Information vs. Entry Costs: What Explains U.S. Stock Market Evolution?

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Abstract

I investigate whether changes in stock market participation costs can explain the long-term increase in the number of U.S. stockholders. I separate these costs into two components: an information cost (the cost of collecting market information), and an entry cost (all other costs, including commissions and fees), and disentangle their general equilibrium implications in a noisy rational expectations economy. While a falling information cost cannot explain the observed increase in stock market participation, a falling entry cost can account for this plus several other features of the U.S. economy, including the falling equity premium, rising return variances, and the boom in passive relative to active investing.

I. Introduction

U.S. stock market participation has increased remarkably over the second half of the 20th century. Starting from a low of 6% in 1952, stock market participation accelerated throughout the 1980s and 1990s and reached 32% in 1989 and 49% in 1998. Yet today, still half of the households do not own any stocks either directly or indirectly. These observations raise two questions. First, why is stock market participation so low? Second, why has it increased over time? These questions are important not only for understanding financial markets, but also for designing fiscal policies and social security systems. This paper sheds light on these issues.

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1These numbers are from the 1989 and 1998 Surveys of Consumer Finances. Vissing-Jørgensen (1998) documents the trend in stock market participation going back to 1952 using the NYSE ownership surveys and IRS tax returns data. Participation rates are even lower in European countries (Guiso, Haliassos, and Jappelli (2001)).

2For example, the famous equity premium and risk-free rate puzzles are tightly related to these questions: in a world where risk sharing is limited, stockholders demand a higher risk premium and nonstockholders, who can only save in bonds, push the risk-free rate down. Abel (2001) estimates that investing some of the Social Security Trust Fund in equity would lead to a substantial fall in aggregate
A simple explanation for these observations relies on the existence of fixed stock market participation costs that have declined over time. Theoretical models commonly appeal to such costs to limit participation (e.g., Merton (1987), Hirshleifer (1988), Allen and Gale (1994), and Abel (2001)). In practice, participation costs encompass a wide range of costs, including trading costs, management fees, and time and money spent keeping up with market developments. Empirically, wealth is by far the most significant determinant of household participation, lending support to a fixed cost explanation. Furthermore, empirical studies show that relatively low participation costs can keep a large fraction of the population away from stocks. For example, Vissing-Jørgensen (2002) estimates that annual fixed costs of U.S.$260 (U.S.$50) can explain why three-quarters (one-half) of nonstockholders do not participate. Whether unobservable information costs account for a large part of these costs is difficult to assess. Nevertheless, there is evidence that they are a strong impediment to owning stocks. Using data from the 1978 Survey of Consumer Financial Decisions, King and Leape (1987) report that more than one-third of households that do not own stocks or mutual funds say it is because they do not know enough about them. Furthermore, participation costs have fallen in the last few decades, thereby attracting more households into the market. This is obvious with respect to trading costs and management fees. In the U.S., the cost of trading individual stocks (including commissions and bid-ask spreads) has dropped dramatically since commissions were deregulated in 1975 (Jones (2002)). Mutual fund fees (including fund expenses, loads, and distribution costs) have also decreased steadily from 2.26% in 1980 to 1.35% in 1998 (Rea, Reid, and Lee (1999)). The picture is less

The outcome of elections may also be affected when the median voter becomes a stockholder (e.g., The New York Times, August 18, 2002, section 4, p. 1).


4The reason these costs are relatively small is that most nonstockholders would invest very little in stocks if they participated. Vissing-Jørgensen’s estimate is actually on the lower end of the spectrum. Saito (1996) and Haliassos and Bertaut (1995) estimate the fixed cost to be, respectively, 1.1 and 1.5 hours per week. Vissing-Jørgensen’s cost of $260 corresponds to about 30 minutes per week at a wage of $10 per hour. Other papers such as Luttmer (1999) and Marshall and Parekh (2000), which estimate the fixed cost that would reconcile a representative agent model with observed asset prices assuming full participation, report larger costs.

5Several European surveys confirm these observations (Guiso and Jappelli (2001), Alessie, Hochguertel, and Van Soest (2001), and Borsch-Supan and Eymann (2001)). Microeconometric studies find that the probability of stock ownership increases with education (e.g., Mankiw and Zeldes (1991) and Haliassos and Bertaut (1995)). In addition, recent papers provide evidence on the role of financial education (Weisbenner (1999), Bernheim and Garret (1996), and Bernheim, Garret, and Maki (1997)) and social interactions (Huberman (2001), Chiteji and Stafford (1999), Duflo and Saez (2002), and Hong, Kubik, and Stein (2002)) for stock ownership. There is also evidence that the cost of becoming aware of individual stocks is significant. Several papers report that small firms earn higher returns controlling for risk and liquidity, a finding that is interpreted as evidence that these stocks are “neglected” (e.g., Banz (1981), Reinganum (1983), Schwert (1983), and Barry and Brown (1984)). Several papers document that investors’ awareness of a stock increases following a listing on a U.S. exchange (Kadlec and McConnell (1994), Foerster and Karolyi (1999), and Miller (1999)), following an addition to the S&P 500 index (Chen, Noronha, and Singal (2004)), with advertising (Grullon, Kanatas, and Weston (2004)), and with brand recognition (Frieder and Subrahmanyam (2002)). Finally, some researchers argue that costly information processing explains some puzzling phenomena such as the “home equity bias” (French and Poterba (1991), Kang and Stulz (1994), and Coval and Moskowitz (1999)) and the “weekend effect” (Lakonishok and Maberly (1990)).
clear for information costs. On one hand, the media have increased their coverage of the stock market since the 1970s and, more recently, the Internet has given investors instant access to information on companies. On the other hand, finding relevant, quality information in this ocean of facts and commentary is a daunting task.

In this paper, I offer a better understanding of how these costs affect stock ownership decisions. For this purpose, I split participation costs into two components. The information cost includes all activities undertaken by investors to improve their assessment of a stock’s (or a fund’s) performance. For example, investors may read the press, listen to radio and TV reports, search the Web, participate in seminars, subscribe to newsletters, join investment clubs, analyze company accounts, or hire financial advisors. The other component, the entry cost, contains all other costs such as trading costs and management fees, time spent filing tax forms, opening brokerage accounts, understanding the principles of stocks, or simply becoming aware of their existence. Though they cannot by themselves limit participation, variable costs are bundled with the entry cost because they can act as powerful deterrents when combined with a small fixed cost.

This paper disentangles the equilibrium implications of entry and information costs on participation and the distribution of returns. Investors have to pay the entry cost to trade an index. To improve their performance, they can collect information about its payoff in exchange for an information cost. Stockholders who do so are called active, while those who do not are passive. As Grossman and Stiglitz (1980) show, information, if acquired, is revealed by the equilibrium price but only partially because some noise traders render the supply of stocks random. Active investors use their private information to adjust their holdings of the index and cash relative to that of passive investors.

A change in any of the costs has two effects. On one hand, it modifies the number of shareholders (the risk sharing effect). On the other hand, it alters the amount of private information collected and, consequently, the level of aggregate risk (the information effect). Moreover, risk and risk sharing go hand-in-hand. Risk sharing is broad when risk is great because investors are attracted by the high equity premium. Symmetrically, risk is great when participation is broad because investors, who compete with many free-riders acquire less information. Greater risk, in turn, increases the mean and variance of returns while broader risk sharing decreases them. Most papers deal either with the risk sharing effect (e.g., Merton (1987) and Hirshleifer (1988)) or with the information effect (e.g., Grossman and Stiglitz (1980) and Verrecchia (1982)), but rarely with both. The

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6The boom in financial reporting is exemplified by Money magazine. Launched in 1972 with an initial circulation of 225,000, it has expanded to reach two million subscribers and 10 million readers today, making it the largest personal finance magazine. In 1972, The New York Times created a separate money and business section. CNBC was created in 1989 and is now the world’s most popular business channel.

7Variable costs are incurred by all stockholders and, in that respect, resemble more the entry cost than the information cost. They cannot be explicitly incorporated in the model because they create an inaction region, i.e., a price range in which investors do not trade. Hence, variable costs break the linearity of the equilibrium price and render the model intractable.

8Stein (1987) and Hau (1998) analyze the informational effects of increased participation, taking both effects into account. However, in these models, the information content of prices is either exogenous as in Stein (1987) or only partially endogenous as in Hau (1998), where traders are assumed to
main contribution of this paper is to analyze the interplay between these effects and its consequences given falling entry and information costs.

The results of the model are twofold. First, a low information cost discourages investors from participating. Hence, a falling information cost leads to lower participation. Second, a falling entry cost leads to (i) a rise in participation, (ii) a fall in the equity premium, (iii) a rise in return variances, and (iv) a boom in passive investing relative to active investing. These four implications are consistent with postwar U.S. data.

The first result is that a low information cost need not induce households to buy stocks. When all stockholders are active, the information cost operates just like the entry cost, with a lower information cost leading to higher participation. However, when both active and passive stockholders coexist in equilibrium, a fall in the information cost increases the participation of the active but decreases the participation of the passive stockholders so that the net effect is a fall in participation. Indeed, as she faces a lower information cost, the marginal passive investor becomes active and trades more aggressively. The risk premium and risk both fall (the information effect), but the effect on the risk premium is stronger so the gains from trade to other agents deteriorate. It follows that the marginal stockholder, who does not benefit from the lower information cost because she is passive, exits the market (the risk sharing effect). This result essentially rests on the differentiating effect the information cost has on investor groups: unlike the entry cost, a falling information cost privileges active stockholders at the expense of passive stockholders.

The second result is that, as the entry cost falls, the participation of passive investors increases at the expense of the active.9 Overall participation increases, the expected return falls, and the price is less informative. Indeed, the fall in the entry cost induces the marginal nonstockholder to enter (passive), thus increasing the price and forcing incumbent investors to reduce their holdings (the risk sharing effect). With less scope to benefit from private information, the marginal active investor becomes passive (the information effect). If the information effect dominates the risk sharing effect, then the variance of returns rises. Empirically, Fama and French (2002) estimate that the expected return on stocks declined in the second half of the century while Campbell, Lettau, Malkiel, and Xu (2001) document that the variance of individual stock returns has more than doubled from 1962 to 1997.10 Furthermore, index funds have soared in the last few decades rel-

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9This result echoes Keynes’s (1989), p. 159 explanation for why the 1929 stock market crash was more severe in the U.S. than in the U.K.:

It is usually agreed that casinos should, in the public interest, be inaccessible and expensive. And perhaps the same is true of stock exchanges. That the sins of the London Stock Exchange are less than those of Wall Street may be due, not so much to differences in national character, as to the fact that to the average Englishman Throgmorton Street is, compared with Wall Street to the average American, inaccessible and very expensive. The jobber’s “turn,” the high brokerage charges and the heavy transfer tax payable to the Exchequer, which attend dealings on the London Stock Exchange, sufficiently diminish the liquidity of the market to rule out a large proportion of the transaction characteristics of Wall Street.

10Campbell, Lettau, Malkiel, and Xu (2001) document an increase in individual stock return variances but no change in market return variance. Though the present model only considers one stock
ative to active funds. As Table 1 shows, mutual funds assets increased by a factor of 16 from 1989 to 1999, while index funds assets grew over a hundredfold over the decade. These observations are all consistent with a fall in the entry cost.

<table>
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<th>TABLE 1</th>
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<td><strong>U.S. Equity Mutual Funds and Index Funds</strong></td>
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<td>Avg. annual growth rate</td>
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Data are from the Investment Company Institute.

The model builds on Grossman and Stiglitz’s (1980) partially revealing rational expectations equilibrium. As in Verrecchia’s (1982) extension, investors can choose any level of precision for their information. This paper differs from the classical framework in two ways. First and most important, I assume that investors have to pay a fixed cost, the entry cost, to participate in the stock market. This adds another margin (whether to hold any stocks) to the margins Grossman and Stiglitz (how many stocks to hold and whether to purchase information) and Verrecchia (how much information to acquire) describe. In equilibrium, agents are separated into three groups: nonstockholders, passive stockholders, and active stockholders. How informative prices depend on the number of active stockholders, which itself depends on the total number of stockholders. Symmetrically, the number of stockholders is determined by the informativeness of prices. Because of this interaction, each cost generates both a risk sharing and an information effect.

Second, I assume that agents have quadratic rather than constant absolute risk aversion utility (CARA or exponential). While both preferences capture the trade-off between mean and variance, which lead to the same portfolio choices and allow for a closed-form solution to the model, a limitation of CARA preferences is that investors collect the same amount of information regardless of the numbers of shares supplied and stockholders in equilibrium. In contrast, the demand for information under quadratic preferences increases with the number of shares each investor expects to hold, reflecting the increasing returns to scale representing the market as a whole, the model can easily be extended to many stocks, with a common component in their payoffs. Assuming the common component is revealed (either by a public signal or by the market index) leads to a constant market return variance and increasing idiosyncratic return variances, in agreement with the data.

Though the first index fund was offered in 1976 by Vanguard, agents could invest passively before then by directly holding a limited number of stocks to mimic the market.

In addition, in recent years stock prices and volatility surged while many new participants entered the stock market, in particular, small inexperienced investors. The average number of shares per trade on the Nasdaq has fallen steadily from 1,813 shares in January 1995 to 705 in February 2000. Institutional investors typically trade in blocks of 5,000 to 10,000 shares or more, so a large number of smaller orders is required to reduce the average trade size. See Chordia, Roll, and Subrahmanyam (2001) for evidence on the NYSE.
displayed by the production of information.\textsuperscript{13} As a consequence, the price is less informative and return variance increases in response to a fall in entry costs. As mentioned previously, the evidence supports this prediction. Furthermore, I model absolute risk tolerance as an increasing function of initial wealth, which is also a well-documented empirical fact.

The results obtain, fully or in part, under a number of generalizations. In particular, they are robust to the addition of a trading round. This generalization is of interest because information in the model allows investors to learn about the stock earlier. Better information shifts risk forward, increasing the gains from trade in early periods. Nevertheless, I find that the results on participation are not altered.

The remainder of the paper is organized as follows. Section II describes the economy and defines the equilibrium concept. Section III characterizes the equilibrium. Section IV analyses the effect of the costs on the participation rate. Section V concludes. The Appendix features proofs of theorems and simulations.

II. The Economy

The model is in the spirit of Grossman and Stiglitz (1980) and Verrecchia (1982). There are three periods, a planning period \((t = 0)\), a trading period \((t = 1)\), and a consumption period \((t = 2)\).

A. Investment Opportunities

Two assets are traded competitively in the market, a riskless asset and a risky asset (the stock). The stock represents the equity market as a whole, which investors attempt to time.\textsuperscript{14} The riskless asset is in perfectly elastic supply and has a gross rate of return of \(R\). The risky asset has a price \(P\) and a random payoff \(II\). As is customary in models where information can be extracted from the equilibrium price, two sources of risk are needed to make the extraction problem nontrivial and to preserve the incentives to purchase private information. The second source of risk is the supply of stocks \(\theta + eP\), where \(\theta\) is a random shock to the supply and \(e\) \((0 \leq e < \infty)\) captures the elasticity of the supply with respect to price. This elasticity represents actions taken by companies to benefit from expected price changes. When prices are expected to increase, companies issue more shares and invest in more projects. When prices are expected to fall, companies buy back shares and scale down their investments. By assumption, \(\theta\) and \(II\) are jointly normally distributed and independent,

\[
\begin{pmatrix}
\theta \\
II
\end{pmatrix} \sim N \left\{ \left( \begin{array}{c} E(II) \\ E(\theta) \end{array} \right), \left( \begin{array}{cc} \sigma_{II}^2 & 0 \\ 0 & \sigma_{\theta}^2 \end{array} \right) \right\}.
\]

\textsuperscript{13}See the discussion in Section IV.C. In addition, Peress (2004) shows that locally quadratic preferences are a good approximation for a wide class of preferences when risk is small.

\textsuperscript{14}For concreteness, the stock may be viewed as a share of a mutual fund. Most mutual funds are specialized in equity or bonds. In 1998, there were over 7,000 mutual funds in the U.S.; hybrid funds accounted for only 7% of all funds and managed only 9% of the industry’s assets according to the Investment Company Institute.
There is an entry cost in participating in the stock market, \( F \). This cost has to be paid by any agent buying or selling stocks, and as mentioned in the Introduction, it represents trading costs and management fees, time spent filing tax forms, opening brokerage accounts, understanding the principles of stocks, or becoming aware of their existence. There is no cost associated with trading the riskless asset.

**B. Information Structure**

Agents may spend time and resources to forecast the performance of the stock market. Agent \( j \) may purchase a signal \( S_j \) about the payoff \( \Pi \),

\[
S_j = \Pi + \varepsilon_j,
\]

where \( \{\varepsilon_j\} \) is independent from \( \Pi, \theta \), across agents and from the distribution of wealth. Let \( x_j \) denote the precision of agent \( j \)'s signal. I assume that \( \varepsilon_j \) is normally distributed,

\[
\varepsilon_j \sim N \left( 0, \frac{1}{x_j} \right).
\]

The signal costs \( C(x_j) \) dollars where \( C \), the information cost, is increasing and strictly convex in the precision level. Specifically, I assume that \( C(0) = 0, C'(.) \geq 0 \), and \( C''(.) > 0 \) on \( \]0, \infty[ \), and \( \lim_{x \to \infty} C'(x) = +\infty \). These assumptions ensure the existence of an interior solution. They capture the idea that each extra piece of information is more costly than the previous one, possibly because they are correlated.\(^{15} \)

For example, the cost function \( C(x) \equiv c_0 + \frac{1}{2} c_1 x^2 \) on \( \]0, \infty[ \) for \( c_0 \geq 0 \) and \( c_1 > 0 \) satisfies these assumptions. Agency problems (not modeled here) preclude investors from sharing or selling their private information.

Finally, in a rational expectations equilibrium, agents know that the equilibrium price \( P \) contains some information about the risky payoff \( \Pi \) and they will use it as an informative signal. \( \mathcal{F}_j \) denotes investor \( j \)'s information set: \( \mathcal{F}_j = \{S_j, P\} \) if investor \( j \) is active and \( \mathcal{F}_j = \{P\} \) if she is passive. \( E_j(.) \) refer, respectively, to period 1 and period 0 expectations by investor \( j \), i.e., where the private signal \( S_j \) is distributed with precision \( x_j \).

**C. Investors**

There is a continuum of heterogeneous agents. Their objective is to maximize a mean-variance objective function, \( U(W') \equiv E(W') - (1/(2\tau)) \text{var}(W') \),

\(^{15}\) Suppose that information takes the form of draws from a distribution centered at the unknown value of the payoff \( \Pi \). Suppose further that the errors from these draws are correlated. Specifically, let \( s_i \) be the observed realization of a draw, i.e., \( s_i = \Pi + (\omega + \nu_i) \) where \( \omega \) is independent of \( \Pi \) and \( \theta \) with variance \( \sigma^2_\omega \), and \( \nu_i \) is independent of \( \Pi, \omega, \theta \), and \( \nu_i \) for all \( i' \neq i \) with variance \( \sigma^2_\nu \). Finally, suppose that the investor has to pay a fixed fee per observation. An investor who observes \( m \) such independent draws estimates \( \Pi \) by its sample mean, \((1/m) \sum_{i=1}^m s_i\), and the precision of her estimate,

\[
\left( \text{var} \left( \frac{1}{m} \sum_{i=1}^m s_i \right) \right)^{-1} = \left( \sigma^2_\omega + \frac{\sigma^2_\nu}{m} \right)^{-1},
\]

is increasing and concave in the sample size \( m \). It follows that the cost function is convex in the precision of the estimate of \( \Pi \).
where $W$ is final wealth and $\tau$ is the coefficient of absolute risk tolerance. Agents differ only in their initial endowment of the riskless asset and contemplate the prospect of becoming stockholders. To capture the key influence of wealth on portfolio decisions, I model absolute risk tolerance, $\tau$, as an increasing function of initial wealth, $W$. Numerous empirical studies support the hypothesis of increasing absolute risk tolerance. This preference specification can be interpreted as a local quadratic approximation to any utility function around initial wealth as in Peress (2004). I impose the following limit conditions for convenience: $\lim_{W \to 0} \tau(W) = 0$ and $\lim_{W \to \infty} \tau(W) = \infty$. Let $W_j$ be agent $j$'s initial wealth, and $G$ the cumulative distribution function of $W_j$ on a compact set $[V, \bar{V}]$. To simplify the discussion, I assume that $VV = 0$. The choice variables of an agent are the precision of her private signal $x_j$ and her holdings of equity and bonds, $T_j$ and $B_j$.

D. Timing

The timing is depicted in Figure 1. There are three periods. The first period $(t = 0)$ is the planning period: the agent decides whether to participate in the stock market (and pay $F$) and whether to acquire information (she chooses $x_j$ and pays $C(x_j)$). These decisions are made before either $P$ or possibly $S_j$ is observed. The second period $(t = 1)$ is the trading period. The investor observes her private $S_j$ with the precision $x_j$ she chose in the previous period. At the same time, markets open and she observes the equilibrium price. She uses the public and private signals to compute $E_j(I \mid F_j)$ and $var_j(I \mid F_j)$ and then chooses her holdings of the risky assets, $T_j$. The third period $(t = 2)$ is a pure consumption period: the agent consumes the proceeds from her investments, $W_j$.  

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16Peress (2004) shows how to solve a similar equilibrium under general preferences by making a small risk approximation. Essentially, preferences are approximated locally around initial wealth by an envelope of quadratic functions.

17This particular timing ensures that the decisions to participate and to acquire information depend neither on the signal $S_j$ nor on the price $P$ and hence that the equilibrium price is linear in $I$ and $\theta$. Indeed, once she has paid $F$, an agent will always find it optimal to hold a position in stocks (whether long or short) regardless of her signal because she is almost risk neutral for small risks.
E. Equilibrium Concept

1. Individual Maximization
When agent \( j \) does not enter the stock market, she receives the expected utility from her riskless investment, \( RW_j \). When she does enter the stock market, her maximization problem must be solved in two stages, working from the trading period back to the planning period. In the trading period \( (t = 1) \), she observes \( P \) and \( S_j \) (where \( x_j \), the precision of \( S_j \), is inherited from the planning period) and then forms her portfolio taking \( P, R, \) and \( C(x_j) \) as given,

\[
\max_{T_j, b_j} E_j \left[ U(W^f ; W_j) \mid \mathcal{F}_j \right] \text{ subject to } IT_j + RB_j = W^f \\
PT_j + B_j = W_j - F - C(x_j).
\]

Note that agents may borrow at rate \( R \) and short stocks if they wish. Call \( v(S_j, x_j, W_j ; P) \) the value function for this problem.

In the planning period \( (t = 0) \), the agent chooses the precision of her private signal to maximize her expected utility averaging over all the possible realizations of \( S_j \) and \( P \) and taking \( C(.) \) as given,

\[
\max_{s_j \geq 0} E_j \left[ v(S_j, x_j, W_j ; P) \right].
\]

Call \( V(W_j) \equiv V_j \) the value function for this problem; \( V_j \) coincides with \( V^A(W_j) \equiv \max_{s_j > 0} E_j \left[ v(S_j, x_j, W_j ; P) \right] \) if the stockholder chooses to be active and with \( V^P(W_j) \equiv E_0 \left[ v(0, 0, W_j ; P) \right] \) if she chooses to be passive. Agent \( j \) decides to participate in the stock market and pay \( F \) if and only if \( V(W_j) > RW_j \).

2. Market Aggregation
The entry and information decisions of an agent depend on the decisions of all other agents in the economy because the gains from trade and private information depend on the price level and its informativeness. Two variables summarize the aggregate behavior of agents. First, let \( n \) be the aggregate risk tolerance of investors participating in the market or market risk tolerance,

\[
n = \int_1 1_{\{V_j > RW_j \}} \tau(W_j) dG(W_j),
\]

where \( 1_{\{V_j > RW_j \}} \) is the indicator function of the inequality \( V_j > RW_j \); \( 1_{\{V_j > RW_j \}} = 1 \Leftrightarrow V_j > RW_j \Leftrightarrow \text{agent } j \text{ enters the stock market (and pays } F) \).

Second, call \( i \) the aggregate precision or informativeness of the price implied by aggregating individual precision choice and \( \mu \), its inverse,

\[
i = \int_1 x_j \tau(W_j) dG(W_j) \text{ and } \mu i = 1,
\]

where \( \mu \) is a measure of the noisiness of the price or noise that will prove useful in the next sections. Private precisions are weighted by risk tolerance because investors transmit their information through their demand for stocks, which is proportional to their risk tolerance. Individual decisions both depend on and determine these aggregate variables, \( i \) and \( n \). The formal definition of an equilibrium follows.
3. Definition of an Equilibrium

A rational expectations equilibrium is given by two demand functions $T_j$ and $x_j$, a price function $P$ of $\Pi$ and $\theta$, and two scalars $n$ and $i$ such that i) $x_j = x(W_j; n, i)$ and $T_j = T(S_j, x_j, W_j; P, n, i)$ solve the maximization problem of an investor taking $P$, $n$, and $i$ as given (equations (2) and (3)), ii) $P$ clears the market for the risky asset, $\int T(S_j, x_j, W_j; P, n, i) dG(W_j) = \theta + eP$, iii) the market risk tolerance $n$ implied by aggregating individual participation decisions equals the level assumed in the investor’s maximization problem, $n = \int 1_{V_j < \xi W_j} \tau(W_j) dG(W_j)$, and iv) the informativeness of the price $i$ implied by aggregating individual precision choices equals the level assumed in the investor’s maximization problem, $i = \int x_r(W_j) dG(W_j)$.

III. Description of the Equilibrium

I now turn to the description of the equilibrium. Theorem 1 describes the equilibrium in the trading period (i.e., gives the price and demand for stocks for a given level of market risk tolerance and aggregate information). Theorem 2 describes the equilibrium in the planning period (i.e., the participation and information acquisition decisions).

A. Price and Demand for Stocks

Theorem 1. Price and Demand for Stocks

Assume the participation and information decisions have been made (i.e., $n$ and $i$ are given). There exists a linear rational expectations equilibrium.

i) The equilibrium price is given by

$$\begin{align*}
\text{RP} &= P_0(\mu, n) + P_{\Pi}(\mu, n)(\Pi - \mu \theta), \\
h_0(\mu) &\equiv \frac{1}{\sigma_0^2} + \frac{1}{\mu^2 \sigma_0^2}, \\
h(\mu, x) &\equiv h_0(\mu) + x, \\
\tilde{h} &\equiv h \left( \frac{i}{n}, - \frac{1}{\mu \sigma_0^2}, \right) + \frac{e}{R n}, \\
(6) \quad P_0 &\equiv \frac{1}{\tilde{h}} \left( \frac{E(\Pi)}{\sigma_0^2} + \frac{E(\theta)}{\mu \sigma_0^2} \right), \\
P_{\Pi} &\equiv \left( 1 - \frac{1}{h_0(\mu)} \right), \quad \text{and} \quad \mu_i \equiv 1.
\end{align*}$$

(7) $P_0 \equiv \frac{1}{\tilde{h}} \left( \frac{E(\Pi)}{\sigma_0^2} + \frac{E(\theta)}{\mu \sigma_0^2} \right)$, $P_{\Pi} \equiv \left( 1 - \frac{1}{h_0(\mu)} \right)$, and $\mu_i \equiv 1$.

ii) The optimal stockholding for a participating investor $j$ with a signal of precision $x_j$ (possibly equal to 0) is given by

$$T_j = \tau(W_j) \frac{E(\Pi | F_j) - \text{RP}}{\text{var}(\Pi | F_j)} = \tau(W_j) \left( \tilde{h} P_0 + \frac{1}{\mu \sigma_0^2}(\Pi - \mu \theta) + x_j S_j - \text{RP} h(\mu, x_j) \right).$$

I briefly highlight a few properties of Theorem 1. The equilibrium price reveals $\Pi - \mu \theta$, a noisy signal for the log-payoff $\Pi$ with error $\mu \theta$. Hence, $\mu$
measures the noisiness of the price system (for a given $\sigma^2_p$). The variables $h_0(\mu)$, $x$, and $h(\mu, x)$ measure the precisions coming, respectively, from the price, the private signal, and both signals (the precisions simply add up), $i/n$ is a measure of the average private information in the market, and $\bar{h}$ is the average precision from both signals, adjusted for the supply elasticity, $e/(nR)$. The equilibrium price results from the combination of signal extraction and compensation for risk. In particular, the risk premium, $E(\Pi - R) = [E(\theta) + eE(\Pi)/R]/(n\bar{h})$, is decreasing in the market risk tolerance $n$ and the adjusted average precision of the stock $\bar{h}$.

B. Participation and Information Decisions

Theorem 2 describes the participation and information decisions. Because the entry and information costs are fixed and absolute risk tolerance is increasing in wealth, wealthier agents are more likely to participate and to acquire information. Accordingly, let $W^{N,A}$, $W^{N,P}$, and $W^{P,A}$ be the levels of wealth that make an agent indifferent between being a nonstockholder and an active stockholder, between being a nonstockholder and a passive stockholder, and between being a passive and an active stockholder, respectively. These wealth thresholds are functions of noise, $\mu$, and market risk tolerance, $n$, and are computed in the Appendix.

**Theorem 2. Participation and Information Decisions**

The equilibrium is of two possible types depending on the relative values of the wealth thresholds $W^{N,A}$, $W^{N,P}$, and $W^{P,A}$:

i) If $W^{P,A} > W^{N,A} > W^{N,P}$ then only agents with wealth above $W^{N,P}$ participate and only agents with wealth above $W^{P,A}$ acquire information. Agents with wealth between $W^{N,P}$ and $W^{P,A}$ participate but are passive. This is the “active-passive equilibrium.”

ii) If $W^{P,A} < W^{N,A} < W^{N,P}$, then only active agents participate and their wealth is above $W^{N,A}$. Agents with wealth below $W^{N,A}$ do not participate. This is the “all-active equilibrium.”

The optimal precision level $x_j$ for an active investor with wealth $W_j$ is given by

$$C'(x_j) = \frac{1}{2R} \tau(W_j)A(\mu, n),$$

where $A$ is increasing in $\mu$ and decreasing in $n$ as shown in the Appendix.

Theorem 2 states that there are only two possible configurations for the thresholds $W^{N,A}$, $W^{N,P}$, and $W^{P,A}$ depending on whether it pays to participate without acquiring information. Each configuration corresponds to an equilibrium type (Figure 2). Agents are ranked according to their initial wealth: poorer agents stay out of the market, agents with intermediary wealth participate and are passive (in the active-passive equilibrium), and wealthier agents participate and are active. It follows that in active-passive equilibrium, $n = \int_{W^{N,P}}^{W^{P,A}} \tau(W_j)dG(W_j)$ and $i = \int_{W^{P,A}}^{W^{N,A}} x(W_j)\tau(W_j)dG(W_j)$, while in an all-active equilibrium $n = \int_{W^{P,A}}^{W^{N,A}} \tau(W_j)dG(W_j)$ and $i = \int_{W^{N,A}}^{W} x(W_j)\tau(W_j)dG(W_j)$, where the integrating bounds, $W^{N,A}$, $W^{N,P}$,
and $W^{PA}$ depend on $n$ and $\mu$ (recall that $\mu i \equiv 1$). Note that degenerate active-passive equilibria are possible. For example, there may be no stockholders (if $\bar{W} < W^{NP}$) or all stockholders may be passive (if $\bar{W} < W^{PA}$).

**FIGURE 2**

Participation and Information Decisions for Different Wealth Levels

![Diagram showing participation and information decisions for different wealth levels in the active-passive equilibrium (left panel) and the all-active equilibrium (right panel).](image)

Figure 2 shows participation and information decisions for different wealth levels in the active-passive equilibrium (left panel) and the all-active equilibrium (right panel).

The function $A$ is defined in section B of the Appendix, and it measures the marginal benefit of private information. Specifically, it is the marginal increase in the expected square Sharpe ratio from a marginal increase in precision. The assumptions on the information cost function guarantee that there exists at most one precision level satisfying the first-order condition (9). The equation, illustrated in Figure 3, states that at the optimum the gain from a small increase in precision is exactly offset by its extra cost. The equation shows that the optimal precision level is increasing with absolute risk tolerance, which by assumption is increasing with wealth. Thus, wealthier investors acquire more information.

Theorem 2 shows that the risk sharing and information effects are intimately related. The extent of risk sharing depends on the level of aggregate information: $W^{NA}$ and $W^{NP}$ are decreasing functions of noise $\mu$, as more information deters participation. Symmetrically, the optimal precision $x_j$ is decreasing in the market risk tolerance $n$. Indeed, the larger is $n$, the greater is the aggregate demand for stocks and hence the greater the price (on average). Therefore, when $n$ increases, an investor owns fewer stocks and finds information less valuable. In this economy, the provision of information depends on market participation.\textsuperscript{18}

The empirical evidence on the distribution of assets across households suggests that both entry and information costs are important in shaping portfolios. Wealth is by far the most significant determinant of household participation, pointing to an entry cost. For example, Bertaut and Starr-McCluer (2000) find that a

\textsuperscript{18}There is no such link under CARA preferences. The first-order condition is then $2RC'(x) = \tau/h(\mu, x)$, from which $E(\theta)$ and $n$ are absent.
doubling of financial wealth leads to a 26% increase in the probability of stock ownership. Equation (9), which establishes that wealthier households acquire more information, provides means of assessing the relevance of the information cost. Its observable implications are threefold. First, survey evidence suggests that information expenditure increases with income. Lewellen, Lease, and Schlabaum (1977) report that the money spent by customers of a large U.S. retail broker on financial periodicals, investment research services, and professional counseling increases with income. Donkers and Van Soest (1999) document, using a survey of Dutch households, that interest in financial matters is strongly positively correlated to income. Second, the model predicts that wealthier households invest a larger share of their wealth in stocks (combining equation (8) with equation (9)). Several papers estimate the elasticity of portfolio shares with respect to wealth to be positive (around 0.1). Third, the model implies that wealthier households achieve a higher risk-adjusted return on their portfolio. To my knowledge, no empirical study of U.S. households examines this question. However, Massa and Simonov (2003) combine several data sets to create a comprehensive sample of Swedish households that includes, in particular, information on wealth (financial and nonfinancial), stockholdings (direct and indirect), and capital gains and losses. They compute the Sharpe ratios for the different wealth groups and find that the ratios are larger for wealthier households. Overall, the evidence

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19 On another front, research in accounting shows that small trades react less to earnings news than large trades, suggesting that wealthier investors (i.e., investors who place large orders) process the news and adjust their orders faster than poorer investors (Cready (1988) and Lee (1992)).

20 See Vissing-Jørgensen (2002), Bertaut and Starr-McCluer (2000), or Perraudin and Sørensen (2000). An alternative explanation for the observed positive elasticity is that relative risk aversion is decreasing with wealth. Peress (2004) shows that this hypothesis does not hold in the data.

21 Yitzhaki (1987) goes only part of the way. He documents that returns and their standard deviation both increase with income but he does not compute Sharpe ratios.
confirms the importance of both the entry and information costs for household portfolio choices. Next, I examine the consequences of changing these costs.

IV. Changing Costs, Participation, and the Distribution of Returns

This section analyzes the effects of changes in the costs, \( F \) and \( C \), on participation and the distribution of returns. Following Theorem 2, the equilibrium is one of two types, depending on whether passive agents participate. Therefore, I begin in Section IV.A with an analysis of how the equilibrium type changes with the costs. I then turn in Section IV.B to the consequences on participation and the distribution of returns. Section IV.C discusses some generalizations of the model.

A. Equilibrium Type

I examine how the equilibrium type changes when the participation costs, \( F \) and \( C \), vary, holding fixed the other exogenous parameters. Let \( c \) be a parameter indexing the family of information costs functions \( C(\cdot, c) \), where a larger \( c \) corresponds to a shift upward in the cost function. For example, if \( C(x) = c_0 + \frac{1}{2}c_1 x^2 \) on \([0, \infty[\) for \( c_0 \geq 0 \) and \( c_1 > 0 \), then a larger \( c_0 \) or \( c_1 \) implies a higher cost for all \( x > 0 \). Figure 4 illustrates how the different types of equilibria are distributed in the \( F-c \) space. There are basically two regions, each corresponding to an equilibrium type. At point A, there is no entry cost (\( F = 0 \)) so everybody participates but the information cost is so large that nobody acquires information. This is a degenerate active-passive equilibrium, where information is symmetric and the stock is valued based on the common priors. As the entry cost \( F \) increases keeping \( c \) constant (horizontal move), the marginal investor (i.e., the least wealthy) exits the market thereby increasing the gains from trade for the remaining stockholders. When \( F \) reaches point B, the wealthiest investors begin to purchase information. The equilibrium is active-passive with active and passive investors coexisting. As the entry cost \( F \) increases to point C, the number of actives rises and the number of passives falls. At point D, there are no passive investors left and the equilibrium becomes an all-active equilibrium. As \( F \) increases further, some active investors have to exit the market. Note that there will always be some investors participating no matter how large \( F \) is because the asset is in fixed supply.

To complete the description, I now increase \( c \) keeping \( F \) fixed (vertical move). When the information cost increases, fewer investors acquire information. At point E, where only active investors participate, the increase in \( c \) forces the marginal active investor to exit. At point C, where both active and passive investors coexist, marginal active stockholders are deterred from acquiring information and some nonstockholders are induced to enter the market. This point is explained in the next section.

B. Participation and the Distribution of Returns

Before I describe in the next two subsections the general equilibrium implications of a fall in costs, it may be insightful to go over two polar cases in partial
equilibrium. At one end of the spectrum, suppose the entry cost falls while the aggregate level of information is held constant (i.e., $\mu$ is constant). The risk sharing effect then operates alone as in Merton (1987) or Hirshleifer (1988). Participation rises while the equity premium and the variance of returns fall. At the other end of the spectrum, suppose the information cost falls while the number of investors is held constant (i.e., $n$ is constant). The only effect is the information effect as in Grossman and Stiglitz (1980) or Verrecchia (1982). The amount of information increases (i.e., $\mu$ and aggregate risk fall) and again the equity premium and the variance of returns fall.

When both the risk sharing and information effects are allowed to operate, the implications are ambiguous. Indeed, investors acquire less information when risk sharing is broader and they are reluctant to participate when information is too precise. The point of Theorems 3 and 4 is precisely to evaluate the net effect of a fall in costs taking into account both the risk sharing and the information effects. To emphasize how the entry and information costs can have different implications, I begin with the active-passive equilibrium. In the all-active equilibrium, agents participate if and only if they acquire information, so a fall in the information cost will increase participation just as a fall in the entry cost does.

1. Active-Passive Equilibrium

Theorem 3 shows that changes in entry and information costs have strikingly different implications on participation and the equity premium in an active-passive equilibrium.
Theorem 3. Changes in the Entry and Information Costs in an Active-Passive Equilibrium

i) If the entry cost falls (while the information cost does not change), passive agents enter the market and active stockholders become passive: participation rises; noise rises (prices are less informative); the equity premium falls; and the variance of returns rises if the information effect dominates the risk sharing effect, and falls otherwise.

ii) If the information cost falls (while the entry cost does not change), some passive stockholders become active while some passive stockholders exit the market: participation falls; noise falls (prices are more informative); the equity premium falls if the risk sharing effect dominates the information effect, and rises otherwise; and the variance of returns falls.

Theorem 3 looks at changes in the equilibrium level of participation induced by an exogenous change in a cost (e.g., $F$), holding fixed all other exogenous variables (e.g., $C$) and allowing all endogenous variables to adjust (e.g., $n$ and $\mu$). Figure 5 illustrates the changes in the wealth thresholds, $W^{NP}$, $W^{NP}$, and $W^{PA}$ triggered by a fall in the costs. Recall that agents with wealth above $W^{NP}$ participate and agents with wealth above $W^{PA}$ acquire information.

FIGURE 5
Effect of a Fall in the Entry and Information Costs

A fall in the entry cost, $F$

$W^{NP}$ $W^{PA}$ $W^*_j$

A fall in the information cost, $c$

$W^{NP}$ $W^{PA}$ $W^*_j$

Figure 5 shows the effect of a fall in the entry and information costs on the thresholds for participation and information acquisition in an active-passive equilibrium. Agents with wealth above $W^{NP}$ participate and agents with wealth above $W^{PA}$ are active.

The key to Theorem 3 is that in an active-passive equilibrium, both active and passive investors participate. Both are directly affected by a change in the entry cost $F$, but only the active investors are directly affected by a change in the information cost $C$. As the entry cost $F$ decreases, the marginal nonstockholder responds by entering the market, passive. Participation, aggregate demand, and price all rise, while the risk premium falls (the risk sharing effect). Furthermore,
the entry of new investors reduces the benefit of private information to incumbent active stockholders who now have fewer stocks in their portfolio. Therefore, they decrease the precision of their signals and the marginal active investor becomes passive. This results in less aggregate information, more noise, and more aggregate risk (the information effect). The information effect dampens the fall in the equity premium, but does not reverse it. This need not be the case for the variance of returns, which could increase if the information effect dominates the risk sharing effect. The Appendix features an example of such a situation. To summarize, a fall in the entry cost induces the entry of passive investors who progressively take over the active investors, pushing up the equilibrium price, noise, and possibly return variance.

The effect of a fall in the information cost $C$ is more complex because it does not affect passive investors directly. Indeed, as $C$ decreases, the marginal passive investor becomes active (information is now sufficiently cheap). She trades more aggressively and the risk premium falls to clear the market. Aggregate risk and the variance of returns are also lower (the information effect). But the marginal stockholder no longer finds it profitable to participate (this investor is passive so she does not benefit from the fall in the cost of information). Consequently, participation falls when the information cost falls. This striking result rests on a key point: the participation of both active and passive investors. The fall in participation dampens the fall in return variance without reversing it. This need not be the case for the equity premium, which may increase if the risk sharing effect dominates the information effect. The Appendix features an example of such a situation.

When both costs fall, their effects on participation work in the opposite directions, neutralizing each other. Their effect on the equity premium and the variance of returns can operate in the same or opposite directions, magnifying or dampening changes. For example, a fall in both costs could lead to a large fall in the equity premium and an increase in the variance of returns if the information effect dominates the risk sharing effect. The net effect ultimately depends on the relative magnitude of the fall in the costs.

I conjecture that the results obtain in a dynamic version of the model where the stock return does not only involve next period’s dividend payment but also the stock’s resale price. In particular, this may create a difficulty for the variance of returns. When more information is acquired, the current price tracks future dividends and prices more closely, thereby reducing the return variance (as in the static model). But the variance of future prices also increases since future prices track more closely dividends even further into the future. However, because future prices are discounted at the risk-free rate, the former effect dominates the later (e.g., Pagano (1986) and West (1988)). As Campbell, Lettau, Malkiel, and Xu (2001), p. 39 put it, “improved information about future cashflows increases

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22This happens if the average supply of stocks, $E(\theta)$, is large or if the demand for information, $x_t$, is sufficiently sensitive to changes in the marginal benefit of information, $A$. The latter in particular happens when the marginal cost of information is almost flat, i.e., $C''$ is small as the first-order condition (9) shows. In this case, a small decline in $A$ generates a large fall in private precision.

23This happens in particular if absolute risk tolerance has a big slope, i.e., if $\tau'$ is large as equation (9) shows. In this case a fall in participation results in a large decline in aggregate risk tolerance.
the volatility of the stock price, but it reduces the volatility of the stock return because news arrives earlier, at a time when the cashflows in question are more heavily discounted.” I now turn briefly to the all-active equilibrium.

2. All-Active Equilibrium

In an all-active equilibrium, all stockholders are active so \( F \) and \( C \) play similar roles. To keep things simple, I focus on parallel shifts in the cost of information and do not consider changes in the marginal cost of information.\(^{24}\) I suppose \( C(x) \equiv c_0 + \tilde{C}(x) \) on \([0, \infty[\) and \( C(0) \equiv 0 \), where \( \tilde{C} \) satisfies the assumptions made previously on \( C \) (i.e., \( \tilde{C}^{\prime}(\cdot) \geq 0 \), \( \tilde{C}^{\prime\prime}(\cdot) > 0 \), and \( \lim_{x \to \infty} \tilde{C}^{\prime}(x) = +\infty \). I examine the consequences of changes in \( c_0 \) and \( F \) keeping \( \tilde{C} \) unchanged. For example if \( C(x) \equiv c_0 + \frac{1}{2}c_1x^2 \), then I analyze the implications of a change in \( c_0 \) keeping \( c_1 \) constant. Clearly \( c_0 \) and \( F \) are interchangeable in an all-active equilibrium where investors participate if and only if they acquire information.

*Theorem 4. Changes in Entry and Information Costs in an All-Active Equilibrium*

Suppose the entry or information costs fall. Then, participation rises. Furthermore, if the information effect is weak, then noise falls (prices are more informative), the equity premium falls, and the variance of returns falls.

As entry or information costs drop, the marginal nonstockholder is lured into the market and she chooses to be active. Therefore, participation rises and risk sharing broadens. The information effect is itself ambiguous! On one hand, the new active stockholders contribute to aggregate information. On the other hand, information is less profitable as in the active-passive equilibrium so incumbent stockholders respond to the new entrants by reducing the precision of their signals. The effects on noise, the equity premium, and the variance of returns are ambiguous. If the information effect is weak, i.e., incumbent stockholders reduce their precision by less than the gain from new stockholders, then aggregate information rises and noise, the equity premium, and return variance fall. If the information effect is strong enough, the opposite happens. The equity premium falls by a smaller amount and may even increase.\(^{25}\)

C. Generalizations

The results are robust to a number of generalizations. First, lower noise in the model allows investors to know more about the stock earlier, thereby shifting risk from the consumption to the trading period. This may result in more risk and larger gains from trade in early periods. Accordingly, I insert a period between the planning and trading periods to allow stockholders to trade before signals are observed. I assume for simplicity that risk tolerance is a function of initial wealth and does not change over time. Under mean-variance preferences, the expected utility in the first trading period is linear in the level of wealth at the start of the

\(^{24}\)A fall in the marginal cost of information increases the number of active agents, just like a fall in the entry cost does, but it also increases the precision of private signals, which are now cheaper.

\(^{25}\)As in the active-passive equilibrium, this occurs if the demand for information is sufficiently sensitive, i.e., if \( C^{\prime\prime} \) is small. In this case, a small decline in the marginal benefit of information, \( A \), generates a large fall in private precision (equation (9)).
period. Effectively, the agent is risk neutral. The risk premium equals zero and the agent is indifferent to whether there is more or less risk in this period. Hence, the results in the paper obtain by construction because of the preference structure.

More generally, there are two reasons why better information does not lure investors into the market. First, better information reduces the gains from trade in the second trading period as in the two-period model. Second, better information can in fact reduce the variance of the price in the second trading period and therefore the risk premium in the first trading period. Indeed, price is a function of the two sources of risk, the terminal payoff and noise. Better information not only increases dependence on the first source (thereby increasing the price variance and risk premium), but also decreases dependence on the second (thereby reducing the price variance and risk premium).

Second, the results extend to other sources of heterogeneity. The only requirement is that the characteristics that differentiate agents affect their demand for stocks. For example, high risk aversion, high information costs (a proxy for little education), high exposure to background risk, or low correlation between traded and non-traded assets all reduce the demand for stocks.

Finally, the effect of costs on participation obtain under CARA preferences. In particular, a fall in the information cost reduces participation in an active-passive equilibrium. Again, the mechanism merely relies on the coexistence of two groups of stockholders, passive and active. A falling information cost benefits the active, who acquire more information and more stocks, but it harms the passive, who face a less advantageous risk-return trade-off. Therefore, the marginal stockholder (who is passive) exits the market. The only difference under CARA preferences is that the entry cost no longer generates an information effect. The demand for information is unrelated to the expected supply of shares and to the market risk tolerance (and therefore to participation). This implies, in particular, that the variance of returns always falls when the entry cost falls.

V. Conclusion

Stock market participation has increased remarkably over the second half of the 20th century. Yet, only half of U.S. households own stocks today. The low participation rate and its increase are often rationalized by high and declining participation costs. This paper splits these costs into two components, an information

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26 There are several reasons why quadratic preferences may be more appropriate than CARA to address the issues raised in this paper. First, on an intuitive level, the demand for information should depend on the extent of risk sharing. Indeed, the production of information displays increasing returns to scale because its cost, unlike its benefit, is independent of the investment scale. So, investors expecting to trade less should acquire less information. CARA preferences imply that the number of active investors and the precision of their signals do not change in an active-passive equilibrium as the stock supply falls or the number of investors grows (the first-order condition for precision is $2\text{RC}^{-1}(\tau) = \tau / h(\mu, \tau)$, from which $E(\theta)$ and $n$ are absent). In contrast, quadratic preferences imply that they fall (A increases with $E(\theta)^2$ and decreases with $n$). Second, on a theoretical level, quadratic preferences can be seen as a second-order approximation to general preferences when risk is small. Peress (2004) establishes the conditions for such an approximation to be valid. From this point of view, CARA utility appears to be a knife-edge case, while quadratic utility captures features of more general preferences. Finally, on an empirical level, the increase in return volatility derived under quadratic preferences corresponds to the evidence reported in Campbell, Lettau, Malkiel, and Xu (2001). CARA preferences do not lead to such a volatility increase.
and an entry cost, to disentangle their general equilibrium implications on participation and the distribution of returns. The information cost consists of the time and resources spent by investors to predict stock performance and the entry cost includes all other costs such as trading costs and fees. I solve a Grossman and Stiglitz (1980) economy where information is purchased and partially revealed by the price and investors have to pay an entry cost to hold stocks.

The model shows that the equilibrium can be of two possible types depending on the levels of the information and entry costs. When the entry cost is low or the information cost is high, both active and passive stockholders, i.e., those who purchase a private signal and those who do not, coexist (the active-passive equilibrium). Otherwise, only active investors hold stocks (the all-active equilibrium). The main model assumes that agents differ in their wealth and that absolute risk tolerance increases with wealth. It follows that poorer agents stay out of the market, agents with intermediary wealth enter but are passive (the active-passive equilibrium), and wealthier agents enter and are active. Furthermore, wealthier stockholders acquire more information.

I then analyze the consequence of falling costs. The active-passive equilibrium yields more interesting results because the three groups of stockholders react very differently. A fall in the entry cost induces the entry of passive investors who progressively take over the active, pushing up the equilibrium price, noise, and possibly return variance. As argued in the Introduction, the U.S. has witnessed in the postwar period a fall in trading costs and management fees, an increase in stock market participation, a fall in expected returns, an increase in individual return variances, and a surge in passive investing relative to active investing.

Interestingly, a fall in the information cost increases the participation of the active investor but decreases the participation of the passive investor so that the net effect is a fall in participation. The reason is that, as she faces a lower information cost, the marginal passive investor becomes active and consequently trades more aggressively. It is no longer profitable for the marginal stockholder to participate. She does not benefit from the lower information cost because she is passive, and she exits the market. At the same time, there is more aggregate information and less noise so the variance of returns declines. This striking result rests on a key point, the joint participation of both active and passive investors, and is shown to obtain in an economy with a different source of heterogeneity and preferences.

The model could be extended to study the effect of disclosure requirements and financial education on the dispersion of stock ownership. These are means for government agencies and regulatory bodies to indirectly control the cost of acquiring information. In particular, one could examine how accounting rules (US GAAP vs. German or French) affect equity ownership, as currently debated in several European countries. This issue is also of interest to corporations that often prefer a broad investor base to a concentrated one.

Exploring further the decision to own stocks, it would be interesting to study a stockholder’s choice of direct vs. indirect ownership (i.e., individual stocks vs. mutual and pension funds). Indeed, an important change in the U.S. financial sector has been the institutionalization of the stock market. This trend started

27Heaton and Lucas (1999) argue that the increase in diversification induced by greater indirect ownership can explain in part the stock market boom of the 1990s.
after World War II but has accelerated in the last several decades. For example, the share of financial wealth allocated to indirect equity increased from 13% to 31.4% over the last 10 years while the share of individual stocks increased only from 15.6% to 23% (1989 and 1998 Surveys of Consumer Finances). One could extend the model by allowing investors to acquire information on different assets, each associated with a different information technology. Direct stockholders would gather information on individual stocks, which is expensive since it involves firm-specific information while fundholders would acquire information on the market as a whole, which is cheap. Alessie, Hochguertel, and Van Soest (2001) provide evidence consistent with this view. They show that the probability of owing mutual funds increases with income but less than the probability of owing individual stocks does. Such a model would contribute to understanding portfolio management and its consequences on stock prices.

Appendix

A. Proofs of Theorems

Proof of Theorem 1. Price and Demand for Stocks. To prove Theorem 1, guess that the equilibrium price is given by equations (6)-(8) and solve for the optimal portfolio of a stockholder (recall that the information choice is taken as given at this stage). The first step in the investor’s problem is to estimate the mean and variance of the stock’s payoff using the equilibrium price (or equivalently $\xi \equiv \Pi - \mu \theta$) and her private signal $S_i$. Then I turn to portfolio choices and finally to market clearing.

Signal Extraction. For the price function given in equation (6) ($P$ is linear in $\Pi$ and $\theta$), agent $j$’s conditional mean and variance of $\Pi$ are

$$\text{var} (\Pi | \mathcal{F}_j) = \frac{1}{h(\mu, x_j)} = \frac{1}{h_j} \quad \text{and} \quad E(\Pi | \mathcal{F}_j) = a_0 + a_0 \xi + a_0 S_j$$

where $a_0 h_j = \frac{E(\Pi)}{\sigma_\Pi^2} + \frac{E(\theta)}{\mu \sigma_\theta^2} = P_0 \tilde{h}, \quad a_0 h_j = \frac{1}{\mu^2 \sigma_\theta^2}, \quad \text{and} \quad a_0 S_j \equiv x_j.$

Intuitively, $\text{var}_j(\Pi | \mathcal{F}_j)$ falls as the precision of either the private signal, $x_j$, or the precision of the public signal, $1/\mu$, increase. Similarly, $E_j(\Pi | \mathcal{F}_j)$ is a weighted average of priors and public and private signals where the weight on the private signal (the public signal) is increasing in $x_j$ (in $1/\mu$). If the investor does not acquire private information, set $x_j = 0$ and $S_i$ vanishes from the equations.

Portfolio Choice. Under mean-variance preferences, the optimal position in the stock is given by

$$\frac{\tau(W_j) E_j(\Pi | \mathcal{F}_j) - RP}{\text{var}_j(\Pi | \mathcal{F}_j)}.$$

Plugging in the above expression for $E_j(\Pi | \mathcal{F}_j)$ and $\text{var}_j(\Pi | \mathcal{F}_j)$ leads to equation (8).

Market Clearing. The equilibrium price clears the market for stocks. Aggregating equation (8) over all participating investors yields the aggregate demand for stocks,

$$\int T_i = n \left[ \frac{E(\Pi)}{\sigma_\Pi^2} + \frac{E(\theta)}{\mu \sigma_\theta^2} + \frac{1}{\mu^2 \sigma_\theta^2} (\Pi - \mu \theta) \right] + \Pi i - RP(n h_0 + i),$$

where the term $\Pi i$ comes from applying the law of large numbers to the sequence of independent random variables $\{\tau_j x_j\}$. Formally, I follow He and Wang (1995) in defining a charge space $(\mathcal{J}, \mathcal{P}(\mathcal{J}), m)$, where $\mathcal{P}(\mathcal{J})$ is the collection of subsets of $\mathcal{J}$, and
where $\mathbf{r}_i \left(X_j \in \mathbf{F}_j \right)$ is investor $j$'s Sharpe ratio, a function of $S_j$ and $P$ (and $x_j$). $E_i(z_j^2)$ is the squared Sharpe ratio investor $j$ expects to achieve in the planning period, which no longer depends on $S_j$ and $P$ but is still a function of $x_j$. Integrating over the distributions of $S_j$ and $P$ yields

$$
E_i \left(z_j^2 \right) = h(x_j)A - 1,
$$

where $A(\mu, n) \equiv \left(1/(nh)^2\right)[E(\sigma^2) + h_0n^2 + 2n(e/R + 1/\mu)]$ and $z \equiv \theta + e\Pi / R$.

$A$ measures the marginal benefit of private information. Differentiating $A$ and rearranging yields

$$
\frac{\partial A}{\partial n} = \frac{2}{(nh)^2} \left[ E(\sigma^2)^3 h_0 + \frac{\sigma^2}{\sigma^2_{\tilde{\theta}}} \left( \frac{e\sigma^2_{\tilde{\theta}}}{R\mu\sigma^2_{\tilde{\theta}}} - 1 \right) \right] < 0 \quad \text{and} \quad \frac{\partial A}{\partial \mu} = \frac{2}{(nh)^2\mu^2} \left[ E(\sigma^2) + \left( \frac{2\sigma^2}{\sigma^2_{\tilde{\theta}}} + 1 \right) \frac{n h_0 n^2}{\mu \sigma^2_{\tilde{\theta}}} + \frac{3n^2}{\mu^2 \sigma^2_{\tilde{\theta}}} + \frac{2en}{R\mu\sigma^2_{\tilde{\theta}}} + e \right] > 0,
$$

so $A$ is an increasing function of $\mu$ (bigger $\mu$ implies that prices are less revealing) and a decreasing function of $n$ (smaller $n$ implies larger gains from trade).

To solve for the optimal precision level, I maximize the expression in (10) with respect to $x_j$ taking $\mu$ and $n$ (hence $A$) as given. If there is an interior solution, $x_j$, it must satisfy the first-order condition $2RC'(x_j) = \tau(W_j)A$. By assumption, $C$ is convex in $x_j$ so the right-hand side is increasing while the left-hand side is constant. This implies that a unique interior solution exists if and only if $C'(0) < \tau(W_j)A/2R$. Otherwise the solution will be the corner 0. Therefore, there exists a level of wealth, $W^{P,A}(\mu, n) \equiv \tau^{-1}(2RC'(0)/A)$, such that investors acquire information if and only if $W_j > W^{P,A}$. Therefore, only the richest investors purchase information. Note that if $C$ is continuous around 0, then $W^{P,A} \equiv x(W^{P,A}) = 0$.

**Participation.** By definition, $W^{N,A}(\mu, n)$ and $W^{N,P}(\mu, n)$ are the levels of wealth that make an agent indifferent between being a nonstockholder and an active stockholder, $V^{A}(W^{N,A}, \mu, n) = RW^{N,A}$, and between being a nonstockholder and a passive stockholder, $V^{P}(W^{N,P}, \mu, n) = RW^{N,P}$. $W^{N,P}$ can be solved explicitly, $W^{N,P} \equiv \tau^{-1}(2RF/(h_0 A - 1))$, while $W^{N,A}$ is defined together by $R \left[ F + C(\sigma^2_{\tilde{\theta}}) \right] = \frac{\tau}{2}(W^{N,A}) \left[ h(\sigma^2_{\tilde{\theta}})A - 1 \right]$ and
2RC'(x^{NA}) = \tau(W^{NA})A. Substituting out W^{NA} yields an implicit equation in x^{NA}, F + C(x^{NA}) = C'(x^{NA})[h(x^{NA}) - (1/A)].

**Proof of Theorem 3. Changes in the Entry and Information Costs.** I consider first the active-passive equilibrium and then the all-active equilibrium.

1. **Active-Passive Equilibrium.** In an active-passive equilibrium, W^{PA} > W^{NA} > W^{NP}; agents with wealth above W^{NA} participate and agents with wealth above W^{PA} acquire information. Agents with wealth between W^{NP} and W^{PA} participate but do not acquire information. Hence, \( n = \int_{W^{NP}}^{W^{PA}} \tau(W_j)dG(W_j) \) and \( i = \int_{W^{PA}}^{W^{NP}} x(W_j)\tau(W_j)dG(W_j) \).

Changes in aggregate risk tolerance \( n \) and noise \( \mu \):

**Information aggregation equation.** Differentiating the equation above defining the aggregate precision \( i \) in an active-passive equilibrium yields

\[
\frac{di}{dp} = -\tau(W^{PA})x(W^{PA})g(W^{PA})dW^{PA} + \int_{W^{PA}}^{W^{NP}} dx(W_j)\tau(W_j)dG(W_j),
\]

where the first term comes from differentiating the upper bound in the integral (extensive margin) and the second from differentiating the integrand (intensive margin). Differentiating the expression for \( W^{PA} \) yields

\[
dW^{PA} = \frac{\tau(W^{PA})}{\tau'(W^{PA})} \left[ -\frac{dA}{A} + \frac{1}{C'(0)} \frac{\partial C'(0)}{\partial c} dc \right].
\]

Recall that \( (\partial A)/(\partial \mu) > 0 \) while \( (\partial A)/(\partial n) < 0 \), so \( W^{PA} \) is decreasing in \( \mu \) and increasing in \( n \) holding \( c \) constant. Differentiating equation (9) delivers

\[
dx_j = \frac{1}{C''(x_j)} \left[ \frac{1}{2R} \tau(W_j) \left( \frac{\partial A}{\partial \mu} dm + \frac{\partial A}{\partial n} dn \right) - \frac{\partial C'(x_j)}{\partial c} dc \right],
\]

where \( x_j \) is increasing in \( \mu \) and decreasing in \( n \) holding \( c \) constant. Finally recall that, in equilibrium, \( \mu i \equiv 1 \), so \( dm = -\mu^2 di \). Putting all these elements together,

\[
(12) \quad B_n dn + B_\mu d\mu = B_c dc,
\]

where

\[
J \equiv \int_{W^{NP}}^{W^{PA}} \tau(W_j)^2 \frac{2R}{2C''(x_j)}dG(W_j) + \frac{\tau(W^{PA})^2 x(W^{PA})g(W^{PA})}{A \tau'(W^{PA})} > 0,
\]

\[
B_n = \frac{\partial A}{\partial n} \mu^2 J > 0, \quad B_\mu \equiv 1 + \frac{\partial A}{\partial \mu} \mu^2 J > 0,
\]

and

\[
B_c = \mu^2 \int_{W^{PA}}^{W^{NP}} \tau(W_j) \left( \frac{\partial C'(x_j)}{\partial c} \right) dG(W_j) + \frac{2R \mu^2}{A \tau'(W^{PA})} \frac{\partial C'(0)}{\partial c} > 0.
\]

**Aggregate risk tolerance equation.** Similarly, differentiate the equation defining \( n \) to obtain

\[
dn = -\tau(W^{NP})g(W^{NP})dW^{NP}
\]

and the equation \( W^{NP} \equiv \tau^{-1}((2RF)/(h_0A - 1)) \) to obtain

\[
dW^{NP} = -\frac{2RF}{(h_0A - 1)^2 \tau'(W^{NP})} \left[ \left( h_0 \frac{\partial A}{\partial \mu} - \frac{2A}{\mu^2 \sigma^2} \right) d\mu + h_0 \frac{\partial A}{\partial n} dn \right]
\]

\[
+ \frac{2R}{(h_0A - 1) \tau'(W^{NP})} dF.
\]

Plugging back leads to

\[
D_n dn + D_\mu d\mu = D_c dF,
\]

(13)
where
\[ D_n = -\frac{\tau'(W^N P)}{\tau(W^N P)g(W^N P)} \left( \frac{h_0 A - 1}{2RF} \right) + h_0 \frac{\partial A}{\partial n} < 0, \]
\[ D_F = \frac{2R}{h_0 A - 1} > 0, \] and \[ D_\mu = h_0 \frac{\partial A}{\partial \mu} - \frac{2A}{\mu^2 \sigma_h^2} > 0. \]

Together equations (12) and (13) constitute a system of two linear equations in two unknowns \(dn\) and \(d\mu\), with solutions
\[ dn = \left( \frac{B_n D_F}{A} \right) dF - \left( \frac{D_n B_n}{A} \right), \]
\[ d\mu = \left( \frac{B_n D_F}{A} \right) dF + \left( \frac{D_n B_n}{A} \right). \]

This very useful relation can be established by replacing the coefficients with their expressions and dropping the term in
\[ \frac{\tau'(W^N P)}{\tau(W^N P)g(W^N P)} \]
since it is positive. It is then sufficient to show that
\[ h_0 \left( -\frac{\partial A}{\partial n} \right) \left( \frac{h_0 A - 1}{\mu^2 \sigma_h^2} \right) > -\frac{\partial A}{\partial n} \left( \frac{\partial A}{\partial \mu} \right) > -\frac{\partial A}{\partial n} \frac{n^2 J}{\left( 1 + \frac{\partial A}{\partial \mu} \mu^2 J \right)}. \]

This inequality is trivially satisfied since
\[ \frac{\partial A}{\partial \mu} > 0, \quad D_\mu = h_0 \frac{\partial A}{\partial \mu} - \frac{2A}{\mu^2 \sigma_h^2} > 0, \quad \text{and} \quad J > 0. \]

From the signs of the different coefficients, it follows that
\[ \frac{dn}{dc} > 0, \quad \frac{d\mu}{dc} > 0, \quad \frac{dn}{dF} < 0, \quad \text{and} \quad \frac{d\mu}{dF} < 0. \]

Thus, noise (\(\mu\)) and aggregate risk tolerance (\(n\)) increase if the information cost rises or if the entry cost falls.

Changes in participation \(N\), the equity premium \(Q\), and the variance of returns \(v^2\):

Participation. In an active-passive equilibrium, the participation rate, \(N\), is given by the number of stockholders with initial wealth above \(W^N P: N = G(W^N P)\), so \(dN/dF = 1/\tau(W^N P)dn\). It follows that \(dN/dF > 0\) and \(dF/dF < 0\). Thus, participation \(N\) increases if the information cost rises or if the entry cost falls.

Equity premium. The (unconditional) equity premium is given by
\[ Q = E(\Pi - RP) = \frac{E(\omega)}{nh}. \]

Hence
\[ \frac{\partial Q}{\partial n} = -\frac{Q^2 h_0}{E(\omega)} < 0 \quad \text{and} \quad \frac{\partial Q}{\partial \mu} = \frac{Q^2}{E(\omega)\mu^2} \left( 1 + \frac{n^2}{\mu^2 \sigma_h^2} \right) > 0. \]

It follows that
\[ dQ = \frac{\partial Q}{\partial n} dn + \frac{\partial Q}{\partial \mu} d\mu = \frac{1}{\Delta} \left[ \left( \frac{\partial Q}{\partial n} B_n + \frac{\partial Q}{\partial \mu} B_\mu \right) - \left( \frac{\partial Q}{\partial n} B_n + \frac{\partial Q}{\partial \mu} B_\mu \right) \right] dF. \]
Therefore,
\[
\frac{dQ}{dF} = \frac{1}{\Delta} \left( \frac{\partial Q}{\partial n} B_\mu - \frac{\partial Q}{\partial \mu} B_n \right) D_F > 0.
\]
The sign follows from
\[
- \frac{\partial Q}{\partial n} / \frac{\partial Q}{\partial \mu} > - \frac{\partial A}{\partial n} / \frac{\partial A}{\partial \mu} > -B_n/B_\mu.
\]
Similarly,
\[
\frac{dQ}{dc} = - \frac{1}{\Delta} \left( \frac{\partial Q}{\partial n} D_\mu - \frac{\partial Q}{\partial \mu} D_n \right) B_c.
\]
This expression has an ambiguous sign,
\[
\frac{dQ}{dc} > 0 \iff -D_n/D_\mu > - \frac{\partial Q}{\partial n} / \frac{\partial Q}{\partial \mu},
\]
where \(-D_n/D_\mu\) is larger when the risk sharing effect is stronger (e.g., when \(\tau'(W^{N^T})\) is greater). In other words, the equity premium falls with the entry cost. It only falls with the information cost if the risk sharing effect dominates the information effect.

**Variance of returns.** The variance of returns is given by \(v \equiv \text{var}(P - RP) = A - Q^2\). Hence,
\[
\frac{\partial v}{\partial n} = \frac{\partial A}{\partial n} - 2Q \frac{\partial Q}{\partial \mu} \quad \text{and} \quad \frac{\partial v}{\partial \mu} = \frac{\partial A}{\partial n} - 2Q \frac{\partial Q}{\partial \mu}.
\]
It follows that
\[
dv = \frac{\partial v}{\partial n} dn + \frac{\partial v}{\partial \mu} d\mu = \frac{1}{\Delta} \left[ \left( \frac{\partial v}{\partial n} B_\mu + \frac{\partial v}{\partial \mu} B_n \right) D_F dF - \left( \frac{\partial v}{\partial n} D_\mu + \frac{\partial v}{\partial \mu} D_n \right) B_c dc \right].
\]
Therefore,
\[
\frac{dv}{dc} = - \frac{1}{\Delta} \left( \frac{\partial v}{\partial n} D_\mu - \frac{\partial v}{\partial \mu} D_n \right) B_c > 0.
\]
The sign follows from
\[
- \frac{\partial v}{\partial n} / \frac{\partial v}{\partial \mu} < - \frac{\partial A}{\partial n} / \frac{\partial A}{\partial \mu} < -D_n/D_\mu,
\]
which itself follows from
\[
- \frac{\partial Q}{\partial n} / \frac{\partial Q}{\partial \mu} > - \frac{\partial A}{\partial n} / \frac{\partial A}{\partial \mu}. \quad \text{Similarly,} \quad \frac{dv}{dF} = \frac{1}{\Delta} \left( \frac{\partial v}{\partial n} B_\mu - \frac{\partial v}{\partial \mu} B_n \right) D_F.
\]
The sign of this expression is ambiguous:
\[
\frac{dv}{dF} < 0 \iff -B_n/B_\mu > - \frac{\partial v}{\partial n} / \frac{\partial v}{\partial \mu},
\]
where \(-B_n/B_\mu\) is larger when the information effect is stronger (e.g., when \(C''\) or \(\tau'(W^{P^A})\) are lower). The variance of returns falls with the information cost. It increases when the entry cost falls if the information effect dominates the risk sharing effect. It decreases otherwise.

2. **All-Active Equilibrium.** In an all-passive equilibrium, \(W^{P,A} < W^{N,A} < W^{N,P}\): only agents with wealth above \(W^{N,A}\) participate and they acquire information. Hence,
\[
n = \int_{W^{N,A}} W^{N,A} \tau(W_j) dG(W_j) \quad \text{and} \quad i = \int_{W^{P,A}} W^{P,A} \tau(W_j) dG(W_j).
\]

**Changes in aggregate risk tolerance \(n\) and noise \(\mu\):**
Information aggregation equation. Differentiating the equation above defining the aggregate precision \( i \) in an all-passive equilibrium yields

\[
di = -\tau(W^{N,A})x(W^{N,A})g(W^{N,A})dW^{N,A} + \int_{W^{N,A}}^{W_{W}} dx(W_{i})\tau(W_{i})dG(W_{i}),
\]

where the first term comes from differentiating the upper bound in the integral (extensive margin) and the second from differentiating the integrand (intensive margin). As above, differentiating equation (9) delivers

\[
dx_{i} = \frac{1}{C'(x_{i})} \left[ \frac{1}{2R} \tau(W_{i}) \left( \frac{\partial A}{\partial \mu} d\mu + \frac{\partial A}{\partial n} dn \right) - \frac{\partial C'(x_{i})}{\partial c} dc \right].
\]

Differentiating the two relations defining \( W^{N,A} \),

\[
R \left[ F + C(x^{N,A}) \right] = \frac{1}{2} \tau(W^{N,A}) \left[ h(x^{N,A})A - 1 \right] \quad \text{and} \quad 2RC'(x^{N,A}) = \tau(W^{N,A})A,
\]

yields

\[
dW^{N,A} = \frac{1}{A\tau(W^{P,A})} \left[ \tau'(W^{P,A}) \left( \frac{\partial A}{\partial \mu} d\mu + \frac{\partial A}{\partial n} dn \right) + 2R\frac{\partial C'(0)}{\partial c} dc \right].
\]

Finally, in equilibrium \( \mu_{i} \equiv 1 \), so \( d\mu = -\mu^{3} di \). Putting all these elements together,

\[
B'_{j}dn + B'_{\mu}d\mu = B'_{c}dc + B'_{F}dF,
\]

where

\[
J' \equiv \frac{(h_{0} + x(W^{N,A}))}{\left[ (h_{0} + x(W^{N,A}))A - 1 \right]} \frac{\tau(W^{N,A})}{\tau'(W^{N,A})} \left( \frac{\partial A}{\partial \mu} d\mu + \frac{\partial A}{\partial n} dn \right) + \int_{W^{N,A}}^{W_{W}} \frac{\tau(W_{i})}{2RC'(x_{i})} dG(W_{i}) > 0,
\]

\[
B'_{\mu} \equiv \frac{1}{\mu^{3}} + \frac{\partial A}{\partial \mu} J' - \frac{2\tau(W^{N,A})}{\left[ (h_{0} + x(W^{N,A}))A - 1 \right]} \frac{\tau'(W^{N,A})}{\mu^{3}\sigma_{\theta}^{2}} A > D'_{\mu}B'_{F}/R > 0
\]

\[
(D'_{\mu} \text{ is defined below}),
\]

\[
B'_{c} \equiv \frac{\partial C(x(W^{N,A}))}{\partial c} \frac{2\tau(W^{N,A})}{\left[ (h_{0} + x(W^{N,A}))A - 1 \right]} \frac{\tau'(W^{N,A})R}{\tau(W^{N,A})}
\]

\[
+ \int_{W^{N,A}}^{W_{W}} \frac{\tau(W_{i})}{C'(x_{i})} \frac{\partial C'(x_{i})}{\partial c} dG(W_{i}) > 0, \quad \text{and}
\]

\[
B'_{F} \equiv \frac{2R\tau(W^{N,A})}{\left[ (h_{0} + x(W^{N,A}))A - 1 \right]} \frac{\tau'(W^{N,A})}{\tau(W^{N,A})} > 0.
\]

I assume that \( C(x) = c_{0} + \tilde{C}(x) \) and consider changes in \( c_{0} : \frac{\partial \tilde{C}(x)}{\partial c_{0}} = 0 \) and \( \tilde{C}(x)/c_{0} = 1 \) for all \( x \). Hence, \( B'_{c} \equiv 1 + RB'_{F} > 0 \).

Aggregate risk tolerance equation. Similarly, differentiate the equation defining \( n \) in to obtain

\[
dn = -\tau(W^{N,A})g(W^{N,A})dW^{N,A}.
\]

Plugging back the expression for \( dW^{N,A} \) delivers

\[
D'_{j}dn + D'_{\mu}d\mu = D'_{c}dc + D'_{F}dF,
\]

where
where

\[ D'_n = -\frac{\tau(W^{N,A})}{2\tau(W^{N,A})g(W^{N,A})} \left[ (h_0 + x(W^{N,A}))A - 1 \right] \]

\[ + \frac{1}{2} (h_0 + x(W^{N,A})) \tau(W^{N,A}) \frac{\partial A}{\partial n} < 0, \]

\[ D'_r = R > 0, \quad D'_\mu = \left[ \frac{1}{2} (h_0 + x(W^{N,A})) \frac{\partial A}{\partial \mu} - \frac{A}{\mu^2 \sigma^2} \right] \tau(W^{N,A}) \]

\[ > \left[ \frac{1}{2} h_0 \frac{\partial A}{\partial \mu} - \frac{A}{\mu^2 \sigma^2} \right] \tau(W^{N,A}) > 0, \quad \text{and} \]

\[ D'_c = \frac{R \partial C(x(W^{N,A}))}{\partial c} dc > 0. \]

Focusing on changes in \( c \), simplifies the last expression, which reduces to \( D'_c = R \).

Together equations (14) and (15) constitute a system of two linear equations in two unknowns \( dn \) and \( dp \). Its solution is \( dn = (B'_n D'_r - B'_n B'_r / \Delta') df - (D'_n B'_n - B'_n B'_n / \Delta') dc \) and \( dp = (D'_r B'_r - B'_r D'_r / \Delta') df - (B'_r B'_r - D'_r B'_r / \Delta') dc \) where \( \Delta' \equiv B'_n D'_n - B'_n B'_n < 0 \). To show that \( \Delta' < 0 \), replace the coefficients with their expressions, simplify and check the sign of each term. Clearly, \( \frac{dp}{df} < 0 \) and \( \frac{dp}{dc} > 0 \) if

\[ \frac{\tau(W_j)^2}{2RC''(x_j)} dG(W_j) < x(W^{N,A}). \]

This happens when the contribution to aggregate information from new stockholders \((x(W^{N,A}))\) is larger than the loss on the part of incumbent stockholders (the left-hand side of (16)). In other words, the inequality ensures that the information effect is weak. This happens in particular when \( C'' \) is large as the demand for information is insensitive to changes in \( n \) or \( \mu \). Thus, aggregate risk tolerance \((n)\) increases when the information or entry costs fall. Noise \((\mu)\) falls only when the information effect is weak.

Changes in participation \( N \), the equity premium \( Q \), and the variance of returns \( \nu \):

Participation. In an all-active equilibrium, the participation rate, \( N \), is given by the number of stockholders with initial wealth above \( W^{N,A} \): \( N = 1 - G(W^{N,A}) \) so \( dN = 1 / (\tau(W^{N,A})) dn \). It follows that \( (dN)/(dc) < 0 \) and \( (dN)/(df) < 0 \). Thus participation \((N)\) increases if the information or the entry cost fall.

Equity premium. Proceeding as above,

\[ dQ = \frac{\partial Q}{\partial n} dn + \frac{\partial Q}{\partial \mu} d\mu = \frac{1}{\Delta'} \left[ \frac{\partial Q}{\partial n} (B'_n D'_r - D'_n B'_r) + \frac{\partial Q}{\partial \mu} (D'_n B'_r - B'_n B'_r) \right] df \]

\[ - \left( \frac{\partial Q}{\partial n} (B'_n B'_r - B'_n D'_r) + \frac{\partial Q}{\partial \mu} (B'_n B'_r - D'_n B'_r) \right) dc. \]

Therefore,

\[ \frac{dQ}{df} = \frac{1}{\Delta'} \left( \frac{\partial Q}{\partial n} (B'_n D'_r - D'_n B'_r) + \frac{\partial Q}{\partial \mu} (D'_n B'_r - B'_n B'_r) \right) \]

\[ = \frac{R}{\Delta'} \left( \frac{\partial Q}{\partial n} (B'_n - D'_n) + \frac{\partial Q}{\partial \mu} (D'_n - B'_n) \right). \]

Similarly,

\[ \frac{dQ}{dc} = -\frac{1}{\Delta'} \left( \frac{\partial Q}{\partial n} (D'_n B'_r - B'_n D'_r) + \frac{\partial Q}{\partial \mu} (B'_n D'_r - D'_n B'_r) \right). \]
Focusing on changes in $c_0$ again simplifies the calculations,

$$\frac{dQ}{dF} = \frac{dQ}{dc} = \frac{R}{\Delta'} \left( \frac{\partial Q}{\partial n} (B'_{F} - D'_{n}) + \frac{\partial Q}{\partial \mu} (D'_{t} - B'_{n}) \right).$$

The sign is ambiguous but if inequality (16) is satisfied then the information effect is weak and noise falls. As a result, the effect through $\mu$ operates in the same direction as that through $n$ and $dQ/dF = dQ/dc > 0$. In that case, the equity premium falls with the information or the entry costs.

**Variance of returns.** As above,

$$dv = \frac{\partial v}{\partial n} dn + \frac{\partial v}{\partial \mu} d\mu = \frac{1}{\Delta'} \left[ \left( \frac{\partial v}{\partial n} (B'_{F} D'_{F} - D'_{n} B'_{F}) + \frac{\partial v}{\partial \mu} (D'_{t} B'_{F} - B'_{n} D'_{F}) \right) dF \right. - \left. \left( \frac{\partial v}{\partial n} (D'_{t} B'_{F} - B'_{n} D'_{F}) + \frac{\partial v}{\partial \mu} (B'_{t} D'_{F} - D'_{n} B'_{F}) \right) dc \right].$$

**FIGURE 6**

Aggregate Risk Tolerance ($n$), Noise ($\mu$), Mean Equity Premium ($Q$), Variance of Returns ($v$), and Participation ($N$) for Different Levels of the Information Cost ($c$) and No Entry Cost in an Active-Passive Equilibrium. The pictures are drawn for $C(x) = \frac{1}{2} c x^2$, $F = 0$, $\gamma(W) = W/100$, $e = 0$, $\sigma^2_n = E(\sigma) = 1$, $\sigma^2_\mu = E(\sigma) = 0.01$, $W = 100$, and $R = 1$.

Therefore,

$$\frac{dv}{dF} = \frac{1}{\Delta'} \left( \frac{\partial v}{\partial n} (B'_{F} D'_{F} - D'_{n} B'_{F}) + \frac{\partial v}{\partial \mu} (D'_{t} B'_{F} - B'_{n} D'_{F}) \right) \quad \text{and}$$

$$\frac{dv}{dc} = \frac{1}{\Delta'} \left( \frac{\partial v}{\partial n} (D'_{t} B'_{F} - B'_{n} D'_{F}) + \frac{\partial v}{\partial \mu} (B'_{t} D'_{F} - D'_{n} B'_{F}) \right).$$
Focusing on changes in $c_0$,

$$\frac{dv}{dF} = \frac{dv}{dc} = \frac{R}{\Delta'} \left( \frac{\partial v}{\partial \mu} (B'_\mu - D'_\mu) + \frac{\partial v}{\partial \mu} (D'_\mu - B'_\mu) \right).$$

The sign is ambiguous but again under inequality (16), which ensures that the information effect is weak, noise falls, so $dv/dF = dv/dc > 0$. The variance of returns falls with the information or the entry costs.

**FIGURE 7**

Aggregate Risk Tolerance ($n$), Noise ($\mu$), Mean Equity Premium ($\Omega$), Variance of Returns ($\nu$), and Participation ($N$) for Different Levels of the Entry Cost ($F$) in an Active-Passive Equilibrium. The pictures are drawn for $C(x) = 0.0005x^2$, $\tau(W) = W/100$, $\sigma_\omega^2 = E(\sigma) = 1$, $\epsilon = 0$, $\sigma_\theta^2 = E(\theta) = 0.01$, $\bar{W} = 100$, and $R = 1$.

**B. Numerical Example**

I simulate the economy to illustrate the results. I consider the following specifications. The cost of information is $C(x) = h\epsilon x^2 + c_1 x$, the supply of stocks is inelastic ($\epsilon = 0$),
absolute risk tolerance is \( \tau(W) = W/a \), wealth is uniformly distributed on \([0, 100]\), and the number of agents is normalized to 1. It follows that

\[
x_j = \frac{A}{2caR} W_j - \frac{c_1}{c}, \quad W^{P,A} = \frac{2Rc_1a}{A}, \quad W^{N,P} = \frac{2RFa}{h_0A - 1},
\]

\[
W^{N,A} = \frac{2aR}{A} (cx^{N,A} + c_1), \quad x^{N,A} = \frac{1}{A} - h_0
\]

\[
+ \frac{1}{c} \sqrt{\left( \frac{1}{A} - h_0 \right)^2 + 2c \left( F + c_1 \left( \frac{1}{A} - h_0 \right) \right)},
\]

\[
dG(W) = \frac{dW}{W}, \quad n = \frac{1}{2aVW} \left( \bar{V}^2 - W^{N,P} \right), \quad i = \frac{A}{6a^2 c RV} \left( \bar{V}^3 - W^{P,A} \right)
\]

\[- \frac{c_1}{2ac \bar{V}W} \left( \bar{V}^2 - W^{P,A} \right), \quad J = \frac{i}{A}.
\]

I solve for the active-passive equilibrium. The solutions are displayed in Figures 6 and 7 for the following parameter values: \( \sigma^2 = E(\pi) = 1, \sigma^2 = E(\theta) = 0.01, a = 100, \bar{V} = 100, R = 1, \) and \( c_1 = 0 \). Figure 6 shows that participation rises with the cost of information. Figure 7 shows that return variance can rise when the entry cost declines.

References


