Learning About Technologies and Technological Progress

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ABSTRACT

We present a model of financial development and technological progress based on imperfect information. In the model, entrepreneurs innovate more when financiers are better informed about their projects because they expect their successful projects to receive more funding. Conversely, financiers collect more information about projects when entrepreneurs innovate more because the opportunity cost of misinvesting, i.e. of funding unsuccessful projects, is higher. Thus, knowledge about technologies and technological knowledge are mutually reinforcing. The model is consistent with several empirical regularities. Empirical strategies for testing the model are discussed.

JEL classification codes: G20, O31, O4

Keywords: Financial development, growth, technological progress, innovation, capital allocation, learning.

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1 Introduction

What drives differences in income across countries and over time is not fully understood. They do not appear to be explained by differences in factor accumulation as assumed in traditional neoclassical growth models. Rather, most of the heterogeneity stems from differences in total factor productivity (TFP), the residual from a regression of income growth on factor growth. This observation led to the emergence of endogenous growth theory, which investigates the source of TFP. Though the theory describes the discovery of technologies by profit-maximizing agents, their behavior is, to a large extent, determined by the law of motion postulated for technological progress. As Romer (1990, page 84) concedes, “unbounded growth is more like an assumption than a result of the model”. Many scholars argue that the fundamental causes of growth, and in particular those that drive innovation, are to be found in economic institutions, such as legal systems, government policies and financial institutions.

Financial institutions, once considered a “sideshow” (Robinson (1952)), have attracted much attention. There is now extensive evidence that they influence economic growth (see Levine (1997, 2005) for reviews). While considerable progress has also been achieved on the theoretical front, Levine (2005) concludes his survey of the literature asserting that “our understanding of finance and growth will be substantively advanced by the further modeling of the dynamic interactions between the evolution of the financial system and economic growth”. In this paper, we develop a dynamic model in which financial development and technological innovation are mutually reinforcing, thereby promoting economic growth through TFP growth. The model is motivated by the

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Page 85. Levine goes on: “we need additional thought on the co-evolution of finance and growth. Technology innovation, for instance, may only foster growth in the presence of a financial system that can evolve effectively to help the economy exploit these new technologies. Furthermore, technological innovation itself may substantively affect the operation of financial systems by, for example, transforming the acquisition, processing, and dissemination of information. (...) These are mere conjectures and ruminations that I hope foster more careful thinking.” Our model centers on the interplay between technological innovation and financial systems that Levine emphasizes.
following empirical findings, on which we elaborate in the next section (see the references therein).

1. Financial development promotes economic growth and its effect runs through TFP growth, rather than capital growth.

The subsequent three observations shed light on the link from finance to TFP.

2. Financial development stimulates investments in research and development (R&D) and R&D contributes to TFP.

3. Financial development also enhances TFP by improving capital efficiency. Countries with more developed financial sectors allocate capital more efficiently across industries and firms. A more efficient distribution of capital at the micro level translates into higher TFP at the macro level.

4. Financial development improves capital efficiency by alleviating informational problems.

We develop a model that is consistent with these observations. At the center of the model lies imperfect information. Its main insight is that knowledge about technologies and technological knowledge are mutually reinforcing. That is, entrepreneurs innovate more when financiers are better informed about the profitability of their projects because they expect to receive more capital should their projects be successful. Conversely, financiers collect more information about projects when entrepreneurs innovate more because the opportunity cost of misinvesting, i.e. of funding unsuccessful projects while missing out on successful projects, is higher. While several papers display a similar feedback, to the best of our knowledge we are the first to model the interplay between the quality of financiers’ *ex ante* knowledge about technologies and entrepreneurs’ innovation effort.
This insight is embedded into a competitive overlapping-generations model featuring rational agents who conceive risky projects, learn about their prospects and invest in them. Costs have to be incurred both for innovating (what we call “research”) and for learning. Unlike other papers (the related literature is discussed below), the positive feedback between research and learning is not a consequence of risk sharing since risk is fully diversified away, nor moral hazard since efforts are contractible. Instead, it follows from the complementarity between productivity and capital. Expressing output as \( Y = AK \) where \( A \) and \( K \) denote respectively a project’s uncertain return and the amount of capital it attracts, shows that the return on financiers’ funds increases with \( A \) (every unit of capital yields a larger payoff) while the reward for research rises with \( K \) (an invention can be applied on a larger scale). The complementarity between \( A \) and \( K \) leads to a complementarity between research and learning.

Our model is consistent with the aforementioned observations, to the extent that financial development is positively correlated with the quality of financiers’ knowledge about projects.\(^2\) Better-informed agents – who live in economies that are more financially developed – invest capital more efficiently, thus stimulating research and TFP. This in turn makes them wealthier and more willing to learn about investment opportunities. Per capita income grows and its growth is entirely driven by TFP growth. We quantify the contributions of learning and research and their interplay, and show that the growth rate of income in the economy with both learning and research is larger than the sum of the growth rates in the no-learning and in the no-research economy – once again a reflection of the positive feedback between learning and research. This implies for example that an economy that would converge to a steady-state when learning and research do not interact can experience unbounded growth once they do.

\(^{2}\)To keep the model parsimonious, we do not model the financial sector explicitly. But we interpret the amount of resources devoted to analyzing investment opportunities as a proxy for the degree of financial development.
The model yields important insights on the effectiveness of policies aimed at stimulating innovations. First, it suggests that innovation policies, such as research subsidies or tax breaks, have a multiplier effect thanks to the induced improvement in capital efficiency. That is, the total benefit of these policies consists not only of their direct effect but also of their indirect effect on research, obtained through enhanced learning. Second, innovations are also encouraged by policies designed at increasing capital efficiency, such as removing impediments to trading financial assets or improving accounting standards.

We discuss how to evaluate empirically the model’s main predictions regarding the interplay between learning and technological innovation. Specifically, the model predicts that (i) financiers learn more when entrepreneurs perform more research, and that (ii) entrepreneurs perform more research when financiers learn more. Empirically assessing these relationships not only requires proxies for research and learning, but also instruments to treat the endogeneity bias that this two-way relationship generates.\(^3\) We propose strategies to address these issues, such as an event-study approach and the use of instrumental variables. The latter in particular can be implemented within the framework laid out by Rajan and Zingales (1998) and used by several others since, which combines industry and country data.

Our paper lies at the intersection of two streams of theoretical research. First, it is part of the endogenous growth literature (e.g. Romer (1986, 1990), Grossman and Helpman (1991), Aghion and Howitt (1992)). This stream of research models the discovery of technologies by profit-maximizing agents. Ongoing growth is possible thanks to scale effects that lead to technological improvements: as the economy expands, the incentives to innovate strengthen and overcome the

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\(^3\)In the case of prediction (i) for example, a least squares regression of learning on research yields inconsistent estimates because the regression’s residual is correlated with the regressor – research, as implied by the second prediction.
diminishing returns to capital. Second, our paper belongs to the literature on finance and growth. In this literature, imperfections limit the efficient use of resources. These include incomplete information as in Greenwood and Jovanovic (1990) or project indivisibilities as in Acemoglu and Zilibotti (1997). Financial institutions contribute to economic growth by mitigating these imperfections. Again ongoing growth is feasible because the increasing benefit of financial intermediation offsets the diminishing returns to capital. Our paper bridges these two literatures. It centers on information problems that limit the financing and production of innovations. It shows that financial development stimulates technological innovation by improving the allocation of capital to risky projects.

We are not the first to emphasize the interaction between financial development and incentives to innovate under imperfect information. In Bhattacharya and Chiesa (1995), De la Fuente and Marin (1996), Acemoglu and Zilibotti (1999) and Acemoglu, Aghion and Zilibotti (2006), financiers supply capital to entrepreneurs whose effort they can only monitor at a cost. We assume away these moral hazard problems — information is symmetric in our setting, and focus instead on the evaluation of projects. In the model, it is the expectation that capital will be efficiently allocated that encourages innovation. Thus, the information that matters is produced *ex ante* rather than *ex post*. Both kinds of information play an important role in practice. We conjecture that evaluating projects matters most in the early stages of the innovation process, such as when seed capital is raised, while the moral hazard problem is more important in the later stages, once large amounts of capital have been levied.

The paper proceeds as follows. Section 2 reviews the empirical regularities that guide the model.\(^4\) Another strand of research centers on the adverse selection and signalling problems which occur when financiers do not observe entrepreneurs’ ability (e.g. Bhattacharya and Ritter (1983), King and Levine (1993), Ueda (2004), Aghion, Howitt and Mayer-Foulkes (2005)).

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Section 3 describes the economic environment. Section 4 solves for the equilibrium. Section 5 characterizes the dynamics as well as the process of technological progress and financial development. Section 6 discusses empirical strategies for testing the model. The appendix contains proofs and extensions.

2 Empirical Motivation

We review the facts listed in the Introduction that motivate the model.

1. Financial development causes growth by improving TFP.

   A large literature, surveyed by Levine (1997, 2005) shows that financial development promotes economic growth. Country-level, industry-level, firm-level and event-study investigations suggest that financial intermediaries and markets have a large, causal impact on real GDP growth (e.g. Rajan and Zingales (1998), Jayaratne and Strahan (1996), Fisman and Love (2004), Levine and Zervos (1998), Beck, Levine and Loayza (2000)). Moreover, Levine and Zervos (1998) and Beck, Levine and Loayza (2000) find that their relation to TFP growth is strong whereas their link to capital growth is tenuous. Thus, it appears that financial development contributes to growth by improving TFP rather than capital accumulation.

2. Financial development stimulates R&D investments and R&D improves TFP.

   Carlin and Mayer (2003) examine a sample of advanced OECD countries. They show that industries dependent on equity finance invest more in R&D and grow faster in countries with better accounting standards. They do not find a similar increase for investment in fixed assets, or for countries with a large financial sector. This suggests that finance is associated with the funding of new technologies and that informational problems are a serious impediment to providing capital.
Brown, Fazzari and Petersen (2008) also establish a link between equity financing and R&D by analyzing U.S. high tech firms. They estimate that improved access to finance explains most of the 1990’s R&D boom in the U.S.. Credit (Herrera and Minetti (2007)) and venture capital (Kortum and Lerner (2000), Ueda and Hirukawa (2003), Hellmann and Puri (2000)) are also essential to the funding of innovations. Moreover, there exists abundant evidence establishing that R&D is an important determinant of productivity (e.g. Griliches (1988), Coe and Helpman (1995)).

3. Financial development improves allocative efficiency and allocative efficiency improves TFP.

Countries with more developed financial sectors allocate their capital more efficiently. In a cross-country study, Wurgler (2000) documents that they increase investments more in their growing industries, and decrease investments more in their declining industries, than countries with less developed markets.\(^5\) Event-studies report similar findings. Bekaert, Harvey and Lundblad (2001, 2005), Bertrand, Schoar and Thesmar (2007), Galindo, Schiantarelli and Weiss (2007), Chari and Henry (2008) show that countries that liberalize their financial sector allocate capital more efficiently. Henry (2003) and Henry and Sasson (2008) document in addition a rise in TFP. This is not surprising given that allocative efficiency is an important determinant of TFP (Caballero and Hammour (2000), Jeong and Townsend (2007), Restuccia and Rogerson (2008) and Hsieh and Klenow (2009)). For example, Hsieh and Klenow (2009) find that TFP would double in China and India if capital and labor were reallocated to equalize their marginal products across plants.


Rajan and Zingales (1998) and others show that the quality of information disclosure, proxied by accounting standards, enhances growth of industries dependent on external finance. Carlin and

\(^5\)Hartmann et al.(2007) find that the same pattern holds among OECD countries.
Mayer (2003) report that information disclosure is associated with more intense R&D in industries dependent on equity finance and that the relation of industry growth and R&D to information disclosure is more pronounced than to the size of the financial sector. Wurgler (2000) finds a positive cross-country relation between the efficiency of investments and the informativeness of stock prices. Herrera and Minetti (2007) show that the quality of banks’ information has a positive influence on the probability that Italian manufacturing firms innovate.⁶

3 The Economy

The economy is composed of two sectors—a final and an intermediate goods sector, and overlapping generations of agents. Each generation consists of a representative entrepreneur who conceives the projects that compose the intermediate sector, and a representative financier who invests in them.

3.1 Agents

The economy is populated by overlapping generations of agents who live for two periods. There is no population growth. Agents are risk neutral. Since there is no aggregate risk, our results extend to any increasing and concave utility function. Agents only consume when old.⁷ Each generation consists of a representative entrepreneur (‘she’) and a representative financier (‘he’).

The entrepreneur creates the technologies that produce intermediate goods. In period t, she conceives a continuum of projects with unit mass indexed by \( n \in [0, 1] \). Their output is determined

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⁶Wurgler (2000) uses a proxy for informativeness developed by Morck, Yeung and Yu (2000). They measure the extent to which stocks move together and argue that prices move in a more unsynchronized manner when they incorporate more firm-specific information. Examining the cross-section of U.S. firms, Durnev, Morck and Yeung (2004) and Chen, Goldstein and Jiang (2007) document that firms make more efficient capital budgeting decisions when their stock price is more informative. Herrera and Minetti (2007) use the duration of the credit relationship to proxy for the quality of a bank’s information about a firm.

⁷Thus the saving rate is exogenously set to one. We make this assumption not only to simplify the model but also because the evidence suggests that financial development enhances growth through higher productivity rather than through higher saving rates (Levine and Zervos (1998), Beck, Levine and Loayza (2000)).
by a technology with constant returns to capital:

\[ \hat{Y}_{t+1}^n \equiv \hat{A}_t^n K_t^n, \]

where \( \hat{Y}_{t+1}^n \) is the quantity of intermediate good produced in period \( t+1 \) by project \( n \) net of capital depreciation, \( \hat{A}_t^n \) is its random productivity (or return) and \( K_t^n \) is the amount of capital invested in the project. Projects are independent from one another (\( \hat{A}_t^n \) independent of \( \hat{A}_t^m \) for any \( m \neq n \)). They are liquidated immediately after production.\(^8\)

Projects succeed with a 0.5 probability. Successful projects yield a return \( \bar{A}_t \), while unsuccessful projects yield \( A_t \) where \( \bar{A}_t > A_t > 0 \). The entrepreneur sets projects’ return in case of success and failure, \( \bar{A}_t \) and \( A_t \), but has no influence on their probability of success. Creating a continuum of independent projects with returns \( \bar{A} \) and \( A \) requires a research cost \( \epsilon_A(\bar{A} + A) \) where \( \epsilon_A \) is continuous, increasing, convex, \( \epsilon_A(0) = e_A'(0) = 0 \) and \( e_A'(+\infty) = +\infty \), and where \( \epsilon_A(A) \equiv 1 + \epsilon_A''(A)/e_A'(A) > 1 \) denotes one plus the elasticity of \( e_A \) with respect to \( A \).\(^9\) Under this formulation, returns in case of success and failure are perfect substitutes in terms of their cost. Moreover, there are no technological spillovers across generations, i.e. a generation’s technological achievements does not make it easier for the next generation to innovate (\( e_A \) is stationary). We refer to \( \bar{A}_t \) and \( A_t \) as the research effort, or productivity. The entrepreneur raises the capital required to operate her technologies from the financier.

The financier is employed in the final good sector, to which he supplies labor inelastically for a competitive wage \( w_t \). His labor endowment is normalized to 1 so that aggregate labor supply equals 1. He invests his labor income in the projects set up by the entrepreneur. He allocates \( K_t^n \)

\(^8\)Assuming projects are liquidated just after production simplifies the dynamics of the economy and allows to focus on the early stage of a firm’s development. It is well known that young firms, because they have little retained earnings, are more dependent on external financing than mature firms. Several empirical studies confirm that financial development fosters growth mainly through the former (Rajan and Zingales (1998), Beck, Demirgüç-Kunt and Maksimovic (2005), Brown, Fazzari and Petersen (2008)).

\(^9\)For example, \( \epsilon_A(A) \equiv A^{\epsilon_A}/\epsilon_A \) where \( \epsilon_A \) is a constant strictly larger than 1 satisfies these conditions.
units of capital to project \( n \) in period \( t \). At the time of investment, the financier does not know which projects will succeed. Instead, he receives an imperfect signal \( S_t \) that identifies the successful projects. The signal is right, i.e. accurately reveals successful projects, with a probability \( q_t \) but wrong with a probability \( 1 - q_t \). Observing a signal of precision \( q \) requires a learning cost \( e_q(q) \), where \( e_q \) is continuous, increasing, convex, \( e_q(1/2) = e'_q(1/2) = 0 \) and \( e'_q(1) = +\infty \), and where \( \varepsilon_q(q) = 1 + q e''_q(q) / e'_q(q) > 1 \) denotes one plus the elasticity of \( e'_q \) with respect to \( q \).\(^{10}\) \( q = 1/2 \) corresponds to an uninformative signal, and \( q = 1 \) to a perfect signal. Unlike research, learning does not affect projects’ productivity. Instead, it allows to match capital with projects more efficiently.

We refer to \( q_t \) as the learning effort or the precision of information.

Our assumptions call for some remarks. First, effort levels (in research and learning) are contractible. That is, they are determined \( ex \ ante \) cooperatively by the entrepreneur and the financier. As a result, the first-best outcome is achieved. Incomplete information rather than asymmetric information drives the findings of the paper.\(^{11}\) Second, the assumption that the production of intermediary goods displays constant returns to scale makes our findings starker but is not essential – our main findings continue to hold under decreasing and increasing returns to scale. Third, we can assume without loss of generality that the research and learning efforts are identical across projects as long as intermediate technologies are linear in capital and risk is fully diversified away.

Their cost for the entire continuum is therefore adequately represented by \( e_A(\overline{A}_n + \underline{A}_n) \) and \( e_q(q_t) \).\(^{12}\)

\(^{10}\)For example, \( e_q(q) = \int_{1/2}^q [(x - 1/2)/(1 - x)]^\theta \, dx \) for \( \theta > 0 \), which implies \( e'_q(q) = [(q - 1/2)/(1 - q)]^\theta \), satisfies these conditions.

\(^{11}\)Initially, i.e. at the time they choose their efforts, the entrepreneur and the financier are equally ignorant about which projects will be successful. While information asymmetries may also be present or emerge later, the focus of the paper is on the interplay between \( ex \ ante \) efforts during the initial contracting phase. Moreover, the fact that efforts are contractible implies that multiple equilibria do not arise.

\(^{12}\)That is \( \overline{A}_n = \overline{A}, \underline{A}_n = \underline{A} \) and \( q^n = q \) for all \( n \in [0,1] \). Indeed, the expected profit conditional on the signal is linear in the amount invested in each project so only projects that receive a success signal and have the highest expected conditional return, \( E(\overline{A}_n \mid S_n) = q^n \overline{A}_n + (1 - q^n) \underline{A}_n \), are funded (problem 1 below). It follows that, given a set of projects to develop and evaluate, agents choose the same learning and research efforts across all projects (problem 2 below). If the financier learns about a subset of projects only, then the entrepreneur only researches projects in that subset and we simply assume these projects comprise the initial set of projects. The important
Finally, our findings obtain if, instead of controlling projects’ productivity, the entrepreneur sets their probability of success, as shown in Appendix D. They also obtain under a more general cost structure, \( e_A(\overline{A}, A) \), as long as \( e_A \) is increasing and convex in each variable.

### 3.2 Technologies

The economy is composed of two competitive sectors, a final and an intermediate good sector. The intermediate goods sector is made up of the projects conceived by the entrepreneur and funded by the financier. The intermediate goods are used as inputs in the production of the final good. The final good is produced according to a riskless technology,

\[
G_t \equiv L_t^{1-\beta} Y_t^\beta,
\]

where \( G_t \) is final output, \( L_t \) is labor, \( Y_t \) is the employment of intermediate good and \( 0 < \beta < 1 \) is the factor share of the intermediate good in the production of the final good. The final good is used as the numeraire. Many identical firms compete in the final good sector and aggregate to one representative firm.

### 3.3 Timing

The timeline is depicted in Figure 1. An agent lives one period as a young agent (either as an entrepreneur or as a financier) and one period as an old agent (as a consumer). At the start of a period, the entrepreneur and the financier determine their research and learning efforts. We stress that the entrepreneur chooses her research effort before the financier observes his signal. The timing makes it clear that research is not driven by the expectation that a particular project will succeed, but by the expectation that capital will be efficiently deployed (i.e. that successful projects, whichever they are, will be well funded). After earning a wage and observing his signal, the assumption is that a large enough number of projects is conceived so that risk can be diversified away. Acemoglu and Zilibotti (1997) study what determines the optimal number of projects to develop.
the financier distributes his wage across the projects. In the following period, the young become
old, the successful projects are revealed, final goods are produced and agents consume their share
of profits.

3.4 Equilibrium Concept

We define an equilibrium by three conditions.

1. Market clearing in the intermediate goods sector

Final goods producers maximize their profit. Since labor and intermediate goods trade in compet-
titive markets and aggregate labor supply equals 1, the following equilibrium factor prices obtain in
period $t+1$:

$$w_{t+1} = (1 - \beta)(Y_{t+1})^\beta \quad \text{and} \quad \rho_{t+1} = \beta(Y_{t+1})^{\beta-1},$$

where $\rho_{t+1}$ denotes the price of intermediate goods in period $t+1$, and $Y_{t+1}$ sums up output over
all projects:

$$Y_{t+1} = \int_n \hat{A}_t^n K_t^n.$$

Since there is no aggregate risk in this economy, $w_{t+1}$, $\rho_{t+1}$ and $Y_{t+1}$ are deterministic.

2. Capital allocation

After observing his signal $S_t$, the financier distributes his capital $w_t$ across the projects offered by
the entrepreneur to maximize total expected profits, taking $\rho_{t+1}$, $\mathbf{A}_t$, $\mathbf{A}_0$ and $q_t$ as given:

$$\pi(q_t, \mathbf{A}_t, \mathbf{A}_0, w_t) \equiv \max_{\{K^n_t\}_{n \in [0,1]}} \mathbb{E} \left( \rho_{t+1} \int_n \hat{A}_t^n K_t^n | S_t \right) \quad \text{subject to} \quad \int_n K_t^n = w_t. \quad (1)$$

As with output, $\pi(q_t, \mathbf{A}_t, \mathbf{A}_0, w_t)$ (or $\pi_t$ for short) is deterministic.

3. First-best effort levels
The entrepreneur and the financier determine cooperatively their effort levels (before the signal $S_t$ is observed) to maximize the *ex ante* total surplus, which equals the expected profit net of effort costs, taking the price of intermediate goods $\rho_{t+1}$ as given (the model is agnostic about how this surplus is shared between the entrepreneur and the financier):

$$\max_{q_t, \overline{A}_t, \underline{A}_t} \pi(q_t, \overline{A}_t, \underline{A}_t, w_t) - e_A(q_t) - e_A(\overline{A}_t + \underline{A}_t).$$

(2)

### 4 Equilibrium Characterization

We solve for the learning and research efforts for a given level of income. We start with investments and production, then move to learning and research, first for any given intermediate goods’ price, then in equilibrium. The next section will describe the dynamics of learning, research and income.

#### 4.1 Capital Allocation and Production

We determine the optimal investment plan of a financier who earns a wage $w_t$ and observes a signal $S_t$ (problem 1). The financier maximizes a linear objective – he is a risk-neutral price-taker and the projects generate constant returns to scale, so he allocates all his capital to the projects deemed successful by his signal and none to the others. A fraction $q_t$ of successful projects are correctly identified yielding $q_t \overline{A}_t w_t$ units of intermediate goods in total, while a fraction $1 - q_t$ of unsuccessful projects are incorrectly considered successful yielding $(1 - q_t) \underline{A}_t w_t$ in total. Because all risk is idiosyncratic, factor prices, $w_{t+1}$ and $\rho_{t+1}$, aggregate production, $Y_{t+1} = [q_t \overline{A}_t + (1 - q_t) \underline{A}_t]w_t$, and expected profits, $\pi_t = \rho_{t+1} Y_{t+1}$, that are deterministic.

#### 4.2 Learning and Research for a Given Intermediate Goods’ Price

The financier and entrepreneur solve problem 2. The perfect substitutability of $\overline{A}_t$ and $\underline{A}_t$ in the cost of research ($e_A$ is a function of their sum, $\overline{A}_t + \underline{A}_t$) implies that the entrepreneur prefers to
concentrate her effort on improving returns in case of success, i.e. she chooses a corner solution and sets $A_t$ to 0. The optimal $q_t$ and $A_t$ solve the first-order conditions:

$$\rho_{t+1} \bar{A}_t w_t = e'_q(q_t)$$

and

$$\rho_{t+1} q_t w_t = e'_A(A_t).$$

The optimal learning effort equates the marginal benefit of learning ($\partial E(\pi_t)/\partial q_t = \rho_{t+1} \bar{A}_t w_t$ on the left-hand side of equation 3) to its marginal cost ($e'_q(q_t)$ on the right-hand side). Given that the former is constant in $q_t$ while the latter increases and spans the entire real line, there exist a unique solution and it is interior. Similarly, the entrepreneur’s research effort $\bar{A}_t$ is uniquely defined by equation 4 which equates the marginal benefit of research ($\partial E(\pi_t)/\partial A_t = \rho_{t+1} q_t w_t$ on the left-hand side) to its marginal cost ($e'_A(\bar{A}_t)$ on the right-hand side).

Higher price and income, $\rho_{t+1}$ and $w_t$, stimulate learning and research because they expand revenues without affecting costs (a higher wage implies larger investments and output, and a higher price larger revenues for a given output). Moreover, the precision of the financier’s information $q_t$ rises with the entrepreneur’s research effort $\bar{A}_t$. Indeed, determining the successful projects is more useful when they outperform the others by a large amount. Conversely, the entrepreneur’s research effort is higher when the financier is better informed, i.e. $\bar{A}_t$ increases with $q_t$. This is because successful projects, whichever they are, are expected to attract more capital, and hence to generate more revenues.

4.3 Learning and Research in Equilibrium

To close the model, we substitute the expression for $\rho_{t+1} = \beta(q_t \bar{A}_t w_t)^{\beta-1}$ in the first-order conditions 3 and 4. This leads to the following proposition characterizing the equilibrium.
Proposition 1: In equilibrium, the learning and research efforts, $q_t$ and $A_t$, are the unique solutions to the following system of equations:

$$\frac{\beta A_t^\beta w_t^\beta}{q_t^{1-\beta}} = e'_q(q_t) \quad (5)$$

and

$$\frac{\beta q_t^\beta w_t^\beta}{A_t^{1-\beta}} = e'_A(A_t) \quad (6)$$

Equation 5 is illustrated in Figure 2, in which the solid curve represents the benefit of information and the dotted curve its cost. The equilibrium precision choice is located at the (unique) intersection of the two curves. To describe the main properties of learning and research in equilibrium, it is convenient to define $\varphi_q$ and $\varphi_A$, the inverse functions of $q^{1-\beta}e'_q(q)$ and $A^{1-\beta}e'_A(A)$. The assumptions on $e_q$ and $e_A$ imply that $\varphi_q$ and $\varphi_A$ are increasing. We can express equations 5 and 6 as equations 7 and 8:

$$q_t = \varphi_q(\beta(\overline{A_t}w_t)^\beta) \quad (7)$$

and

$$A_t = \varphi_A(\beta(q_tw_t)^\beta). \quad (8)$$

The properties of learning and research are summarized in the following corollary.

Corollary 2:

- In equilibrium, the learning effort increases with income holding the research effort fixed, and with the research effort holding income fixed. Formally, $\frac{\partial q_t}{\partial w_t} \geq 0$ and $\frac{\partial q_t}{\partial A_t} \geq 0$.

- Conversely, the research effort increases with income holding the learning effort fixed, and with the learning effort holding income fixed. Formally, $\frac{\partial A_t}{\partial w_t} > 0$ and $\frac{\partial A_t}{\partial q_t} > 0$.

Corollary 2 illustrates the main contribution of this paper, namely the interplay between research and learning. Equation 8 reveals that research is promoted when the financier is better-informed because the entrepreneur knows that she will receive more funds for her successful projects. Conversely (equation 7), a higher productivity encourages the financier to learn by magnifying the
opportunity cost of misinvesting, i.e. of funding unsuccessful projects while missing out on successful
projects. Thus, knowledge about technologies and technological knowledge are mutually reinforcing.

The positive feedback between research and learning follows from the complementarity between
productivity $\tilde{A}_t^n$ and capital $K_t^n$ in the production of intermediate goods. Since $Y_t^{n+1} = \tilde{A}_t^n K_t^n$, the
return on the financier’s funds increases with $\tilde{A}_t^n$ because every unit of capital is more productive
the larger $\tilde{A}_t^n$. Similarly, the reward for innovating rises with $K_t^n$ because an invention is applied
on a larger scale. Thus, the complementarity between $\tilde{A}_t^n$ and $K_t^n$ leads to a complementarity
between learning and technological innovation (research). Formally, the average quantity of goods
produced by a project can be broken down into the contributions of projects’ average productivity,
$E(\tilde{A}_t^n) = (\overline{A}_t + \underline{A}_t)/2$, of the average stock of capital per project, $E(K_t^n) = w_t$, and of the quality
of the match between projects and capital, captured by $cov(\tilde{A}_t^n, K_t^n) = (q_t - 1/2)(\overline{A}_t - \underline{A}_t)w_t$:

$$E(\tilde{A}_t^n K_t^n) = E(\tilde{A}_t^n)E(K_t^n) + cov(\tilde{A}_t^n, K_t^n)$$
$$= (\overline{A}_t + \underline{A}_t)/2 * w_t + (q_t - 1/2)(\overline{A}_t - \underline{A}_t)w_t$$
$$= [q_t\overline{A}_t + (1 - q_t)\underline{A}_t]w_t$$

This expression simplifies to $E(\tilde{A}_t^n K_t^n) = q_t\overline{A}_t w_t$ when $\underline{A}_t = 0$. Thus, average output increases
with the product of the precision of the financier’s information $q_t$ with the return on the successful
project $\overline{A}_t$.

Corollary 2 states not only that research and learning feed on each other, but also that they are
more intense in a wealthier economy. Again, this is because an expansion in the scale of investments
increases the benefits of learning and research, not their costs. We show next that the converse is
also true, i.e. that learning and research make the economy wealthier.
5 Financial Development, Technological Progress and Growth

We now discuss the dynamics of income, research and learning and their interplay along the economy’s growth path.

5.1 The Sources of Growth

Income in period $t+1$, $w_{t+1}$, is related to the precision of information $q_t$ and productivity $A_t$ in period $t$ through the following equation:

$$w_{t+1} = (1 - \beta)Y_{t+1} = (1 - \beta) \cdot w_t^\beta \cdot TFP_A(A_t) \cdot TFP_q(q_t)$$

where $TFP_A(A_t) \equiv \bar{A}_t^\beta$ and $TFP_q(q_t) \equiv q_t^\beta$.

Equation 9 shows that both technological knowledge and knowledge about technologies contribute to income growth. It nests previous findings that analyze separately their growth impact.

On one hand, the endogenous growth literature focuses on the $TFP_A$ term. It acknowledges that technology can be improved by purposeful activity, such as R&D. In this context, growth stems from a scale effect, in that an expansion of either the size of the population, or the stocks of knowledge, physical or human capital improves the incentive to innovate (see Jones (2005) for a recent
discussion). In our framework, this channel can be identified by freezing $q_t$. The resulting dynamics of income, obtained from substituting equation 8 into equation 9, are described by the following equation,

$$w_{t+1} = c_2 w_t^\beta [\varphi_A(c_1 w_t^\beta)]^\beta$$

where $c_1$ and $c_2$ are positive constants. As an illustration, we suppose that the cost of research is given by $e_A(A) \equiv A^{\varepsilon_A}/\varepsilon_A$ where $\varepsilon_A$ is a positive constant. Income at $t+1$, $w_{t+1}$, is then proportional to $w_t^{\beta \varepsilon_A/(\varepsilon_A - \beta)}$ so the economy grows at a rate $\beta \varepsilon_A/(\varepsilon_A - \beta) - 1 = -(1 - \beta) + \beta^2/(\varepsilon_A - \beta)$, where the term $-(1 - \beta) < 0$ represents the (declining) contribution of capital accumulation and $\gamma_A \equiv \beta^2/(\varepsilon_A - \beta)$ is the contribution of technological progress. The economy grows without bound if and only if $1/\varepsilon_A > 1/\beta - 1$, i.e. if $\varepsilon_A$ is low and $\beta$ large. In that case, the diminishing returns to capital are overcome by strengthening incentives for research, obtained through an expansion of the investment scale $w_t$. The present paper can be viewed as an attempt to relate these incentives to the state of the financial sector, namely the quality of investment knowledge $q_t$.

On the other hand, the financial development literature highlights the role of frictions that reduce the efficiency of investments. Examples include information limitations (e.g. Greenwood and Jovanovic (1990)) and investment indivisibilities (e.g. Acemoglu and Zilibotti (1997)). Since the cost of alleviating such frictions is (to a first approximation) unrelated to the size of the capital stock unlike its benefit, richer economies are less constrained and invest more efficiently. This effect is present in our model and can be identified by freezing the return $A_t$ and substituting equation 7 into equation 9. Income then evolves according to

$$w_{t+1} = c_3 w_t^\beta [\varphi_q(c_4 w_t^\beta)]^\beta,$$

where $c_3$ and $c_4$ are positive constants. For example, if the cost of learning is given by $e_q(q) \equiv \cdots$
\[ (q^\varepsilon - 0.5q^\varepsilon)/(\varepsilon_q \text{ where } \varepsilon_q \text{ is a positive constant, then income is proportional to } w_t^{\beta \varepsilon_q/(\varepsilon_q - \beta)} \text{ so income grows at a rate } \beta \varepsilon_q/(\varepsilon_q - \beta) - 1 = -(1 - \beta) + \beta^2/(\varepsilon_q - \beta).^{13} \] Again, ongoing growth is possible if and only if the improvement in the capital allocation as a result of more intense learning (the \( \beta^2/(\varepsilon_q - \beta) \) term) is stronger than the diminishing return to capital (the \( -(1 - \beta) \) term). This happens if \( 1/\varepsilon_q > 1/\beta - 1 \), i.e. if \( \varepsilon_q \) is low and \( \beta \) large. An alternative view of our paper is that it shows how the incentive to mitigate investment inefficiencies depends on the level of the technology, i.e. how \( q_t \) depends on \( \overline{A}_t \).

Importantly, since learning influences research, it contributes to economic growth not only directly through its effect on \( TFP_q \) but also indirectly through its impact on \( TFP_A \). To capture its total effect, one needs to take into account its positive influence on entrepreneurs’ incentive to innovate. Conversely, the full benefit of research consists of its direct effect through \( TFP_A \) and its indirect effect through \( TFP_q \). This point is illustrated in the next section which describes the dynamics of income and provides a decomposition of the growth rate of income into its various sources. It has important implications for the effectiveness of policies aimed at stimulating innovations. First, it suggests that innovation policies, such as research subsidies or tax breaks, have a multiplier effect thanks to the improvement in capital efficiency. Second, innovations are also encouraged by policies designed at increasing capital efficiency, such as removing impediments to trading financial assets or improving accounting standards.\(^{14} \)

### 5.2 Dynamics

We now characterize the dynamics of income. A steady-state equilibrium satisfies equations 5, 6 and 9 together with the condition \( w_{t+1} = w_t \). A trivial solution to this system obtains when income

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\(^{13}\text{We assume for the purpose of this illustration that the solution to equation 5 is interior. It is not guaranteed with this parametrization of } \varepsilon_q \text{ given that } \varepsilon_q'(1/2) > 0 \text{ and } \varepsilon_q'(1) < +\infty.\)

\(^{14}\text{Consistent with this view, Bertrand, Schoar and Thesmar (2007) find that the 1980s deregulation of the French banking sector lead to an increase in productivity in industries more dependent on bank financing.}\)
equals zero and neither learning nor research take place \((w_t = \overline{A}_t = 0 \text{ and } q_t = 1/2)\). Non-trivial steady-states may also exist. A sufficient condition for the existence and unicity of a non-trivial steady-state is that:

\[
\frac{1}{\varepsilon_A(\overline{A})} + \frac{1}{\varepsilon_q(\overline{q})} \neq \frac{1}{\beta} - 1 \text{ for all } \overline{A} > 0 \text{ and } 1 > q > 1/2. \tag{10}
\]

We label with a * steady-state quantities, and \(\varepsilon_A^* \equiv \varepsilon_A(\overline{A}^*) \) and \(\varepsilon_q^* \equiv \varepsilon_q(q^*)\) one plus the elasticities of the marginal costs evaluated at the steady-state \((\varepsilon_A^* > 1 \text{ and } \varepsilon_q^* > 1)\). We denote \(w_0\) the wage at time \(t = 0\) and \(\gamma \equiv d\ln(w_{t+1}/w_t)/d\ln(w_t)\) the growth rate of income. The following proposition characterizes the dynamics of the economy.

**Proposition 3**: Assume condition 10 holds. The economy admits two steady-state equilibria, 0 and \(w^* > 0\).

1. If \(\frac{1}{\varepsilon_A^*} + \frac{1}{\varepsilon_q^*} < \frac{1}{\beta} - 1\), then \(w^*\) is a stable steady-state while 0 is not. Income converges to \(w^*\).

2. If \(\frac{1}{\beta} > \frac{1}{\varepsilon_A^*} + \frac{1}{\varepsilon_q^*} > \frac{1}{\beta} - 1\), then 0 is a stable steady-state while \(w^*\) is not. If \(w_0 > w^*\), then the economy grows without bound. If instead \(w_0 < w^*\), then the economy contracts towards 0.

3. If \(\frac{1}{\varepsilon_A^*} + \frac{1}{\varepsilon_q^*} > \frac{1}{\beta}\), then the economy is unstable and oscillating.

Proposition 3 implies that the economy converges to a steady-state in which income no longer grows when \(1/\varepsilon_A^* + 1/\varepsilon_q^* < 1/\beta - 1\), i.e. for \(\varepsilon_A^*\) or \(\varepsilon_q^*\) large and \(\beta\) low (case 1). Income increases but at a declining rate \((\gamma < 0)\). Thus, learning and research only have a transitory impact on growth. This is because their costs rise at a fast rate with effort levels, while the marginal product of intermediate goods falls rapidly with its employment.

If instead \(1/\beta > 1/\varepsilon_A^* + 1/\varepsilon_q^* > 1/\beta - 1\), which corresponds to \(\varepsilon_A^*\) or \(\varepsilon_q^*\) low and \(\beta\) high but not too much (case 2), then learning and research have a permanent impact and ongoing growth is possible. Income grows without bound if its initial value \(w_0\) exceeds \(w^*\), but shrinks towards 0.

---

15For example if \(\beta < 1/3\), then any increasing convex cost functions \(e_q\) and \(e_A\) satisfy this condition.
otherwise. If we focus on cases 1 and 2 and interpret the learning effort \( q \) and its cost \( \varepsilon_q(q) \) as measures of financial development, then the model predicts that the financial sector develops in tandem with the real economy.\(^{16}\) Finally, if \( 1/\varepsilon_A^* + 1/\varepsilon_q^* > 1/\beta \), i.e. \( \varepsilon_A^* \) or \( \varepsilon_q^* \) very low and \( \beta \) very high (case 3), then the system oscillates and is unstable. The following corollary breaks down the growth rate of income into its various components.

**Corollary 4**: The growth rate of income in a neighborhood of the steady-state \( \gamma \) is given by

\[
\frac{1}{\gamma + 1} = \frac{1}{\beta} - \frac{1}{\varepsilon_A^*} - \frac{1}{\varepsilon_q^*}.
\]

It can be decomposed into \( \gamma = \gamma_K + \gamma_A + \gamma_q + \gamma_{A-q} \) where

- \( \gamma_K \equiv -(1 - \beta) \) is the contribution of capital accumulation to growth,
- \( \gamma_A \equiv \frac{\beta^2}{\varepsilon_A^* - \beta} \) is the contribution of research in the absence of learning,
- \( \gamma_q \equiv \frac{\beta^2}{\varepsilon_q^* - \beta} \) is the contribution of learning in the absence of research,
- \( \gamma_{A-q} \equiv \gamma_A \gamma_q (1 + (\gamma + 1)/\beta) \) is the contribution of the interaction between research and learning.

Corollary 4 characterizes the growth rate of income around the steady-state. It nests the two polar cases we discussed in section 5.1. When \( \varepsilon_q^* = +\infty \), there is no learning – \( q_t \) is frozen as in the “endogenous growth case” and \( \gamma_q = 0 \), so the economy grows at the rate \( \gamma_K + \gamma_A \). If instead \( \varepsilon_A^* = +\infty \), there is no research – \( A_t \) is frozen as in the “financial development case” and \( \gamma_A = 0 \), then the economy grows at the rate \( \gamma_K + \gamma_q \).

Moreover, if we focus on the non-oscillating dynamics (cases 1 and 2 in Proposition 3) and leave aside the neoclassical effect on capital accumulation (represented by the \( \gamma_K \) term), then the

\[^{16}\varepsilon_q^*(q_t) \) can represent the amount of resources devoted to analyzing investment opportunities. Alternatively, we could add to the economy a competitive intermediary who invests funds on behalf of the financier. The intermediary collects information about projects’ returns and is paid a fee to compensate for the disutility of learning. There is free entry in the intermediary sector. Equation 22 in the Appendix implies that \( d\ln(q_t/q^*)/d\ln(w_t/w^*) \) is positive if \( 1/\varepsilon_A^* + 1/\varepsilon_q^* < 1/\beta \).
growth rate of income in the economy with both learning and research is larger than the sum of the
growth rates in the no-learning economy $\gamma_A$ and in the no-research economy $\gamma_q$. Indeed $\gamma_{A-q} > 0$
and $\gamma - \gamma_K = \gamma_A + \gamma_q + \gamma_{A-q} > \gamma_A + \gamma_q$. This is again a reflection of the positive feedback between
learning and research. An implication is that the set of parameters for which ongoing growth
is possible expands when learning and research interact. Suppose for example that $\beta = 1/2$ and
$\varepsilon_A^* = \varepsilon_q^* = \varepsilon^*$. The no-learning and the no-research economies each converge to their steady-state
for all values of $\varepsilon^*$ (since $\gamma_K + \gamma_A = \gamma_K + \gamma_q = -(\varepsilon^* - 1)/(2\varepsilon^* - 1) < 0$). When learning and
research both occur, income grows at the rate $\gamma = (2 - \varepsilon^*)/(\varepsilon^* - 1)/2$. If $\varepsilon^* < 2$, then $\gamma > 0$ and
the economy grows without bound. As a comparison, an economy in which learning and research
both take place without interacting grows at the rate $\gamma_K + \gamma_A + \gamma_q = -(\varepsilon^* - 3/2)/(2\varepsilon^* - 1)$ which
is positive over a narrower range of parameters, namely only if $\varepsilon^* < 3/2$.
We also note that $\gamma_{A-q}$ can be of the same magnitude as $\gamma_A$ and $\gamma_q$ for reasonable parameter
values. As an illustration, suppose that $\beta = 1/2$ and $\varepsilon_A^* = \varepsilon_q^* = 1.4$. These elasticities imply
that increasing the research or learning efforts by 1% costs approximately 1.4% more. Under these
parameters values, $\gamma_A = \gamma_q = 27\%$ while $\gamma_{A-q} = 25\%$.

5.3 No-Learning Regime

In the model, the financier always learns. This is because the cost of learning is assumed to satisfy
$\epsilon'_q(1/2) = 0$. Empirically however, financial institutions only emerge once a critical level of wealth
or technological knowledge have been reached. In this section, we assume that $\epsilon'_q(1/2) > 0$ and
show that learning only takes place for sufficiently developed economies. The following proposition
shows how the financier’s learning decision is altered.

**Proposition 5:** The financier learns if and only if his income exceeds the threshold $w$ given by:

$$w \varphi_A(2^{-\beta} \beta w^\beta) = \epsilon'_q(1/2)/(2^{1-\beta} \beta)^{1/\beta}. \quad (11)$$

22
In this case, his learning effort $q_t$ solves equations 5 and 6.

The proposition is easily understood by inspecting Figure 2. The optimal learning effort is located at the intersection of the declining solid curve (the marginal benefit of learning) and an increasing curve (its marginal cost). If $e'(1/2) = 0$ (the situation considered in the previous sections), then the two curves intersect exactly once. In this case, $w = 0$ and the financier learns regardless of his income. If instead $e'(1/2)$ is large relative to income, the cost curve lies entirely above the benefit curve and the financier does not learn. As income increases, the benefit curve slides upward and eventually crosses the cost curve. Thus, when $e'(1/2) > 0$, the financial sector develops only if the economy is sufficiently rich or technologically advanced.

A variety of dynamic patterns are possible when $e'(1/2) > 0$. If $w_0 < w < w^*_A$ where $w^*_A$ is the steady-state level of income in the no-learning economy, then income grows initially at the rate $\gamma_K + \gamma_A$ and accelerates once learning takes place to grow at the higher rate $\gamma$. If instead $w_0 < w_A^* < w$, then a “no-learning trap” appears: income grows until it reaches $w_A^*$ but it is not high enough to make learning beneficial. Thus, poor countries never learn and remain in a poverty trap, while rich countries can grow without bound. In this situation, policies designed to develop the financial sector and in particular the information environment such as improving accounting standards and corporate transparency, can help escape the no-learning trap. Policies targeted at research can also work if they can push steady-state income $w_A^*$ beyond the threshold $w$.

6 Empirical Content of the Model

We provide in Section 2 a list of observations that motivate the model. In this section, we connect more formally the model to recent empirical studies, and then suggest empirical strategies for evaluating directly the model’s predictions regarding the interplay between learning and research.
6.1 Link to Recent Empirical Studies

Our framework is useful to interpret the methodology used in recent empirical studies, which exploit variations across countries and industries to assess the impact of finance on growth. Fisman and Love (2004), extending the work of Rajan and Zingales (1998), argue that countries are financially segmented but subject to common global shocks to growth opportunities. They document that country pairs in which both countries are more financially developed exhibit more correlated growth rates across sectors, consistent with the view that financial development improves the allocation of resources to the most productive uses. Our framework can be easily extended to multiple financially-segmented countries with different capital stocks and learning efforts but identical projects (reinterpreted as industries). It implies that the correlation between output in countries \( i \) and \( j \), conditional on common productivity shocks, \( \text{corr}(Y_{it}, Y_{jt} | \tilde{A}_n, n \in [0,1]) \), equals \( \sqrt{q_{it}/(1-q_{it})} \ast \sqrt{q_{jt}/(1-q_{jt})} \), which is indeed larger when both countries are more developed (we interpret the learning effort as measures of financial development).

Using a different approach, Wurgler (2000) regresses, for each country, investment growth on value-added growth across industries and finds that the slope coefficient rises with the country’s degree of financial development. In our model, a regression of investments \( K_{it}^n \) on output \( Y_{i,t+1}^n \) \((n\ indexes\ industries)\ yields\ a\ slope, \( \text{cov}(Y_{i,t+1}^n, K_{it}^n)/\text{var}(Y_{i,t+1}^n) = 1/\tilde{A}_t/(2 - q_t), \) increasing in the learning effort, holding constant the research effort. This property reflects the improved investments made in economies with superior information. It is consistent with Wurgler (2000)’s findings to the extent that a country’s degree of financial development proxies for the financier’s learning effort, and that the research effort is controlled for in the regression with GDP per capita (which is included in Wurgler’s regressions). Our framework yields an additional testable prediction, namely that the slope coefficient is also decreasing in the research effort, holding constant the learning effort.
Intuitively, the ratio of the standard deviation of output to that of capital rises with the research effort because of the complementarity between productivity and capital, which then reduces the slope coefficient. This prediction can be tested by regressing the slope coefficient on measures of research activity such as the ratio of R&D expenditures to GDP, controlling for the degree of financial development.

6.2 Empirical Strategies for Testing the Model

The two predictions at the hart of the model are that (i) the financier learns more when the entrepreneur does more research (equation 5), and conversely that (ii) the entrepreneur performs more research when the financier learns more (equation 6). If we assume that the marginal costs of research and learning display constant elasticities, $\varepsilon_q$ and $\varepsilon_A$, then equations 5 and 6 become:

$$\ln(q_t) = \frac{1}{\varepsilon_q - \beta} (\ln \beta + \beta \ln(A_t) + \beta \ln(w_t)),$$

(12)

and

$$\ln(A_t) = \frac{1}{\varepsilon_A - \beta} (\ln \beta + \beta \ln(q_t) + \beta \ln(w_t)).$$

(13)

Testing these relationships requires measures of the learning and research efforts, though neither is directly observable. Various proxies have been used in the literature: data on security analysts can help quantify the learning effort, (e.g. the number of analysts covering a firm or a sector relative to its size, or the accuracy of analysts' earnings forecasts\textsuperscript{17}, while data on R&D expenditures or patenting activity can be used to measure the intensity of innovation. Importantly, testing equations 12 and 13 also calls for a treatment of the biases that these equations induce. First, any shock to capital will stimulate learning and research independently, thereby generating a spurious correlation between them. Moreover, the two-way relationship generates an endogeneity bias. In the case of equation 12 for example, a least squares regression of learning on research (assuming

\textsuperscript{17}See Veldkamp (2009, Chapter 10) for a description of various measures of the quality of information.
both are accurately measured) yields inconsistent estimates because the regression’s residual is correlated with the regressor (research) as suggested by equation 13. Similarly for equation 13, the regressor (learning) is correlated with the residual from the regression of research on learning.

One strategy for addressing these issues is to exploit unanticipated changes in public policies or regulations. For example, Jayaratne and Strahan (1996) and Bertrand, Schoar and Thesmar (2007) report an improvement in allocative efficiency following banking reforms in the U.S. and in France. Equation 13 implies that R&D and patenting activities should also be higher post deregulation, an implication that neither study examines. Conversely, changes to the innovation environment (e.g. changes to the tax treatment of investments in innovations or to immigration laws affecting researchers) can help pin down the impact of research on learning (equation 12).

An alternative approach is to use the framework laid out by Rajan and Zingales (1998) which combines industry and country data, together with instrumental variables. In practice, there are large variations in the use research across sectors for reasons that pertain to their structure rather than to the financing of innovation (e.g. knowledge-based vs. other industries). One would expect the interaction between learning and research to be stronger in sectors that rely more on research. This intuition is consistent with the model to the extent that \( \beta \), the factor share of the intermediate good in the production of the final good, is interpreted as a measure of an industry’s research intensity. Indeed \( \beta \) equals \( \rho_t Y_t / G_t \), the ratio of expenditures in the “research good” (the good that uses research as an input) to output. If we allow \( \beta \) to differ across industries and assume again constant elasticities for the marginal costs of research and learning, assumed further to be identical across industries and countries, then equations 12 and 13 become:

\[
\ln(q_{t,i}^{i,k}) = \frac{1}{\varepsilon_q - \beta^*} \left( \ln \beta^* + \beta^* \ln(A_t^{i,k}) + \beta^* \ln(w_t^{i,k}) \right),
\]
where the indices \( i \) and \( k \) label respectively an industry and a country, \( \beta^i \) is the research intensity in industry \( i \) and \( q_{it}^{i,k} \), \( A_{it}^{i,k} \) and \( w_{it}^{i,k} \) are the learning effort, the research effort and the stock of capital in industry \( i \) and country \( k \). These equations show that research exerts a stronger influence on learning in sectors that rely more on the “research good” \( \partial \ln(q_{it}^{i,k})/\partial \ln(A_{it}^{i,k}) = \beta^i/(\varepsilon_q - \beta^i) \) increases with \( \beta^i \). Learning also exerts a stronger influence on research in these sectors \( \partial \ln(A_{it}^{i,k})/\partial \ln(q_{it}^{i,k}) = \beta^i/(\varepsilon_A - \beta^i) \) increases with \( \beta^i \). One way to measure industries’ reliance on research is to use U.S. data as a benchmark, as Rajan and Zingales (1998) do for industries’ dependence on external finance. For example, the average ratio of R&D expenditures to sales across all U.S. firms in an industry closely matches the model’s definition of \( \beta^i \), which equals spending in the intermediate good as a fraction of sales of the final good, \( \rho_{it}^{i,k} Y_{it}^{i,k}/G_{it}^{i,k} \). Alternatively, the ratio of intangibles to fixed assets can be employed as in Claessens and Laeven (2003). 18

Instrumental variables can be used to address the endogeneity issues. An instrument that identifies the influence of research on learning (respectively of learning on research) needs to be correlated with research (respectively learning) but uncorrelated with learning (respectively research). The law and finance literature suggests several predetermined legal indices that could be employed. 19 For example, a index of patent rights is arguably uncorrelated with learning but positively correlated with research (Kanwar and Evenson (2005)), while an index of information disclosure (accounting standards) is arguably correlated with learning (Lang and Lundholm (1996),

\[ \ln(A_{it}^{i,k}) = \frac{1}{\varepsilon_A - \beta^i} \left( \ln \beta^i + \beta^i \ln(q_{it}^{i,k}) + \beta^i \ln(w_{it}^{i,k}) \right), \]  (15)
Hope (2003)) but not with research. Alternatively, research may be instrumented thanks to the proportion of R&D performed by the public sector since it is plausibly uncorrelated with learning but negatively correlated with research. Indeed, public R&D expenditures crowd out private R&D expenditures (Goosbee (1998)) and our theory does not apply to R&D performed by the public sector to the extent that they are not driven by expected profits. The set of regressions can be specified as follows:

\[ \text{Analyst}^{i,k} = Cst + \Phi^A Z^{A,k} \times \text{Reliance on Research}^{i} + \text{Country dummies}^{k} + \text{Industry dummies}^{i} + \varepsilon^{i,k}, \]

\[ \text{R&D}^{i,k} = Cst + \Phi^q Z^{q,k} \times \text{Reliance on Research}^{i} + \text{Country dummies}^{k} + \text{Industry dummies}^{i} + \nu^{i,k}, \]

where again each country is labeled by index \( k \) and each industry by index \( i \). \( \text{Analyst}^{i,k} \) and \( \text{R&D}^{i,k} \) denote respectively measures of the learning and research intensities in a given country-industry pair, and \( Z^{A,k} \) and \( Z^{q,k} \) denote country-specific instruments respectively for research (e.g. patent rights or R&D performed by the public sector) and for learning (e.g. accounting standards in a country). The country and industry dummies allow to factor out the forces that affect industries and countries on average and to focus on deviations from these averages.\(^{20}\)

7 Conclusion

We develop a model of financial development and technological progress. Its main insight is that knowledge about technologies and technological knowledge are mutually reinforcing. That is, entrepreneurs innovate more when financiers are better informed about their projects because they expect to receive more funding should their projects be successful. Conversely, financiers collect

\(^{20}\text{For example, the country dummies will neutralize the impact of a country’s financial structure, i.e. whether it is market- or bank-based. Indeed, a limitation of using analyst data is that it only incorporates learning about public firms. To the extent that the tendency for firms to go public varies across industries and countries independently – i.e. if firms in industry } i \text{ are more likely to list in country } k \text{ than in country } k' \text{ so should firms in industry } i', \text{ the industry and country dummies included as regressors will absorb these variations and the estimates will be unbiased.}
more information about projects when entrepreneurs innovate more because the opportunity cost of misinvesting, i.e. of allocating capital to unsuccessful projects while missing out on successful projects, is larger.

This positive feedback promotes economic growth and leads to a variety of dynamic patterns. Its beneficial impact is permanent in some cases leading to unbounded income growth, but only transitory in others – when the marginal costs of learning and research are highly elastic and the factor share of capital is low. Poverty traps can also emerge in which only countries that are sufficiently wealthy enjoy the benefits of the feedback. New predictions are derived from the model. They can be tested using an event study or an instrumental variables approach.
A Proof of Proposition 1

We start with the financier’s investment decision. He allocates his wage \( w_t \) across the continuum of projects conceived by the entrepreneur, guided by his signal \( S_t \). At this stage, he takes as given the price of intermediate goods \( p_{t+1} \), the projects’ returns in case of success and failure, \( \overline{A}_t \) and \( A_t \), and the precision of his signal \( q_t \). We denote \( K_t^+ \) the amount of capital allocated to a project deemed successful by the signal, and \( K_t^- \) the amount allocated to a project deemed unsuccessful. There are 1/2 projects in each category. For example, projects deemed successful consist of the \( q_t/2 \) projects that are truly successful and correctly identified, and of the \((1 - q_t)/2 \) projects that are unsuccessful but incorrectly identified, leading to \( q_t/2 + (1 - q_t)/2 = 1/2 \) projects in total. The budget constraint imposes that \( K_t^+/2 + K_t^-/2 = w_t \). Output equals \( Y_t = (q_t/2)\overline{A}_tK_t^+ + [(1 - q_t)/2]\overline{A}_tK_t^- + (q_t/2)A_tK_t^- + [(1 - q_t)/2]A_tK_t^+ \), where the four terms represent respectively the production of successful projects that correctly and incorrectly identified, and the production of unsuccessful projects that correctly and incorrectly identified. Profits follow:

\[
\pi_t = \rho_{t+1}Y_t = \rho_{t+1} \left\{ q_t/2 \left[ \overline{A}_tK_t^+ + A_tK_t^- \right] + (1 - q_t)/2 \left[ A_tK_t^+ + \overline{A}_tK_t^- \right] \right\} = \rho_{t+1} \left\{ (q_t - 1/2)(\overline{A}_t - A_t)K_t^+ + [q_tA_t + (1 - q_t)\overline{A}_t]w_t \right\},
\]

after substituting in the budget constraint. Maximizing this expression with respect to \( K_t^+ \) (provided that the signal is informative, that is \( q_t > 1/2 \)) yields \( K_t^+ = 2w_t \) and \( K_t^- = 0 \), i.e. the financier invests all his capital in the projects revealed by the signal and none to the others. The expected profit simplifies to

\[
\pi_t = \rho_{t+1} \left[ q_t\overline{A}_t + (1 - q_t)A_t \right] w_t \quad \text{(16)}
\]

once the optimal investment plan is set. The price of intermediate goods follows from their output:

\[
\rho_{t+1} = \beta(Y_{t+1})^{\beta - 1} = \beta \left[ q_t\overline{A}_t + (1 - q_t)A_t \right]^{\beta - 1} w_t^{\beta - 1}. \quad \text{(17)}
\]

We turn to the determination of the learning and research efforts. The financier and the entrepreneur who exert efforts \( \overline{A}_t \), \( A_t \) and \( q_t \) expect a surplus of \( \pi_t - e_q(q_t) - e_A(\overline{A}_t + A_t) = \rho_{t+1} \left[ q_t\overline{A}_t + (1 - q_t)A_t \right] w_t - e_q(q_t) - e_A(\overline{A}_t + A_t) \). They maximize this expression with respect to \( \overline{A}_t \), \( A_t \) and \( q_t \), taking the price of intermediate goods \( \rho_{t+1} \) as given. Since \( \overline{A}_t \) and \( A_t \) are perfect substitutes in the cost of research, the optimum with respect to \( A_t \) is the corner solution \( A_t = 0 \) as long as \( q_t > 1/2 \) (since \( q_t > (1 - q_t) \)). The first-order conditions with respect to the learning and research efforts can be written as: \( \rho_{t+1}\overline{A}_tw_t = e_q'(q_t) \) and \( \rho_{t+1}q_tw_t = e_A'(\overline{A}_t) \). They admit a unique solution and it is interior because \( e_q'(1/2) = e_A'(0) = 0 \), \( e_q'(1) = e_A'(\infty) = +\infty \) and \( e_A' \) and \( e_q' \) are monotonic. These conditions correspond to equations 3 and 4. In equilibrium, the price of intermediate goods \( \rho_{t+1} \) is given by equation 17. Substituting this expression into the first-order conditions for \( \overline{A}_t \) and \( q_t \) leads to equations 5 and 6, which characterize the equilibrium effort levels.
B Proof of Proposition 3 and Corollary 4

We first prove the existence of steady-states, and then describe the transition thereto. A steady-state equilibrium is characterized by the following system of equations, obtained from setting $w_{t+1} = w_t = w^*$ in equations 5, 6 and 9:

$$\begin{align*}
    w^* &= (1 - \beta)w^*A^{-1}q^* \\
    \beta q^* &= \frac{A^{1-\beta}e_A(A)}{A^{1+\beta}} q^*
\end{align*}$$

(18)

A trivial solution is $w^* = A^* = q^* = 0$. Assuming $w^*$, $A^*$ and $q^*$ are strictly positive, we can take logs and write the system as

$$\begin{align*}
    -w^* + \frac{1}{\beta} \ln(q^*) + \frac{1}{\beta} \ln(A^*) &= - \frac{1}{\beta} \ln(\beta) \\
    -w^* - \ln(q^*) + \frac{1}{\beta} \ln[A^{1-\beta}e_A(A^*)] &= \frac{1}{\beta} \ln(\beta) \\
    -w^* + \frac{1}{\beta} \ln[q^{1-\beta}e_q(q^*)] - \ln(A^*) &= \frac{1}{\beta} \ln(\beta)
\end{align*}$$

The system’s Jacobian matrix, $J$, is defined as

$$J = \begin{pmatrix}
    -1 & \frac{1}{\beta} & \frac{1}{\beta} \\
    -1 & -1 & \frac{1}{\beta} (\varepsilon_A^* - \beta) \\
    -1 & \frac{1}{\beta} (\varepsilon_q^* - \beta) & -1
\end{pmatrix}$$

(19)

where we use the fact that $\partial \ln[A^{1-\beta}e_A(A^*)]/\partial \ln(A^*) = \varepsilon_A^* - \beta$ and $\partial \ln[q^{1-\beta}e_q(q^*)]/\partial \ln(q^*) = \varepsilon_q^* - \beta$. The determinant of $J$ satisfies $\beta^2 (1 - \beta) \varepsilon_A^* \varepsilon_q^* \det J = 1 - \beta - (1/\varepsilon_A^* + 1/\varepsilon_q^*)$. Because we assume that $1/\varepsilon_A(A^*) + 1/\varepsilon_q(q) - 1/\beta + 1$ never equals, $\det J \neq 0$ for all $A^* > 0$ and $1 > q^* > 1/2$. It follows that there exists a unique non-trivial steady-state.

To study the dynamics, we log-linearize the system around its steady-state. Taylor-series expansions yield $\ln[e_q'(q_t)] \approx \ln[e_q'(q^*)] + \varepsilon_q^* [\ln(q_t) - \ln(q^*)]$ and $\ln[e_A'(A_t)] \approx \ln[e_A'(A^*)] + \varepsilon_A^* [\ln(A_t) - \ln(A^*)]$. We substitute these expressions into equations 5 and 6 after taking logs and using conditions 18 characterizing a steady-state and obtain:

$$\ln(A_t/A^*) \approx \frac{\beta}{\varepsilon_A^* - \beta} [\ln(q_t/q^*) + \ln(w_t/w^*)],$$

(20)

and

$$\ln(q_t/q^*) \approx \frac{\beta}{\varepsilon_q^* - \beta} \left[\ln(A_t/A^*) + \ln(w_t/w^*)\right].$$

(21)

Solving for $\ln(A_t/A^*)$ and $\ln(q_t/q^*)$ yields

$$\ln(q_t/q^*) \approx \frac{\beta \varepsilon_A^*}{x} \ln(w_t/w^*) \quad \text{and} \quad \ln(A_t/A^*) \approx \frac{\beta \varepsilon_q^*}{x} \ln(w_t/w^*),$$

(22)

where $x \equiv (\varepsilon_A^* - \beta)(\varepsilon_q^* - \beta) - \beta^2$. $x$ is positive if and only if $1/\varepsilon_A^* + 1/\varepsilon_q^* < 1/\beta$. Finally, we take the log of equation 9 and use conditions 18 to write:
\[ \ln(w_{t+1}/w^*) \approx \beta \ln(q_t/q^*) + \beta \ln(\overline{A}_t/\overline{A}) + \beta \ln(w_t/w^*). \]  

Substituting the expressions for \( \ln(\overline{A}_t/\overline{A}) \) and \( \ln(q_t/q^*) \) back into this equation leads to

\[ \ln(w_{t+1}/w^*) \approx (\gamma + 1) \ln(w_t/w^*), \]  

where \( \gamma = \beta \varepsilon^*_A \varepsilon^*_q / \varepsilon \) is identical to the expression provided in Corollary 4. Income grows if \( \gamma > -1 \) (i.e. \( 1/\varepsilon^*_A + 1/\varepsilon^*_q < 1/\beta \)), at a declining rate if \( \gamma < 0 \) and at an expanding rate if \( \gamma > 0 \). The former occurs if \( 1/\varepsilon^*_A + 1/\varepsilon^*_q < 1/\beta - 1 \) and the latter if \( 1/\varepsilon^*_A + 1/\varepsilon^*_q > 1/\beta - 1 \). If instead \( \gamma < -1 \) (i.e. \( 1/\varepsilon^*_A + 1/\varepsilon^*_q > 1/\beta \)), then income oscillates. The cycles are unstable because \( \gamma < -1 \) implies that \( \gamma < -2 \).

C Proof of Proposition 5

The proof proceeds like that of Proposition 1. The first-order condition with respect to the learning effort, \( \rho_{t+1} \overline{A}_t w_t = e'_q(q_t) \), still admits a unique solution since \( e'_q \) is monotonic but it is interior only if \( e'_q(1/2) < \rho_{t+1} \overline{A}_t w_t \). Otherwise the financier does not learn: \( q_t^1 = 1/2 \) if \( e'_q(1/2) > \rho_{t+1} \overline{A}_t w_t \).

The first-order condition with respect to the research effort is given by equation 4. In equilibrium, the learning and research efforts are the unique solutions to the following system of equations:

\[
\begin{align*}
q_t &= 1/2 & \text{if } e'_q(1/2) > 2^{\beta-1} \beta \overline{A}_t^\beta w_t^\beta \\
\frac{\beta \overline{A}_t^\beta w_t^\beta}{q_t^{1-\beta}} &= e'_q(q_t) & \text{if } e'_q(1/2) < 2^{\beta-1} \beta \overline{A}_t^\beta w_t^\beta \\
\text{and } \frac{\beta q_t^3 w_t^\beta}{\overline{A}_t^{1-\beta}} &= e'_A(\overline{A}_t)
\end{align*}
\]

Thus learning only takes place if \( e'_q(1/2) < 2^{\beta-1} \beta \overline{A}_t^\beta w_t^\beta \). The wealth threshold \( w \) above which learning takes place is obtained by plugging equation 8 into equation ??, which leads to equation 11 presented in Proposition 5.

D Controlling Projects’ Probability of Success

We solve here a version of the model in which the entrepreneur controls projects’ probability of success rather than their return. Projects’ returns are exogenously set to 1 in case of success and 0 in case of failure. Creating projects with a success probability \( p \) requires a research effort \( e_p(p) \) where \( e_p \) is continuous, increasing, convex, \( e_p(0) = e'_p(0) = 0, e'_p(1) = +\infty \) and \( e_p(p) \equiv (1 + \rho_p^n)/e_p(p)/e'_p(p) > 0 \).

The financier allocates \( K_t^+ \) units of capital to a project deemed successful by his signal, and \( K_t^- \) to a project deemed unsuccessful. The former consist of successful projects correctly identified –
there are \( p_t q_t \) of them, and unsuccessful projects incorrectly identified – in number \((1-p_t)(1-q_t)\), leading to \( p_t q_t + (1-p_t)(1-q_t) \) projects in total. Similarly, the number of projects deemed unsuccessful equals \((1-p_t)q_t + p_t(1-q_t)\). The budget constraint imposes that \([p_t q_t + (1-p_t)(1-q_t)] K_t^- + [(1-p_t)q_t + p_t(1-q_t)] K_t^+ = w_t\). Output equals \( Y_t = p_t q_t K_t^- + p_t(1-q_t) K_t^+\), where the first term represents the production of correctly identified successful projects, and the second term that of incorrectly identified successful projects. Substituting in the budget constraint and maximizing this expression leads to \( K_t^+ = w_t/[p_t q_t + (1-p_t)(1-q_t)]\), \( K_t^- = 0\), \( Y_{t+1} w_t \omega_t \) and \( \pi(p_t, q_t, w_t) = \rho_{t+1} w_t \omega_t \) where \( \omega_t \equiv p_t q_t/[p_t q_t + (1-p_t)(1-q_t)]\). We note that these expressions match those obtained in the main model if \( p_t = 1/2 \) and \( \mathcal{A}_t = 1 \) (equations 16 and 17).

Maximizing the surplus, \( \pi(p_t, q_t, w_t) - e_q(q_t) - e_p(p_t) \), with respect to \( p_t \) and \( q_t \), taking the price of intermediate goods \( \rho_{t+1} \) as given yields the first-order conditions:

\[
\rho_{t+1} w_t (1 - \omega_t)^2/(1 - p_t)^2 = \epsilon'_q(q_t) \quad \text{and} \quad \rho_{t+1} w_t (1 - \omega_t)^2/(1 - q_t)^2 = \epsilon'_p(p_t).
\]

Finally, substituting \( \rho_{t+1} = \beta(\omega_t w_t)^{\beta-1} \) into these equations leads to the conditions characterizing the equilibrium:

\[
\frac{\beta w_t^\beta (1 - \omega_t)^2}{\omega_t^{1-\beta}(1 - p_t)^2} = \epsilon'_q(q_t) \quad \text{and} \quad \frac{\beta w_t^\beta (1 - \omega_t)^2}{\omega_t^{1-\beta}(1 - q_t)^2} = \epsilon'_p(p_t).
\]

We assume from now on that \( p_t \) is small, i.e. projects are very risky (this happens in particular if \( e_p \) is very strongly convex). The existence of an interior solution to equations 25 is guaranteed for any \( \rho_{t+1} \) if \( e_q \) is sufficiently convex, namely if \( \epsilon_q(q) > (1+q)/(1-q) \) for \( 1 > q > 1/2 \). Equations 26 simplify to

\[
\beta w_t^\beta p_t^{1-\beta} = q_t^{1-\beta}(1-q_t)^{1+\beta} \epsilon'_q(q_t) \quad \text{and} \quad \beta w_t^\beta q_t^{1-\beta} = p_t^{1-\beta}(1-q_t)^{1+\beta} \epsilon'_p(p_t).
\]

The equilibrium properties of learning and research are the same as in the main model (Corollary 2): the financier learns more when the entrepreneur carries out more research (\( q_t \) increases with \( p_t \) holding \( w_t \) fixed), while the entrepreneur carries out more research when the financier learns more (\( p_t \) increases with \( q_t \) holding \( w_t \) fixed).

The dynamic system is characterized by equations 26 and 9, and the initial income \( w_0 \). It admits two steady-states, one of which is zero. The transition to the steady-states \( \hat{\omega}^* > 0 \) is governed by the following equation:

\[
\ln(w_{t+1}/\hat{\omega}^*) \approx (\hat{\gamma} + 1) \ln(w_t/\hat{\omega}^*), \quad \text{where} \quad \frac{1}{\hat{\gamma} + 1} = \frac{1}{\beta} - \frac{1}{\epsilon_p^*} - \frac{1}{\epsilon_q^* - (1+\epsilon_q^*)q^*}.
\]

If \( 1/\epsilon_p^* - 1/[\epsilon_q^* - (1+\epsilon_q^*)q^*] < 1/\beta \), then income grows (note that the second term is positive since \( \epsilon_q^* > (1+q^*)/(1-q^*) \) by assumption). Its growth rate declines if \( \hat{\gamma} < 0 \) (\( \hat{\omega}^* \) is a stable steady-state) and expands if \( \hat{\gamma} > 0 \) (unbounded growth). The former occurs if \( 1/\epsilon_p^* - 1/[\epsilon_q^* - (1+\epsilon_q^*)q^*] < 1/\beta - 1 \) and the latter if \( 1/\epsilon_p^* - 1/[\epsilon_q^* - (1+\epsilon_q^*)q^*] > 1/\beta - 1 \). These dynamic patterns are similar to those derived in the main model.
References


**Figure 1: Timing.**

<table>
<thead>
<tr>
<th>Generation $t$</th>
<th>Young</th>
<th>Old</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Entrepreneur and financier choose:</strong></td>
<td>• Earns wage $w_t$</td>
<td>• Intermediate goods are produced</td>
</tr>
<tr>
<td>• Research effort $A_t$</td>
<td>• Observes signal $S_t$</td>
<td>• Final goods are produced</td>
</tr>
<tr>
<td>• Learning effort $q_t$</td>
<td>• Invests $K^a_t$ across projects</td>
<td>• Agents consume</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Generation $t+1$</th>
<th>Young</th>
<th>Old</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Entrepreneur and financier choose:</strong></td>
<td>• Earns wage $w_{t+1}$</td>
<td>• Intermediate goods are produced</td>
</tr>
<tr>
<td>• Research effort $A_{t+1}$</td>
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<td>• Final goods are produced</td>
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<td>• Invests $K^a_{t+1}$ across projects</td>
<td>• Agents consume</td>
</tr>
</tbody>
</table>
Figure 2: The learning effort. The picture displays the marginal benefit of learning (solid curve) and its marginal cost (dotted and dashed curves) as a function of the learning effort $q_t$. The dotted curve assumes that $e'_q(q) = [(q-1/2)/(1-q)]^{0.4}$ and the dashed curve that $e'_q(q) = [(q-1/2)/(1-q)]^{0.4} + 1$. The optimal learning effort lies at the intersection of the solid and dotted curves in the first case, and in the corner ($q_t = 1/2$) in the second case. The other parameters are $\beta = 1/2$ and $w_t = A_t = 1$. 

\[ e'_q(q) = \frac{(q_0q_t - 1/2)}{(1-q_t)}^{0.4} \]

\[ e'_q(q) = \frac{(q-1/2)}{(1-q)}^{0.4} + 1 \]