because one has to keep track of the changes in the distribution of wealth (an object of infinite dimension) and solve for the hedging demands they induce. Another interesting, yet less complex direction for future research is, to extend the model to a multiasset environment and study the distribution of different *types* of stocks across households. Suppose that one can acquire information about individual stocks and that stocks are associated with different information technologies, some cheap and some expensive. In this setup, one can study the distribution of directly versus indirectly held equity (i.e., individual stocks versus mutual funds), large versus small capitalization,

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[24] Reversing the argument, Coval and Moskowitz (2001) document that U.S. mutual fund managers tend to hold local stocks and that these stocks subsequently outperform. They conclude that managers are informed rather than biased to familiar stocks.

foreign versus domestic, or existing versus newly issued stocks. In a multi-stock model, investors acquire more information about large firms than they do about small ones because they account for a larger fraction of their wealth. Empirically the production of private information increases with firm size. For example, Atiase (1985), Freeman (1987) and Collins, Kothari, and Rayburn (1987) show that stock prices of large firms antici-pate accounting earnings announcements earlier than that of small firms. Similarly, Bhushan (1989) shows that the number of financial analysts following a firm, a proxy for total resources spent on private information acquisition, increases with firm size. Furthermore, recent studies show that correlations among individual stocks are smaller in richer countries than in poorer ones, using both cross-country data [Morck, Yeung, and Yu (2000)] and U.S. time-series data [Campbell et al. (2001)]. An inter-pretation in line with the multiasset extension of the model is that there is more information collection about individual stocks as economies develop and investors become wealthier.

Appendix A: Proof of Theorem 1 (Price and Demand for Stocks)

To prove Theorem 1, guess that the equilibrium price is given by Equations (5) to (6) and solve for the optimal portfolio of a stockholder (recall that the information choice is taken as given at this stage). The first step in the investor's problem is to estimate the mean and vari-ance of the stock's payoff using the equilibrium price (or equivalently $\xi \equiv \pi - \mu\theta$) and her private signal S_j. The results of this gaussian signal extraction problem are summarized below.

A.1 Signal extraction

For the price function given in Equation (5), the formulas for the conditional mean and variance of $\ln \Pi$ are for agent j:

$$V_j(\ln \Pi \,|\, \mathcal{F}_j) = V_j(\pi z \,|\, \mathcal{F}_j) = \frac{z}{h_j}$$

$$\text{and} \quad E_j(\ln \Pi \,|\, \mathcal{F}_j) = a_{0j}z + a_{\xi j}\xi z + a_{S_j}S_j,$$

where

$$a_{0j}h_j \equiv \frac{E(\pi)}{\sigma_\pi^2} + \frac{iE(\theta)}{\sigma_\theta^2}, \quad a_{\xi j}h_j \equiv \frac{i^2}{\sigma_\theta^2}, \quad \text{and} \quad a_{S_j}h_j \equiv x_j.$$

Intuitively, $V_j(\ln \Pi \,|\, \mathcal{F}_j)$ decreases as the precision of the private signal, x_j, or the precision of public signals, i, increase. Similarly $E_j(\ln \Pi \,|\, \mathcal{F}_j)$ is a weighted average of priors, public and private signals, where the weight on the private signal (on the public signal) is increasing in x_j in i. Note that if the investor does not acquire any private information, $x_j = 0$ and S_j vanishes from the equations. In this setup, as in He and Wang (1995), the infinite regress problem does not arise, because that higher-order expectations, i.e., expectations about the expectations of others, can be reduced to first-order expectations, i.e., to expectations about the payoff conditional on private and public information.

A.2 Portfolio choice

To compute the optimal portfolio, maximize the expected utility from final wealth with respect to α. This leads to the usual Euler equation: $E_j[U'(W_{2j})(\frac{\Pi - P}{P} - r^f z) \,|\, \mathcal{F}_j] = 0$. Expand

$U'(W_{2j})$ around W_{0j}: $U'(W_{2j}) \approx U'(W_{0j}) + U''(W_{0j})\Delta W$, where $\Delta W \equiv W_{2j} - W_{0j} \approx r^f W_{0j} z + p W_{0j}\alpha_{0j}z + \alpha_j W_{0j}(\frac{\Pi - P}{P} - r^f z) - C(x_j)z$. In all the approximations that follow, the terms in z^2 and higher orders are neglected. Since $\frac{\Pi - P}{P} - r^f z$ is of order z at least, plugging back into the Taylor expansion for the Euler expansion leads to

$$U'(W_{0j})E_j\left[\frac{\Pi - P}{P} - r^f z \,\middle|\, \mathcal{F}_j\right] + U''(W_{0j})\alpha_j W_{0j}E_j\left[\left(\frac{\Pi - P}{P} - r^f z\right)^2 \,\middle|\, \mathcal{F}_j\right] \approx 0.$$

Furthermore, $E_j[\frac{\Pi - P}{P} - r^f z | \mathcal{F}_j] \approx E_j(\pi z | \mathcal{F}_j) + \frac{1}{2}V_j(\pi z | \mathcal{F}_j) - pz - r^f z$ and $E_j[(\frac{\Pi - P}{P} - r^f z)^2 |$ $\mathcal{F}_j] \approx V_j(\pi z | \mathcal{F}_j)$. Solving for the optimal share yields $\alpha_j = \frac{U'(W_{0j})}{-W_{0j}U''(W_{0j})} \frac{E_j(\pi z | \mathcal{F}_j) + \frac{1}{2}V_j(\pi z | \mathcal{F}_j) - pz - r^f z}{V_j(\pi z | \mathcal{F}_j)}$. Plugging in the above expression for $E_j(\pi z \mid P, S_j)$ and $V_j(\pi z \mid P, S_j)$, using the definition of τ and noting that at the order 0 in z, $\frac{\tau(W_{0j})}{W_{0j}} = \frac{\tau(W_{1j})}{W_{1j}}$, leads to Equation (7). The convergence of this approximation is demonstrated in Appendix E.

A.3 Market clearing
The equilibrium price clears the market for the stock. Aggregating Equation (7) over all investors yields the aggregate demand for the stock:

$$\int_j \frac{W_{0j}\alpha_j}{P} = n\left[\frac{E(\pi)}{\sigma_\pi^2} + \frac{iE(\theta)}{\sigma_\theta^2} + \frac{1}{2} + \frac{i^2}{\sigma_\theta^2}(\pi - \mu\theta) + \pi i - (p + r^f)(nh_0 + i)\right],$$

where the term πi comes from applying the law of large numbers for independent but not identically distributed random variables to the sequence $\{\tau_j x_j \varepsilon_j\}$. Formally, I follow He and Wang (1995) in defining a charge space $(\mathcal{J}, \mathcal{P}(\mathcal{J}), m)$, where $\mathcal{P}(\mathcal{J})$ is the collection of subsets of $\mathcal{J}$, and $m: \mathcal{P}(\mathcal{J}) \to \mathcal{R}^+$ is a finitely additive measure. Let the set of natural numbers represent the set of investors $\mathcal{J} = \{1, 2, \ldots, L\}$. A measure (charge) is given by $m(A) = \lim_{L \to \infty} \frac{1}{L}\#(A \cap \{1, 2, \ldots L\}), \forall A \in \mathcal{J}$, where $\#(\cdot)$ denotes the number of set elements. For an indicator function $\mathrm{IN}(\cdot)$, the aggregate demand of a set $\mathcal{T} \subseteq \mathcal{J}$ of investors is $\int_{j \in \mathcal{T}} X_j dm(j) = \lim_{L \to \infty} \frac{1}{L}\sum_{j=1}^L X_j \, \mathrm{IN}\,(j \in T)$. Applying this definition to $\{\tau_j x_j \varepsilon_j\}$, a sequence of independent random variables with the same mean zero (conditional on π) but different (finite) variances $\tau_j^2 x_j$ leads to $\int \tau_j x_j \varepsilon_j = 0$ and $\int \tau_j x_j S_j = \int \tau_j x_j (\pi z + \varepsilon_j) = \pi z \int \tau_j x_j + \int \tau_j x_j \varepsilon_j = \pi z i$. Finally, equating aggregate demand to aggregate supply θ yields the equilibrium price given by Equations (5) and (6).

Appendix B: Proof of Theorem 2 (Demand for Information)

In order to solve for the information choice, we need to compute the expected utility of a stockholder who acquires a signal of precision x_j. The expected utility of an agent investing a fraction α_j in stocks can be approximated at the order z by

$$E_j(U(W_{2j}) \mid \mathcal{F}_j) \approx U(W_{0j}) + U'(W_{0j})\{W_{0j}r^f z + p W_{0j}\alpha_{0j}z - C(x_j)z + W_{0j}\alpha_j$$
$$[E_j(\pi z \mid \mathcal{F}_j) + \frac{1}{2}V_j(\pi z \mid \mathcal{F}_j) - pz - r^f z]\} + \frac{1}{2}U''(W_{0j})W_{0j}^2\alpha_j^2 V_j(\pi z \mid \mathcal{F}_j).$$

Plugging the formula for α_j [Equation (7)] into this expression yields

$$E_j[v(S_j, x_j, W_{0j}; P)] = U(W_{0j}) + U'(W_{0j})z[\tfrac{1}{2}\tau(W_{0j})E_j(\lambda_j^2) + r^f W_{0j} + E_j(p)W_{0j}\alpha_{0j} - C(x_j)], \qquad (12)$$

where $\lambda_j \equiv \frac{E_j(\pi z | \mathcal{F}_j) + \frac{1}{2}V_j(\pi z | \mathcal{F}_j) - pz - r^f z}{\sqrt{V_j(\pi z | \mathcal{F}_j)}}$ is investor j's Sharpe ratio, a function of S_j and P (and x_j). $E_j(\lambda_j^2)$ is the squared Sharpe ratio investor j expects to achieve in the planning period. It no

longer depends on S_j and P, but it is still a function of x_j. Integrating over the distributions of S_j and P yields

$$E_j(\lambda_j^2) \equiv \varphi(x; i) = h(i; x_j)A + \frac{1}{4h(i; x_j)} + q - 1, \qquad (13)$$

where $A(i) \equiv \frac{h_0 n^2 + 2in + \sigma_\theta^2}{(n\bar{h})^2} + q^2$ and $q(i) \equiv \frac{E(\theta)}{n\bar{h}} - \frac{1}{2h}$. qz is the unconditional risk premium, that is, the excess return on the stock: $qz \equiv E(\ln\frac{\Pi}{P}) - r^f z \equiv (E(\pi) - E(p) - r^f)z$. Note that all agents agree ex ante (i.e., in the planning period) on the value of the risk premium regardless of the private information they will receive in the next period. As a function of x, $\varphi'(x; i) = A - \frac{1}{4h(i;x)_z^2}$ is increasing, concave, and converges to a horizontal asymptote A. As a function of x again, φ is convex and either increasing or U-shaped depending on the sign of $\varphi'(0)$. φ can be at first decreasing in precision x because it measures the expected squared instantaneous Sharpe ratio and information is about the logarithm of the payoff and not about the payoff itself. However, over the range of precisions that an investor effectively chooses, φ is increasing (see below). Under the assumption $\sigma_\pi^2 \leq 2$ (see note 17), φ' is a decreasing function of i (larger i implies that prices are more revealing): $\frac{\partial \varphi'}{\partial i} = \frac{\partial A}{\partial i} + \frac{i}{(h_0 + x_j)^3 \sigma_\theta^2} \leq \frac{\partial A}{\partial i} + \frac{i}{h_0^3 \sigma_\theta^2} \leq 0$.

To solve for the optimal precision level, one maximizes the expression in Equation (12) with respect to x_j taking i (hence A, h_0, and $\bar{h}$) as given. To simplifiy the exposition, define $Q(x, W_0) \equiv \frac{1}{2}\tau(W_0)\varphi(x) - C(x)$. The utility expected by an agent with precision x and initial endowments W_{0j} and α_{0j} is, from Equation (12), a linear function of $Q(x, W_{0j})$ and the optimal precision level, $x(W_{0j})$, simply maximizes $Q(x, W_{0j})$. This optimum can be interior ($x > 0$) or the corner zero ($x = 0$). If there is an interior solution, it must satisfy the first-order condition given by Equation (8) and the second-order condition given by Equation (9). Note that these conditions imply that φ' is positive at the optimum and that φ is indeed increasing in the precision. These equations may admit zero, one or many solutions depending on the level of initial wealth and the shape of the marginal cost. To analyze all possibilities, I distinguish two cases, $\varphi'(0) > 0$ and $\varphi'(0) \leq 0$.

B.1 $\varphi'(0) > 0$
Recall from the definition of φ that $\varphi'(0) = A - \frac{1}{4h_0^2}$. One can check that a lower bound for Ah_0^2 is $\frac{1}{\sigma_\pi^2} + \frac{E(\theta^2)}{n^2}$. Hence a sufficient condition for $\varphi'(0) > 0$ is $\frac{1}{\sigma_\pi^2} + \frac{E(\theta^2)}{n^2} > \frac{1}{4}$. There are several configurations depending on the investor's wealth and the shape of the function C.

If $\frac{dQ(0, W_0)}{dx} = \frac{1}{2}\tau(W_0)\varphi'(0) - C'(0) > 0$, then Q increases in a neighbourhood of $x = 0$ so the optimum is interior. This occurs if and only if $W_0 > \tau^{-1}\left(\frac{2C'(0)}{A - \frac{1}{4h_0^2}}\right)$. On the other hand, if $\frac{dQ(x, W_0)}{dx} = \frac{1}{2}\tau(W_0)\varphi'(x) - C'(x) < 0$ for all $x \geq 0$, then Q is a decreasing function of x so the optimum is the corner $x = 0$. This occurs in particular if $W_0 < \tau^{-1}\left(\frac{2C'(0)}{A}\right)$, since C' is increasing and $\varphi'(x) \leq A$, $\forall x$. For levels of wealth between these two bounds, Q increases, then eventually decreases again (recall that $\lim_{x\to\infty} C'(x) = +\infty$ by assumption). Hence Q has a local maximum, achieved for a precision of $x_l(W_0)$ (several maxima are possible if Q increases and decreases several times). The optimum is interior if and only if $Q(x_l(W_0), W_0) > Q(0, W_0)$. The envelope theorem implies that $\frac{dQ(x_l(W_0), W_0)}{dW_0} = \frac{1}{2}\tau'(W_0)[\varphi(x_l) - \varphi(0)]$. To assess the sign of this expression, note that the function $\varphi(x) - \varphi(0)$ is a convex U-shaped curve with two roots, 0 and $x_0 \equiv (\frac{1}{4} - Ah_0^2)/(Ah_0)$. In the case we are considering, $(\varphi'(0) > 0)$, $x_0 \leq 0$, so $\varphi(x) - \varphi(0)$ is positive for all positive x. Hence $Q(x_l(W_0), W_0)$ is increasing in W_0. In words, the utility achieved at the interior optimum increases with W_0. Therefore there exists a unique number $W_0^*(i)$ such that $Q(x_l(W_0), W_0) > Q(0, W_0)$ if and only if $W_0 > W_0^*$ regardless of α_0. This means that only the richest investors purchase information.

W_0^* is defined implicitly by Equation (8), Equation (9), and by equating $Q(x, W_0^*)$ to $Q(0, W_0^*)$. Nevertheless, W_0^* can be bounded: $\tau^{-1}\left(\frac{2C'(0)}{A}\right) < W_0^* < \tau^{-1}\left(\frac{2C'(0)}{A - \frac{1}{4h_0^2}}\right)$. These bounds

show that if $C'(0) = 0$, then $W_0^* = 0$ and all investors are informed. On the other hand, if $C'(0) > 0$, then informed and uninformed investors coexist. Furthermore, if $C'(0)$ is too large, then $W_0^* > \overline{\mathcal{W}_0}$ and no investor is informed. Finally, note that by imposing further assumptions, one can explicitly compute the threshold W_0^*. For example, assuming that $\sigma_\pi^2 \leq 2$, $(\frac{1}{\sigma_\pi^2} + \frac{E(\theta^2)}{n^2} - \frac{1}{4})C''(0) \geq 2C'(0)$ and $C'''(\cdot) \geq 0$ on $[0, \infty]$ ensures that investors who are so poor that $\frac{dQ(0, W_0)}{dx} < 0$ do not acquire any information (because $\frac{d^2 Q(0, W_0)}{dx^2} < 0$). One can then show that $W_0^* = \tau^{-1}\left(\frac{2C'(0)}{A - \frac{1}{4h_0^2}}\right)$.

B.2 $\varphi'(0) \leq 0$
As in the previous case, a sufficient condition for the optimum to be $x = 0$ is $W_0 < \tau^{-1}\left(\frac{2C'(0)}{A}\right)$. The envelope theorem shows that $\frac{dQ(x_I(W_0), W_0)}{dW_0}$ is of the sign of $\varphi(x_I) - \varphi(0)$. Again the function $\varphi(x) - \varphi(0)$ has two roots, 0 and $x_0 \equiv (\frac{1}{4} - Ah_0^2)/(Ah_0)$. This time $\varphi'(0) < 0$, so $Ah_0^2 < \frac{1}{4}$ and $x_0 > 0$. $\varphi(x) - \varphi(0)$ is negative up to x_0 and then positive. $Q(x_I(W_0), W_0)$ is first decreasing and then increasing in W_0. Like before, there exists a unique number $W_0^*(i)$ such that $Q(x_I(W_0), W_0) > Q(0, W_0)$ if and only if $W_0 > W_0^*$ regardless of α_0. The only difference with the previous case is that $C'(0) = 0$ no longer makes all investors informed.

Appendix C: Proof of Theorem 3 (Unicity)

To show that the equilibrium is unique (within the class of log-linear equilibria), it suffices to show that the aggregate precision i is uniquely defined. Let $\Sigma(i) \equiv i - \int_{W_0^*(i)}^{\mathcal{W}_0} x(W_{0j}, i)\tau(W_{0j})dG(W_{0j})$. Equation (4) defines i implicitly as a root of Σ. Differentiating Σ yields $\Sigma'(i) = 1 + \tau(W_0^*)x(W_0^*)g(W_0^*)\frac{dW_0^*}{di} - \int_{W_0^*}^{\mathcal{W}_0} \frac{dx(W_{0j})}{di}\tau(W_{0j})dG(W_{0j})$, where the second term comes from differentiating the lower bound in the integral and the third from differentiating the integrand. The second term drops out because $x(W_0^*) = 0$ (this follows from the assumption that C is continuous at zero), implying that $\Sigma'(i) = 1 - \int_{W_0^*}^{\mathcal{W}_0} \frac{dx(W_{0j})}{di}\tau(W_{0j})dG(W_{0j})$. Next, differentiating Equation (8) yields $\frac{dx(W_{0j})}{di} = \frac{1}{2}\tau(W_{0j})$ $\frac{\frac{\partial A}{\partial i} + \frac{i}{(h_0 + x_j)^3 \sigma_\theta^2}}{C''(x_j) - \frac{\tau(W_{0j})}{4(h_0 + x_j)^3}}$, which is negative because the numerator is negative (Appendix B) while the denominator is positive [second-order condition of Equation (9)]. Therefore $\Sigma'(i)$ is positive, Σ is monotonic, and i is uniquely defined.

Appendix D: Proof of Lemma 4 (Wealth and Portfolio Shares)

For an uninformed investor, the optimal share α is proportional to relative risk tolerance, $\frac{\tau(W_1)}{W_1}$. Consequently $\varepsilon_\alpha = \varepsilon_\tau - 1$. The precision chosen by a wealthy investor is large, so Equation (8) can be approximated by $C'(x_j) \approx \frac{1}{2}\tau(W_{0j})A$ and the unconditional share by $E(\alpha_j) = \frac{\tau(W_{1j})}{W_{1j}}(u + qx_j) \approx \frac{\tau(W_{1j})}{W_{1j}}qx_j$, where u is a function of the parameters of the model. (If one is interested in the share conditional on π, θ and θ', then u and q are replaced with functions of the random variables of the model.) Differentiating both equations and substituting out x_j yields the approximate share for an investor with large wealth: $\varepsilon_\alpha \approx \varepsilon_\tau - 1 + \frac{\varepsilon_\tau}{\varepsilon_c}$.

Appendix E: Convergence of the Approximation

Small noise expansions are not new to economics and have been used in particular in the real business cycle literature [e.g., Kydland and Prescott (1982), Gaspar and Judd (1997)]. There are two main approaches to deriving approximations for asset demand. The first goes back to Samuelson's (1970) concept of "compact" probabilities, while the other uses bifurcation theory [Judd and Guu (2001)]. Both approaches lead to the same approximation at the

order 0 in z in the Euler equation, which is the order of interest here. I will use Samuelson's compact probabilities.

Samuelson's (1970) objective is to justify mean-variance analysis without resorting to normal distributions or quadratic preferences when risk is small. He shows that the sequence of approximate economies (i.e., economies where preferences have been approximated locally around initial wealth by a quadratic series) converges, as the scaling factor z goes to zero, to the exact no-risk economy (i.e., the economy with the exact preferences but no-risk). More precisely, he shows that the optimal portfolio in the approximate economy converges to the optimal portfolio in the no risk economy as long as the distribution of returns is "compact" (for all z). Let $\mathcal{X}$ be the return on an asset with density function f. Suppose that $E(\mathcal{X}) \approx m + az$ and let $\mathcal{Z}$ be the standardized return $\mathcal{Z} = \mathcal{X} - m$. The distribution of returns $\mathcal{X}$ is compact if [Equation (7) in Samuelson (1970)]:

$$\lim_{z \to 0} \frac{E(\mathcal{Z})}{E(\mathcal{Z}^2)} = \frac{\mathcal{A}}{\mathcal{B}} \quad \text{and} \quad \lim_{z \to 0} \frac{E(\mathcal{Z}^r)}{E(\mathcal{Z}^2)} = \sqrt{z}^{r-2} \mathcal{C}_r \ (r = 3, 4, \dots). \tag{14}$$

Theorem 1 in Samuelson (1970) states that the portfolio share in the exact economy $\alpha(z)$ converges, as z goes to zero, to the solution to the quadratic problem:

$$\max_\alpha \int_0^\infty \left[U(m) + U'(m)(\alpha\mathcal{X} - m + 1 - \alpha) + \frac{1}{2} U''(m)(\alpha\mathcal{X} - m1 - \alpha)^2 \right] f(\mathcal{X}) d\mathcal{X}.$$

In this article, $\mathcal{X} \equiv \frac{\Pi}{P}$. For ease of exposition, I write $E(\pi)$ for $E_j(\pi z \mid \mathcal{F}_j)$ and σ_π^2 for $V_j(\pi z \mid \mathcal{F}_j)$. $E(\frac{\Pi}{P}) \approx 1 + (E(\pi) + \frac{1}{2}\sigma_\pi^2 - p)z$, so $\mathcal{Z} = \frac{\Pi}{P} - 1$. I show next that conditions (14) are satisfied in the model.

I highlight the main steps of the proof. First, $E(\mathcal{X}^r) = \exp[r(\pi - p)z + \frac{r^2}{2}\sigma_\pi^2 z]$. Expanding this expression yields $E(\mathcal{X}^r) = \sum_{j=0}^\infty [r(\pi - p)z + \frac{r^2}{2}\sigma_\pi^2 z]^j/(j!)$. Second, expanding $\mathcal{Z}^r = (\mathcal{X} - 1)^r$ leads to $E(\mathcal{Z}^r) = \sum_{i=0}^r C_r^i(-1)^{r-i}E(\mathcal{X}^i)$. The term in z^j in $E(\mathcal{Z}^r)$ is $\sum_{i=0}^r C_r^i(-1)^{r-i}[i(\pi - p)z + \frac{i^2}{2}\sigma_\pi^2 z]^j/(j!)$. When $r = 1$ or 2, direct calculations imply that $E(\mathcal{Z}^r)$ is proportional to z, so I focus from now on $r \geq 3$. Third, expanding $[i(\pi - p)z + \frac{i^2}{2}\sigma_\pi^2 z]^j$ yields $\sum_{l=0}^j C_j^l(\pi - p)^l(\frac{1}{2}\sigma_\pi^2)^{j-l}\frac{1}{j!}\sum_{i=0}^r C_r^i(-1)^{r-i}i^{2j-l}$. Fourth, $\sum_{i=0}^r C_r^i(-1)^{k-i}i^m = 0$ for all integer $m \leq r - 1$. Indeed, differentiating m times ($m \leq r - 1$) $(a - 1)^r$ with respect to a and then setting $a = 1$ implies that $\sum_{i=0}^r C_r^i(-1)^{r-i}i(i-1)\dots(i-m+1) = 0$. Then, proceeding by forward induction implies that $\sum_{i=0}^r C_r^i(-1)^{r-i}i^m = 0$ for all integer $m \leq r - 1$. Therefore the z^j terms in $E(\mathcal{Z}^r)$ are zero for all j and l such that $2j - l \leq r - 1$ and $l \leq j$, that is, for all $j \leq (r-1)/2$. I have shown that the lowest-order term in $E(\mathcal{Z}^r)$ is $z^{r/2}$ (if r odd, $z^{(r+1)/2}$ if r even) when $r \geq 3$, and z when $r = 1$ and 2. It follows that the lowest-order term in $\frac{E(\mathcal{Z})}{E(\mathcal{Z}^2)}$ is independent of z and that the lowest-order term in $\frac{E(\mathcal{Z}^r)}{E(\mathcal{Z}^2)}$ is at most $\sqrt{z}^{r-2}$. This proves that the distribution is compact and that the approximation converges. The next step is to assess the approximation error. This is done in the following appendix thanks to a numerical illustration.

Appendix F: Quality of the Approximation

To evaluate the quality of the approximation, I solve a simpler version of the model numerically and see how the approximate solution compares to the exact numerical solution. Specifically I focus on the trading period equilibrium and assume investors have CRRA preferences and are all endowed with the same initial wealth and private precision. This assumption simplifies the numerical problem while retaining the essence of the approximation.

The difficulty in solving such a rational expectation problem stems from the fact that the endogenous price belongs to the investors' information sets. To deal with this, I use the projection approach described in Bernardo and Judd (2000). In this approach, the price and

the demand for stocks are approximated by a sum of orthogonal polynomials. Let X be the demand for stocks, $X = \frac{\alpha W_1}{P}$, and $R = 1 + r^f z$ the gross return on the riskless asset:

$$\hat{P}(\pi, \theta) \equiv \sum_{i=0}^{n_d} \sum_{k=0}^{n_d} a_{ik} H_i(\pi) H_k(\theta) \quad \text{and} \quad \hat{X}(P, S) \equiv \sum_{l=0}^{n_d} \sum_{m=0}^{n_d} b_{lm} H_l(P) H_m(S),$$

where H_i is the degree i Hermite polynomial and n_d represents the highest-degree polynomial used in the approximation. Hermite polynomials are used because they are mutually orthogonal with respect to the normal density with mean zero: $\int_R H_i(x) H_k(x) \exp(-x^2) dx = 0$ for all $i \neq k$. The goal is to find the unknown coefficients a_{ik} and b_{lm} that satisfy the first-order and equilibrium conditions. As in Bernardo and Judd (2000), I use the complete set of polynomials rather than the full tensor product to reduce the number of unknowns from $2(n_d + 1)^2$ to $(n_d + 1)(n_d + 2)$.

The investor's first-order condition $E[W_2^{-\gamma}(\Pi - RP) \mid \mathcal{F}_j] = 0$ implies that $E[W_2^{-\gamma}(\Pi - RP)\varphi(S_j)\psi(P)] = 0$ for all continuous bounded functions φ and ψ. Thus it can be approximated numerically with the new conditions

$$E[\widehat{W}_2^{-\gamma}(\Pi - R\hat{P}) H_l(\hat{P}) H_m(S_j)] = 0 \quad \text{for} \quad l = 0, \ldots, n_d \quad \text{and} \quad m = 0, \ldots, n_d - l + 1.$$

The hope here is that a sufficiently small number of projections will yield a useful approximation.
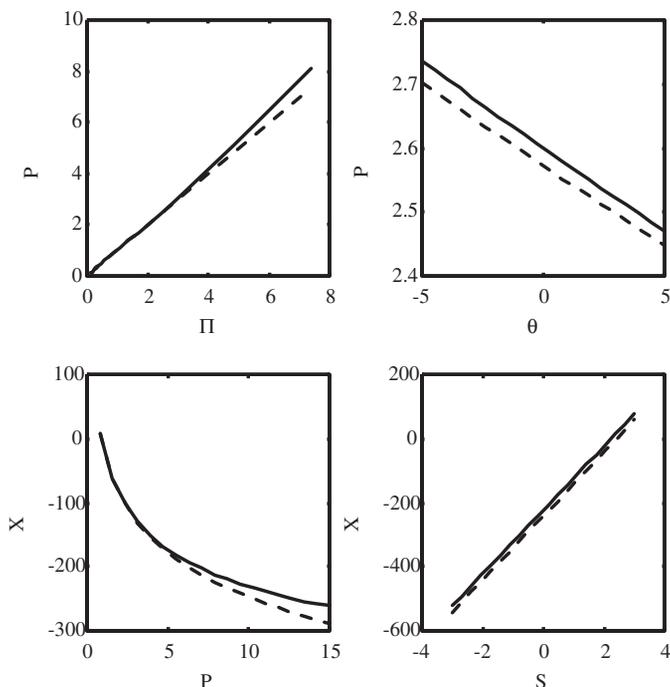
Figure 7
The exact numerical (solid line) and the approximate (dashed line) solutions.
The top panels display the equilibrium price as a function of the payoff (left panel) and the net supply of stocks (right panel). The bottom panels display the demand for stocks as a function of the price (left panel) and the private signal (right panel)
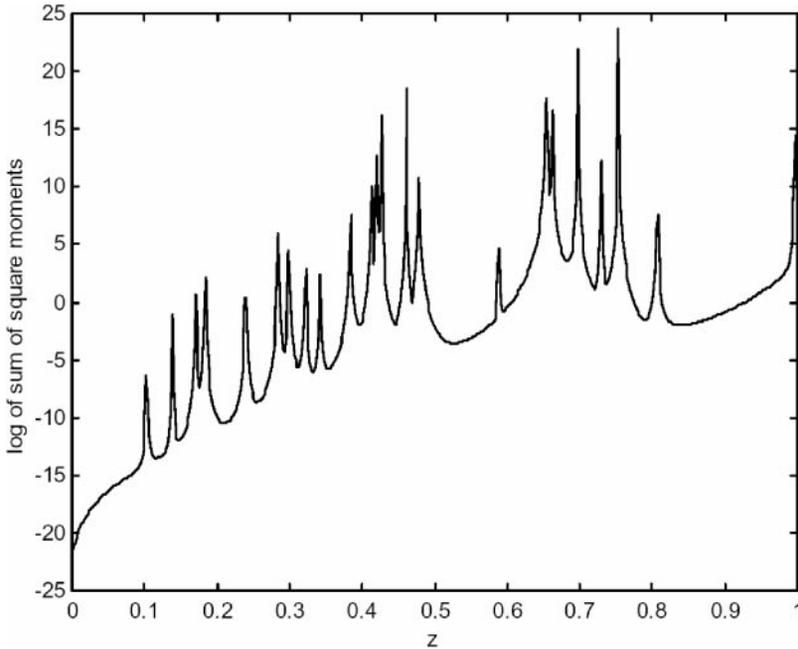
Figure 8
The quality of the approximation: log of mean square moments evaluated at approximate solution for different values of the scaling factor z

As for the market clearing condition $\int_j X(S_j, P)dG(Z_j) = \theta$, it cannot be imposed in each and every state, so it is assumed that the deviations from market clearing are orthogonal to several of the basis polynomials:

$$E\left[\left[\int_j \hat{X}(S_j, \hat{P})dG(Z_j) - \theta\right]H_i(\pi)H_k(\theta)\right] = 0 \quad \text{for } i = 0, \ldots, n_d \quad \text{and} \quad k = 0, \ldots, n_d - l + 1.$$

In this fashion, the equilibrium problem has been reduced to a system of $(n_d + 1)(n_d + 2)$ equations in $(n_d + 1)(n_d + 2)$ unknowns. To compute the expectations, I use Gaussian quadrature techniques (in the pictures, I use 5 grid points to evaluate integrals).

The exact numerical and the approximate solutions for the equilibrium price and demand for stocks are displayed in Figure 7 for the following parameter values: $\sigma_\pi^2 = \sigma_\theta^2 = E(\theta) = 1$, $E(\pi) = x = \gamma = 2$, $W_0 = 10$, $r^f = 0.05$, $z = 0.1$, and $n_d = 3$. To evaluate the approximation error, I plugged the approximate solution into the set of moment conditions and measured how close the conditions are from zero. Figure 8 shows the logarithm of the sum of square moments for the same set of parameters. It is of the order of $\exp(-15)$ or $10e - 7$ for $z = 0.1$. Hence the approximation performs well.

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