Firm Innovation and Financial Analysis: How Do They Interact?

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ABSTRACT

Entrepreneurs innovate more when financiers are better informed about their projects because they expect to receive more funding should their projects be successful. Conversely, financiers collect more information about projects when entrepreneurs innovate more because the opportunity cost of misinvesting, i.e. of missing out on successful projects, is higher. Thus, technological knowledge and knowledge about technologies are mutually reinforcing. We report evidence consistent with this interaction using two quasi-natural experiments that changed, respectively, the innovation incentives and the information environment for U.S. listed firms. A calibration suggests that its contribution to income growth represents more than one third of the total contributions of information collection and innovation. We also estimate that a policy designed to stimulate innovation has an indirect effect through investors’ learning incentives that accounts for a third of the total effect of the policy on firms’ innovation incentives.

JEL classification codes: G20, O31, O4

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1 Introduction

This paper studies the interplay between innovations carried out by firms and the analysis investors make of their prospects. This interplay operates as follows. Entrepreneurs innovate more when financiers are better informed about the profitability of their projects because they expect to receive more capital should their projects be successful. Conversely, financiers collect more information about projects when entrepreneurs innovate more because the opportunity cost of misinvesting, i.e. of funding unsuccessful projects while missing out on successful projects, is higher. Thus, knowledge about technologies (financial analysis) and technological knowledge (firm innovation) are mutually reinforcing. We develop a model to formalize this insight and evaluate it empirically. We find strong evidence that this interaction is at work in reality, and estimate that its contribution to income growth represents from one third to 40% of the total contributions of innovation and financial analysis.

The model is purposely simple. It serves to highlight the ingredients needed to generate our effect and to structure the quantitative analysis. It features competitive rational agents who conceive risky projects, learn about their prospects and invest in them. Costs have to be incurred both for innovating (what we call “research”) and for financial analysis (what we call “learning”). Unlike other papers (the related literature is discussed below), the positive feedback between research and learning is not a consequence of risk sharing since risk is fully diversified away, nor moral hazard since efforts are contractible. Instead, it simply follows from the complementarity between productivity and capital. Expressing output as \( Y = AK^\alpha \) where \( \alpha \) is a positive parameter and \( A \) and \( K \) denote respectively a project’s uncertain return and the amount of capital it attracts, shows that the return on financiers’ funds increases with \( A \) (every unit of capital yields a larger payoff) while the reward for research rises with \( K \) (an invention can be applied on a larger scale). The complementarity between \( A \) and \( K \) leads to the complementarity between research and learning, which is the focus of our study.

Over time, technologies improve along with investors’ information about them and the allocation of capital, leading to total factor productivity (TFP) growth and, in turn, to income growth. We compute the contributions of learning and research and of their interplay, and show that the growth rate of income in the economy with both
learning and research is larger than the *sum* of the growth rates in an economy with research but no learning and in an economy with learning but no research—once again a reflection of the positive feedback between learning and research. This implies for example that an economy that would converge to a steady-state when learning and research do not interact can experience unbounded growth once they do. The model is consistent with stylized facts about the link between finance and growth, to the extent that financial development is positively correlated with the quality of financiers’ knowledge about projects.¹

We evaluate empirically the model’s main predictions in a sample of U.S. publicly-listed firms. Specifically, the model predicts that (i) financiers learn more when entrepreneurs perform more research, and that (ii) entrepreneurs perform more research when financiers learn more. Empirically assessing these relationships requires proxies for research and learning, as well as a methodology to address the endogeneity bias that this two-way relationship generates.² We measure firms’ research effort as their R&D expenditures, and financiers’ learning effort about a firm as the number of financial analysts who follow the firm. To address the endogeneity of the relationships, we resort to difference-in-difference estimates from two quasi-natural experiments that respectively shift firms’ innovation effort and financial sector learning between 1992 and 2006.

The first experiment exploits the staggered implementation of R&D tax credits by U.S. states over that period (Wilson (2009)). After confirming the beneficial effect of these tax incentives to innovation, we show that, following their passage, analysts significantly increase their coverage of firms subject to the tax credits compared to other firms in the country, as predicted by our model. Specifically, we estimate that the sensitivity of analyst coverage to R&D expenditures is 1.2, i.e., a 10% increase in R&D expenditures induces a 12% increase in analyst following, or the addition of about one new analyst. We also test auxiliary implications of the model, specifically that an increase in investors’ learning intensity leads to a more dispersed distribution of both capital and return on capital, as investors channel more (less) funds to the firms they consider more (less) efficient. We

¹To keep the model parsimonious, we do not model the financial sector explicitly. But we interpret the amount of resources devoted to analyzing investment opportunities as a proxy for the degree of financial development.

²In the case of prediction (i) for example, a least squares regression of learning on research yields inconsistent estimates because the regression’s residual is correlated with the regressor—research, as implied by the second prediction.
find empirical support for both predictions: after the passage of an R&D tax credit, new equity proceeds and
the return on assets are significantly more dispersed across firms located in the treatment state, compared to
other states.

Our second experiment uses the identification strategy pioneered by Hong and Kacperczyk (2010) and
extended, among others, by Kelly and Ljungqvist (2012) and Derrien and Kecskes (2013). These last authors
consider closure of and mergers between brokerage houses which lead to the dismissal of analysts, and provide
evidence that the resulting drop in analyst coverage is largely exogenous to firms’ policies. We document that
firms that lose analysts as a result of broker events significantly reduce their R&D expenditures relative to
unaffected firms, in line with our model. Specifically, we estimate that the sensitivity of R&D expenditures to
analyst coverage is 0.3, i.e., a 10% drop in analyst coverage, or the loss of about one analyst, triggers a 3%
decrease in R&D expenditures.

The magnitude of the interaction effect is economically important. We estimate that the indirect effect of
an R&D tax credit on innovation, operating through analysts’ response, is about one third of the size of its
total effect. Going back to the previous examples, a 10% increase in R&D expenditures triggered by an R&D
tax credit increases coverage by about one analyst (12%), which in turn is responsible for around 3.6% of the
total 10% increase in R&D expenditures. We also quantify the contribution of the interaction effect to income
growth. After calibrating the model using parameter estimates derived from our two experiments, we estimate
that it represents from one third to 40% of the total contributions of learning and innovation to income growth.

Our analysis yields important insights on the effectiveness of policies aimed at promoting innovations (e.g.,
research subsidies or tax breaks). First, it suggests that such policies have a multiplier effect thanks to the
induced improvement in capital efficiency. Given our aforementioned estimates, the observed increase in R&D
expenditures triggered by an R&D tax credit is for about two thirds attributable to the direct effect of the tax
credit, and for one third attributable to the indirect effect of enhanced learning by the financial sector, which
further stimulates R&D. Second, policies based on R&D incentives can be rendered more effective by coupling
them with policies designed at increasing capital efficiency, such as encouraging equity research, improving accounting standards, or reducing impediments to trading financial assets.

We are not the first to model the interaction between financial analysis and innovation under imperfect information. In Bhattacharya and Chiesa (1995), De la Fuente and Marin (1996), Acemoglu and Zilibotti (1999) and Acemoglu, Aghion and Zilibotti (2006), financiers supply capital to entrepreneurs whose effort they can only monitor at a cost. In Bhattacharya and Ritter (1983), King and Levine (1993), Ueda (2004), Aghion, Howitt and Mayer-Foulkes (2005), they do not observe entrepreneurs’ ability. We assume away these moral hazard and adverse selection problems, and show instead how the mutually reinforcing influence of learning and research can arise as a first-best outcome in a setting free from contracting frictions and information asymmetry. In our setup, the entrepreneur and the financier coordinate to overcome the uncertainty inherent to the early stages of the innovation process. Though none knows at the time whether an invention will be a success, the entrepreneur needs to know that she will get financial backing should it be so. Only under such an understanding, will she agree to put in the effort needed for a major breakthrough. Conversely, the financier is keener to investigate technologies which have the potential to be breakthroughs.

The paper proceeds as follows. Section 2 presents the model. Section 3 discusses the empirical strategy. Section 4 describes the data. Section 5 displays the empirical results, including a calibration of the model. Section 6 concludes. The appendix contains proofs and extensions of the model.

2 The Model

A simple model describes the interaction between technological innovations and investors’ information about them. It yields novel predictions, which are then tested using quasi-natural experiments, and allows to quantify this interaction. Its main results are presented in the body of the paper, and the derivations are left for the appendix, together with some extensions.
2.1 Setup

The economy is composed of two sectors—a final and an intermediate goods sector—and two types of agents: entrepreneurs who conceive the projects that compose the intermediate sector, and financiers who invest in them. The model features two dates (periods 1 and 2), and is later extended to multiple periods in order to evaluate its implications for long run growth.

2.1.1 Agents

The population consists of a representative entrepreneur (‘she’) and a representative financier (‘he’). Both are risk neutral and only consume in period 2.

**The Entrepreneur.** The entrepreneur creates the technologies that produce intermediate goods. In period 1, she conceives a continuum of projects with unit mass indexed by \( n \in [0, 1] \). Their output is determined by a technology with decreasing returns to capital, \( \hat{Y}^n = \tilde{A}^n(K^n)^\alpha \), where \( K^n \) is the amount of capital invested in project \( n \) in period 1, \( \tilde{A}^n \) is its random productivity (which can be discovered in period 1—see below), \( \hat{Y}^n \) is the quantity of intermediate good it yields in period 2 net of capital depreciation, and \( \alpha \) is a parameter between 0 and 1 which determines the degree of returns to scale.\(^3\)

Projects are independent from one another (\( \tilde{A}^n \) independent of \( \tilde{A}^m \) for any \( m \neq n \)). They succeed with a 0.5 probability. Successful projects yield a productivity \( \tilde{A}^n = \bar{A} \), while unsuccessful projects yield \( \tilde{A}^n = A \) for all \( n \), where \( \bar{A} > A > 0 \). Productivity in case of success and failure, \( \bar{A} \) and \( A \), are chosen by the entrepreneur (who has no influence on the probability of success). Creating a continuum of independent projects with productivity \( \bar{A} \) and \( A \) costs \( e_A(\bar{A} + A) \) in research where \( e_A \) is continuous, increasing, convex, \( e_A(0) = e_A'(0) = 0 \) and \( e_A'(+\infty) = +\infty \). Under this formulation, productivities in case of success and failure are perfect substitutes in terms of their cost. We refer to \( \bar{A} \) and \( A \) as productivity or loosely as the research effort. The entrepreneur raises the capital required to operate her technologies from the financier.

**The Financier.** The financier is endowed with wealth \( w \), which he invests in the projects set up by the

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\(^3\)The model nests the case of constant returns to scale, which can be obtained by driving \( \alpha \) to 1 in the formulas below, noting that \( \lim_{\alpha \to 1}[(q^{1-\alpha} + (1 - q))^{1-\alpha}]^{1-\alpha} = q \).
entrepreneur. He allocates $K^n$ units of capital to project $n$ in period 1. At the time of investment, the financier does not know which projects will succeed. Instead, he receives a continuum of imperfect signals $\tilde{S}^n$ that reveal the successful projects. A signal is right with a probability $q$ but wrong with a probability $1 - q$. That is, out of the 0.5 successful projects (respectively, unsuccessful projects), $q/2$ are accurately identified as successes (respectively, failures), while the remaining $(1 - q)/2$ projects are mislabeled as failures (respectively successes). Observing signals of precision $q$ costs $e_q(q)$, where $e_q$ is continuous, increasing, convex, $e'_q$ is convex, $e_q(1/2) = e'_q(1/2) = e''_q(1/2) = 0$ and $e'_q(1) = +\infty$. $q = 1/2$ corresponds to uninformative signals, and $q = 1$ to perfect signals. As with the entrepreneur, the financier’s chosen effort level applies to all the projects (that is, $q^n = q^m$ for any $m \neq n$). Unlike research, learning does not affect projects’ productivity. Instead, it allows to match capital with projects more efficiently. We refer to $q$ as the precision of information or loosely as the learning effort.

2.1.2 Technologies

The economy is composed of two competitive sectors, a final and an intermediate good sector. The intermediate goods sector is made up of the projects conceived by the entrepreneur and funded by the financier. The intermediate goods are used as inputs in the production of the final good, according to a riskless technology, $G \equiv Y^\beta$, where $G$ is final output (used as the numeraire), $Y$ is the employment of intermediate good and $0 < \beta < 1$ is the factor share of the intermediate good in the production of the final good. Many identical firms compete in the final good sector and aggregate to one representative firm.

2.1.3 Timing

At the start of period 1, the entrepreneur and the financier determine their research and learning efforts. Then the financier observes his signals and distributes his wealth across the projects. In period 2, the successful projects are revealed, final goods are produced and agents consume their share of profits.

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4Empirically, young publicly-traded firms finance their R&D investment almost entirely through retained earnings and external equity (Brown, Fazzari and Petersen (2009)).
2.1.4 Discussion of the Model’s Assumptions

Our model offers a parsimonious description of the interplay between technological innovations and investors’ information about them. An important assumption is that effort levels (in research and learning) are contractible. That is, they are determined ex ante cooperatively by the entrepreneur and the financier. As a result, the first-best outcome is achieved. More generally, there are no information asymmetries in the model: initially, i.e. at the time they choose their efforts, the entrepreneur and the financier are equally ignorant about which projects will be successful. While information asymmetries may also be present or emerge at a later stage, our focus is on the interplay between ex ante efforts during the initial contracting phase in the face of incomplete information.

Other assumptions can be relaxed without significantly altering our findings. They obtain if the entrepreneur controls projects’ probability of success instead of their productivity, as shown in Appendix C.1 in the case of constant returns to scale technologies. They also obtain under a more general cost structure for research, $e_A(\overline{A}, A)$, as long as $e_A$ is increasing and convex in each variable. The assumptions on the cost functions are not necessary but merely sufficient to guarantee the existence and unicity of an equilibrium. For example, the condition $e_q'(1/2) = 0$ can be replaced with $e_q'(1/2) > 0$, thus creating a no-learning regime for low levels of the financier’s wealth, without altering investors’ learning decisions outside of this wealth range, as Appendix C.2 demonstrates. Risk neutrality can be relaxed in favor of any increasing concave utility function since there is no aggregate risk. We could drop the final good sector (its purpose is only to aggregate the output produced by multiple projects) and assume instead that agents derive utility directly from the consumption of intermediate goods.

{The assumption that agents choose an effort equal across projects simplifies the analysis by ensuring that they actually develop a large number of projects rather concentrate their efforts on a few. Alternatively, we could assume that projects admit an upper bound on how much capital they can support.}

5The fact that efforts are contractible implies that multiple equilibria do not arise.
2.2 Equilibrium Characterization

Three conditions define an equilibrium.

1. **Market clearing in the intermediate goods sector.** Final goods producers maximize their profit. Since intermediate goods trade in a competitive market, their equilibrium price in period 2 is \( \rho = \beta Y^{\beta - 1} \), where \( Y = \int_n A^n(K^n)^\alpha \) sums up output over all projects. There is no aggregate risk in this economy, so \( \rho \) and \( Y \) are deterministic.

2. **Capital allocation.** After observing his signals \( \{\tilde{S}^n\}_{n \in [0,1]} \), the financier distributes his wealth \( w \) across the projects offered by the entrepreneur to maximize total expected profits, taking \( \rho, A, A \) and \( q \) as given:

\[
\pi(q, \bar{A}, A, w) \equiv \max_{\{K^n\}_{n \in [0,1]}} E \left( \rho \int_n \tilde{A}^n(K^n)^\alpha \mid \{\tilde{S}^n\}_{n \in [0,1]} \right) \quad \text{subject to} \quad \int_n K^n = w. \tag{1}
\]

As with output, \( \pi \) is deterministic.

3. **First-best effort levels.** The entrepreneur and the financier determine cooperatively their effort levels (before the signals \( \tilde{S}^n \) are observed) to maximize the *ex ante* total surplus, which equals the expected profit net of effort costs, taking the price of intermediate goods \( \rho \) as given (the model is agnostic about how this surplus is shared between the entrepreneur and the financier):

\[
\max_{q, \bar{A}, A} \pi(q, \bar{A}, A, w) - e_q(q) - e_A(\bar{A} + A). \tag{2}
\]

The following proposition, proven in Appendix A, characterizes the equilibrium.

**Proposition 1:** In equilibrium, the learning and research efforts, \( q, A \) and \( \bar{A} \), are the unique solutions to the following system of equations:

\[
\beta w^{\alpha \beta} V(q)^\beta = 2^{\beta(1-\alpha)}(1-\alpha) A^{1-\beta} e'_{A}(\bar{A}) \quad \text{and} \quad A = 0 \tag{3}
\]

and

\[
\beta w^{\alpha \beta} \bar{A}^{\beta} \left[ q^{\frac{\alpha}{1-\alpha}} - (1 - q)^{\frac{\alpha}{1-\alpha}} \right] = 2^{\beta(1-\alpha)}V(q)^{1/(1-\alpha) - \beta} e'_q(q) \tag{4}
\]

where \( V(q) \equiv \left( q^{\frac{1}{1-\alpha}} + (1 - q)^{\frac{1}{1-\alpha}} \right)^{1-\alpha} \).

The financier allocates \( 2w(q/V(q))^{\frac{1}{1-\alpha}} \) units of capital to a project deemed successful by his signal, and \( 2w((1 - q)/(V(q)))^{\frac{1}{1-\alpha}} \) to a project deemed unsuccessful.
The entrepreneur chooses to concentrate her effort on improving returns in case of success, setting $\overline{A}$ to 0. Indeed, increasing the productivity in case of success, $\overline{A}$, is more beneficial than increasing it in case of failure, $\overline{A}$, (because successful projects receive more capital), while costing the same (because the cost of research $\epsilon A$ is a function of the sum $\overline{A} + A$). In equilibrium ($A = 0$), the amounts invested only depend on the learning effort $q$. The research effort does not matter because it scales by an identical factor the values of projects expected to succeed and of those expected to fail. We turn to the properties of the equilibrium.

2.3 Equilibrium Properties

The next proposition describes how learning and research efforts interact in equilibrium.

**Proposition 2**  The research effort is increasing in the learning effort. Conversely, the learning effort is increasing in the research effort.

The first part of the proposition follows from noting that the function $V$ (defined in Proposition 1) is increasing in the learning effort $q$ (for any $q > 1/2$), so that, from equation 3, the research effort $\overline{A}$ increases with $q$, holding wealth fixed. Intuitively, research is promoted when the financier is better-informed because the entrepreneur knows that she will receive more funds should her project succeed. Conversely, equation 4 shows that the learning effort $q$ increases with the research effort $\overline{A}$, holding wealth fixed. Indeed, a higher research effort encourages the financier to learn by magnifying the return differential between successful and failed projects. This raises the opportunity cost of misinvesting, i.e. of funding unsuccessful projects while missing out on successful projects. Thus, knowledge about technologies and technological knowledge are mutually reinforcing.

The positive feedback between research and learning follows from the complementarity between productivity $\tilde{A}^n$ and capital $K^n$ in the production of intermediate goods. Since $\tilde{Y}^n \equiv \tilde{A}^n(K^n)^\alpha$, the return on the financier’s funds increases with $\tilde{A}^n$ because every unit of capital is more productive the larger $\tilde{A}^n$. Similarly, the reward for innovating rises with $K^n$ because an invention is applied on a larger scale. Thus, the complementarity between
\( \tilde{A}^n \) and \( K^n \) leads to a complementarity between learning and research.\(^6\)

The next proposition characterizes the distribution of capital across projects. In Appendix A, we establish:

**Proposition 3**: Capital is more dispersed across projects when the learning effort is higher (holding fixed the research effort and wealth). In contrast, the dispersion of capital does not depend on the research effort (holding fixed the learning effort and wealth). Formally: \( \vartheta \text{Var}(K^n)/\partial q > 0 \) and \( \vartheta \text{Var}(K^n)/\partial \tilde{A} = 0 \)

Intuitively, a better-informed financier chooses more drastic positions: he allocates more funds to projects deemed successes and less to those deemed failures, leading to a more uneven distribution of capital. In the case of constant returns to scale in the intermediate goods sector (i.e., \( \alpha = 1 \)), the dispersion is extreme with projects considered to be failures receiving no funding at all. The research effort, in contrast, has no bearing on the dispersion of capital. The reason is that, in equilibrium (\( \tilde{A} = 0 \)), the amounts invested do not depend on the research effort, \( \tilde{A} \). Our last proposition describes the distribution of returns across projects. In Appendix A, we show:

**Proposition 4**: The return on capital is more dispersed across projects when the learning or research efforts are higher (holding wealth fixed). Formally: \( \vartheta \text{Var}[\tilde{A}^n(K^n)^{\alpha - 1}]/\partial q > 0 \) and \( \vartheta \text{Var}[\tilde{A}^n(K^n)^{\alpha - 1}]/\partial \tilde{A} > 0 \).

As noted in Proposition 3, a better-informed financier channels more capital to more productive projects at the expense of less productive ones. In so doing, he tends to equalize their returns. Proposition 4 establishes that, in spite of this tendency, returns grow more dispersed with the learning effort. An increase in the research effort also increases this dispersion since it magnifies the productivity difference between successful and unsuccessful projects, \( \tilde{A} - \tilde{A} \).

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\(^6\)This intuition is easily formalized in the case of constant returns to scale in the intermediate goods sector (i.e., \( \alpha = 1 \)). In that case, the average quantity of goods produced by a project can be broken down into the contributions of projects’ average productivity, \( E(\tilde{A}^n) = (\tilde{A} + \tilde{A})/2 \), of the average stock of capital per project, \( E(K^n) = w \), and of the quality of the match between projects and capital, captured by \( \text{cov}(\tilde{A}^n, K^n) = (q - 1/2)(\tilde{A} - \tilde{A})w \):

\[
E(\tilde{A}^n K^n) = E(\tilde{A}^n)E(K^n) + \text{cov}(\tilde{A}^n, K^n) = q\tilde{A} + (1 - q)\tilde{A}w = q\tilde{A}w.
\]

Thus, average output increases with the product of the precision of the financier’s information \( q \) with the return on the successful project \( \tilde{A} \).
2.4 Dynamic Extension

We present a dynamic extension of the model in order to evaluate, qualitatively and quantitatively, how the interplay between learning and research influences long term growth. We chain together a sequence of static models. Specifically, the economy is now populated by overlapping generations of entrepreneurs and financiers who live for two periods, as displayed in Figure 1. Projects last two periods and are liquidated immediately after production. The final good sector now employs labor, $L_t$, in addition to intermediate goods, according to the technology $G_t \equiv L_t^{1-\beta}Y_t^{\beta}$. The financier supplies one unit of labor inelastically for a competitive wage $w_t$, which he then invests in the entrepreneur’s projects. Thus, we endogenize the financier’s wealth by assuming it is equal to his labor income $w_t$. The wage in turn equals the marginal product of labor, $(1 - \beta)Y_t^{\beta}$, where $Y_t = \int_n A^n_{t-1}(K_{t-1})^{\alpha}$ is determined by the efforts chosen by agents from the previous generation. There is no population growth. One generation’s research and learning efforts determine the stock of capital for the next generation and hence the productivity of its labor.

2.4.1 The Sources of Growth

Income in period $t+1$, $w_{t+1}$, is related to the precision of information $q_t$ and productivity $A_t$ in period $t$ through the following equation:

$$w_{t+1} = (1 - \beta)Y_{t+1}^{\beta} = (1 - \beta) * w_t^{\beta} * TFP_{q}(q_t) * TFP_{A}(A_t)$$

where $TFP_{A}(A_t) \equiv A_t^{\beta}$ and $TFP_{q}(q_t) \equiv V(q_t)^{\beta}$. Together with equations 3 and 4 (in which all three variables are contemporaneous: $A_t$, $q_t$ and $w_t$) and an initial income level $w_0$, equation 5 describes fully the dynamics of the economy. It identifies the three forces that determine income growth. First, current income matters for next period’s income in the usual neoclassical way because it determines the aggregate capital stock: the marginal product of labor increases with current income but at a declining rate (the $w_t^{\beta}$ term). The other two forces operate through TFP, and can be broken down into the impact of research, $TFP_A$, and learning, $TFP_q$. 

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The first component, $TFP_A$, is the focus of the endogenous growth literature, which acknowledges that technology can be improved by purposeful activity, such as R&D. In our framework, this channel can be identified by freezing $q_t$. Our model relates the incentive to innovate to the state of the financial sector, i.e. it highlights the dependence of the economy’s productivity, $A_t$, on the quality of investment knowledge, $q_t$.

The second component, $TFP_q$, is the focus of the financial development literature, which highlights the role of frictions that reduce the efficiency of investments. Examples include information limitations (e.g. Greenwood and Jovanovic (1990)) and investment indivisibilities (e.g. Acemoglu and Zilibotti (1997)). This channel can be identified in the model by freezing $A_t$. An alternative interpretation of our model is that it shows how the incentive to mitigate investment inefficiencies depends on the level of the technology, i.e. how $q_t$ depends on $A_t$.

Importantly, since learning and research influence each other, they each make both direct and indirect contributions to economic growth. To capture the total effect of learning, one should also take into account its positive influence on entrepreneurs’ incentive to innovate, i.e., its impact on $TFP_A$. Conversely, the full benefit of research consists of its direct effect through $TFP_A$ and its indirect effect through $TFP_q$. This point has important implications for the effectiveness of policies aimed at stimulating innovations. First, it suggests that innovation policies, such as research subsidies or tax breaks, have a multiplier effect thanks to the improvement in capital efficiency. Second, innovations are also encouraged by policies designed at increasing capital efficiency, such as removing impediments to trading financial assets or improving accounting standards.

### 2.4.2 Dynamics

We study the dynamics of income, research and learning and their interplay along the economy’s growth path in the case of constant returns to scale in the intermediate goods sector (i.e., $\alpha = 1$). In that case, equations 3 to 5 simplify to:

\[
\beta q_t^\beta w_t^\beta = A_t^{1-\beta} e_A(A_t) 
\]

(6)

\[
\beta A_t^{1-\beta} w_t^\beta = q_t^{1-\beta} e'_q(q_t) 
\]

(7)

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A steady-state equilibrium satisfies equations 6 to 8 together with the condition \( w_{t+1} = w_t \). A trivial solution to this system obtains when income equals zero and neither learning nor research take place (\( w_t = \overline{A}_t = 0 \) and \( q_t = 1/2 \)). A non-trivial steady-states also exists if

\[
\frac{1}{\varepsilon_A(A)} + \frac{1}{\varepsilon_q(q)} \neq \frac{1}{\beta} - 1 \quad \text{for all } \overline{A} > 0 \text{ and } 1 > q > 1/2,
\]

where \( \varepsilon_A(A) \equiv 1 + A \varepsilon''_A(A)/\varepsilon'_A(A) \) and \( \varepsilon_q(q) \equiv 1 + q \varepsilon''_q(q)/\varepsilon'_q(q) \) denote one plus the elasticity of \( \varepsilon'_A \) and \( \varepsilon'_q \) with respect to \( A \) and \( q \). We label with a * steady-state quantities, and drop the argument to denote the functions evaluated at the steady-state, \( \varepsilon_A \equiv \varepsilon_A(A^*) \) and \( \varepsilon_q \equiv \varepsilon_q(q^*) \). The following proposition characterizes the dynamics of the economy.

**Proposition 5**: Assume returns to scale in the intermediate goods sector are constant (i.e., \( \alpha = 1 \)) and condition 9 holds. The economy admits two steady-state equilibria, 0 and \( w^* > 0 \). The dynamics of the system in a neighborhood of the steady-state are governed by the following equations:

\[
\ln(\overline{A}_t) \approx \frac{\beta}{\varepsilon_A} \left[ \ln(q_t) + \ln(w_t) \right] + \text{cst}, \quad (10)
\]

and

\[
\ln(q_t) \approx \frac{\beta}{\varepsilon_q} \left[ \ln(\overline{A}_t) + \ln(w_t) \right] + \text{cst}'. \quad (11)
\]

\[
\ln(w_{t+1}/w^*) \approx (\gamma + 1) \ln(w_t/w^*), \text{ where } \frac{1}{\gamma + 1} = \frac{1}{\beta} - \frac{1}{\varepsilon_A} - \frac{1}{\varepsilon_q}. \quad (12)
\]

Three cases are possible:

- **Case 1** \((\frac{1}{\varepsilon_A} + \frac{1}{\varepsilon_q} < \frac{1}{\beta} - 1)\): \( w^* \) is a stable steady-state while 0 is not. Income converges to \( w^* \).

- **Case 2** \((\frac{1}{\beta} > \frac{1}{\varepsilon_A} + \frac{1}{\varepsilon_q} > \frac{1}{\beta} - 1)\): 0 is a stable steady-state while \( w^* \) is not. If \( w_0 > w^* \), then the economy grows without bound. If instead \( w_0 < w^* \), then the economy contracts towards 0.

- **Case 3** \((\frac{1}{\varepsilon_A} + \frac{1}{\varepsilon_q} > \frac{1}{\beta})\): the economy is unstable and oscillating.

\( \gamma \) measures the speed of convergence of income to its steady-state, in a neighborhood thereof, conditional on a given level of income. In case 1 \((-1 < \gamma < 0)\), the economy converges to a steady-state in which income no longer grows. Thus, learning and research only have a transitory impact on growth. This is because their costs rise quickly with effort levels (\( \varepsilon_A \) or \( \varepsilon_q \) large), while the marginal product of intermediate goods falls rapidly.
with its employment ($\beta$ low). In case 2 ($\gamma > 0$) instead, learning and research have a permanent impact and ongoing growth is possible. Income grows without bound if its initial value $w_0$ exceeds $w^*$, but shrinks towards 0 otherwise. If we focus on cases 1 and 2 and interpret the learning effort $q$ and its cost $e_q(q)$ as measures of financial development, then the model predicts that the financial sector develops in tandem with the real economy. Finally in case 3 ($\gamma < -1$), the system oscillates and is unstable. The following corollary breaks down the convergence rate of income into its various components.

**Corollary 6**: The speed of convergence of income in a neighborhood of the steady-state $\gamma$ can be decomposed into $\gamma = \gamma_K + \gamma_A + \gamma_q + \gamma_{Aq}$ where:

- $\gamma_K \equiv -(1 - \beta) < 0$ is the contribution of capital accumulation to growth,
- $\gamma_A \equiv \frac{\beta^2}{\varepsilon_A - \beta}$ is the contribution of research in the absence of learning,
- $\gamma_q \equiv \frac{\beta^2}{\varepsilon_q - \beta}$ is the contribution of learning in the absence of research,
- $\gamma_{Aq} \equiv \frac{\beta^2(\beta + \gamma + 1)}{(\varepsilon_A - \beta)(\varepsilon_q - \beta)}$, the residual, is the contribution of the interaction between research and learning.

$\gamma_A$, $\gamma_q$ and $\gamma_{Aq}$ are positive except in the case of oscillating dynamics.

Income grows at the rate $\gamma_K + \gamma_A$ when investors do not learn, and at the rate $\gamma_K + \gamma_q$ when entrepreneurs do not innovate. When learning and research both take place without interacting with one another, it grows at the rate $\gamma_K + \gamma_{Aq}$ ($q_t$ and $A_t$ are replaced with arbitrary constants in, respectively, equations 6 and 7).

Focusing on the non-oscillating dynamics (cases 1 and 2 in Proposition 2) and leaving aside the neoclassical effect on capital accumulation (represented by the term $\gamma_K$), the growth rate of income in the economy with both learning and research, $\gamma_A + \gamma_q + \gamma_{Aq}$, exceeds the sum of the growth rates in the no-learning economy $\gamma_A$ and in the no-research economy $\gamma_q$. This is again a reflection of the mutual influence of learning and research.

We calibrate the model in Section 5.5 to assess the importance for income growth of the interaction effect.

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*ε_q(q_t) can be interpreted as the amount of resources devoted to analyzing investment opportunities. Alternatively, we could add to the economy a competitive intermediary who invests funds on behalf of the financier. The intermediary collects information about projects’ returns and is paid a fee to compensate for the disutility of learning. There is free entry in the intermediary sector. Equation 20 in the Appendix implies that $d\ln(q_t/q^*)/d\ln(w_t/w^*)$ is positive if $1/\varepsilon_A + 1/\varepsilon_q < 1/\beta$.

*Corollary 6 nests the two polar cases we discussed above. When $\varepsilon_q = +\infty$, $q_t$ is frozen as in the “endogenous growth case” and $\gamma_q = 0$, so the economy grows at the rate $\gamma_K + \gamma_A$. If instead $\varepsilon_A = +\infty$, then $A_t$ is frozen as in the “financial development case” and $\gamma_A = 0$, so the economy grows at the rate $\gamma_K + \gamma_q$. 

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2.4.3 The Model’s Fit with Stylized Facts on Finance and Growth

The extended model is consistent with the following four observations documented in the literature on finance and growth (see Appendix A for more details and references). To start with, financial development promotes economic growth and its effect runs through TFP growth, rather than capital growth. The subsequent three observations shed light on the link from finance to TFP. First, financial development stimulates investments in R&D and R&D contributes to TFP. Second, it also enhances TFP by improving capital efficiency. Countries with more developed financial sectors allocate capital more efficiently across industries and firms. A more efficient distribution of capital at the micro level translates into higher TFP at the macro level. Finally, financial development improves capital efficiency (among other ways) by alleviating informational problems.

2.5 Testable Hypotheses

In our empirical work below, we test four specific hypotheses that follow from Propositions 2 to 4. Hypotheses 1 and 2 are direct implications of Proposition 2 and correspond to the central predictions of the paper concerning the mutually reinforcing effect of learning and research. The first states that the entrepreneur performs more research when the financier learns more. The second states the converse, namely that the financier learns more when the entrepreneur does more research.

**Hypothesis 1:** An increase in the learning effort leads to an increase in the research effort.

**Hypothesis 2:** A ni n c r e a s ei nt h er e s e a r c he effort leads to an increase in the learning effort.

Propositions 3 and 4 allow to formulate auxiliary tests of the model based on the cross-sectional distribution of capital and returns, as stated in Hypotheses 3 and 4.

**Hypothesis 3:** An increase in the learning effort leads to a more dispersed distribution of capital across projects.

**Hypothesis 4:** An increase in the learning or research efforts lead to a more dispersed distribution of return on capital across projects.
3 Empirical Strategy

Testing the first two hypotheses concerning the relationships between research and learning calls for a treatment of the biases that a direct OLS estimation of these relationships induce. First, any shock to capital \( w_t \) in the model will stimulate learning and research independently, thereby generating a spurious correlation between them. Moreover, the two-way relationship generates an endogeneity bias. For example, a least squares regression of learning on research yields inconsistent estimates because the regression’s residual is correlated with the regressor (research) as equation 11 shows. Similarly for equation 10, the regressor (learning) is correlated with the residual from the regression of research on learning. Our strategy for addressing these issues is to exploit exogenous changes in public policies or regulations, as is commonly done in the finance and growth literature. We measure firms’ innovation effort with the level of R&D expenditures and proxy for investors’ learning effort about a firm’s prospects using the number of equity analysts covering the firm.\(^9\)

3.1 More innovation leads to more financial analysis

To test whether more innovation by firms leads to more learning by the financial sector, we examine whether analysts’ coverage of firms changed around the staggered implementation of R&D tax credits across U.S. states between 1992 and 2006.\(^10\) These policy changes provide a source of variation in firms’ innovation activities, which is plausibly exogenous to firms’ analyst coverage.

States R&D tax credits proceeded from the implementation of federal tax credits in 1981. Minnesota introduced its own tax credit in 1982, followed by 32 other states as of 2006 (Wilson, 2009). These credits allow firms to reduce their state tax liability by deducting a portion of R&D expenditures from their state tax bill. State taxes are usually based on turnover or business activities (such as the presence of employees or real

\(^9\)We acknowledge that the way we measure information production about innovations is somewhat crude. We resort to equity analysts’ coverage of publicly-traded firms because of data availability. Of course, information is produced by other agents too, such as bankers, bondholders, rating agencies, or wealthy shareholders. Moreover, innovations are also carried out by private firms, which are followed by other information producers, such as venture capitalists, corporate incubators, wealthy individual investors or government agencies. Our estimates remain valid to the extent that the elasticity of information production with respect to innovation and the elasticity of innovation with respect to information production are comparable between information producers on one hand and between public and private firms on the other.

\(^10\)We start the R&D sample in 1992 to align it with our second experiment (brokerage house closures and mergers). The R&D sample stops in 2006 because it is the last year in which state tax credit information is available from Wilson (2009).
We argue that increases in state R&D tax credits provide a plausibly exogenous source of variation in firms’ innovation effort. Indeed, from the point of view of an individual firm, changes in state R&D tax credits are likely to alter firms’ R&D behaviour in ways unrelated to variables (such as technological or market conditions) that could also affect the coverage decision of brokerage firms). Previous analysis of R&D tax credits in the U.S. and elsewhere show that tax credits do encourage changes in R&D expenditures (Hall and Van Reenen, 2000; Wilson, 2009).

Table 1 provides information on state tax credits, including the year in which they were first introduced, their level, and subsequent changes.12 Tax credit rates range from 3% in Nevada and South Carolina to 20% in Arizona and Hawaiī.

Few studies have assessed the impact of state tax credit on firms’ innovation effort. At the state-level, previous research suggests a positive impact of these credits on in-state innovation (Wilson, 2009), and on the number of high-tech establishments in the state (Wu, 2008). Recently, Bloom, Schankerman and Van Reenen (2013) use changes in states and federal tax credits to identify the effect of R&D spillovers of firms located in close geographic and product markets.

We first confirm that increases in tax credits are indeed associated with increases in R&D expenditures for firms headquartered in these states, and then compare the change in analyst coverage of firms located in a state that passed a tax credit with the change in coverage of comparable firms located in states without tax credit changes. The staggered implementation of tax credits across states allows to control for potential endogeneity problems thanks to a difference-in-difference estimation (e.g. Bertrand and Mullainathan (2003)). For example, aggregate shocks contemporaneous to the implementation of a tax credit may influence firms’ analyst coverage and confound the effect of innovation. To the extent that, absent treatment, analyst coverage of firms in

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11 See Heider and Ljungqvist (2014) for more details on state corporate taxes. Some states allow loss-making firms to convert tax credits into cash, and/or to carry these credits forward.

12 We thank Daniel Wilson for making this data available (see http://www.frbsf.org/economic-research/economists/daniel-wilson/).
different states follow similar trends, and that the passage of each state R&D tax credit is not contemporaneous
to other changes in the state that correlate with the analyst coverage of firms headquartered in the state, our
difference-in-difference estimation enables us to isolate the effect of innovation on analyst coverage. In effect,
each year we use any change in analyst coverage of firms headquartered in states which do not experience a
change in R&D tax credit as counterfactual for firms located in states passing an R&D tax credit in that year.
By comparing the changes in coverage of treatment and control firms we obtain an estimate of the causal impact
of innovation on analyst coverage.

We conduct our analysis at the firm level, focusing on manufacturing U.S. listed firms which consider
research and development activities to be material to their business. Whenever a state implements a tax
credit, we compare the change in coverage of firms affected by the state tax credit (treated firms) with the
coverage of firms in other states (control firms). Following Heider and Ljungqvist (2014), to reduce any potential
endogeneity of a state choosing a certain level of tax credit, we do not consider the actual level of the tax credit
granted in the state, but an indicator variable that equals one in the year the state introduces or increases its
R&D tax credit, and zero in other years. Firms’ locations are identified with the location of their headquarters
as reported in Compact Disclosure. Since many firms may locate in Delaware for reasons unrelated to their
operations, we exclude all firms in that state.

As in Heider and Ljungqvist (2014), we estimate our main regression in first differences, because it better
accommodates that firms can be treated multiple times. First-differencing the data controls for all time invariant
characteristics. All regressions include year dummies, and standard errors are clustered at the firm level.

In some specifications we also include lagged time varying controls such as the logarithm of sales and a
dummy variable indicating whether the firm made accounting losses (which affects the firm’s tax liability, and
potentially the benefit of the tax credit). Formally, the regression takes the form:

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13 In practice, we drop firms not reporting or reporting zero R&D expenditures.
14 We do not consider tax credit cuts as very few states implement those over our sample period (60 firm-year observations, versus
620 for tax credit increases). These are treated as untreated observations.
15 When such data is not available, we take the state of the latest headquarters as reported in Compustat. Acharya and Baghai
(2014), citing Howells (1990) and Breschi (2008) suggest that large firms locate their R&D facilities close to the company’s head-
quartes and do not disperse geographically.
\[ \Delta \ln(\text{coverage}_{i,s,t}) = \beta TC_{s,t}^+ + \eta_t + \sum_j \gamma_j \Delta X^j_{i,t-2} + \varepsilon_{it}, \]

where \( TC_{s,t}^+ \) is a dummy variable which takes the values 1 if state \( s \) implemented or increased its R&D tax credit in year \( t - 1 \); \( \eta_t \) are year dummies and \( X^j_{i,t} \) are the firm controls listed above. Our coefficient of interest is \( \beta \) which measures the difference between the change in analyst coverage for firms in the treated state relative to the change in coverage for firms in other states.

That difference-in-differences estimate is robust to many potential confounds. Aggregate time-varying shocks as well as time-invariant firm attributes are captured by the year dummies and the differencing of the data. In addition, we control for time-varying changes in firm characteristics such as size and profitability by directly including these lagged variables in our specifications. One remaining potential concern with the methodology is the finding by Wilson (2009) that, at the state level, a portion of the increase in in-state R&D is due to a decrease in R&D in other states. In our context, it is possible that following the passage of an R&D tax credit firms relocate in high tax credit states at the expense of other states. For example, firms could hire more researchers in states that pass a tax credit, presumably by offering higher compensation and better work conditions to researchers from other states. However, the important point for our analysis is that R&D rises for the firms headquartered in the tax credit states, regardless of where the extra R&D comes from. We show empirically that this is the case. Further, if some firms were simply substituting R&D across states without increasing their overall R&D spending, this would bias our estimates towards not finding an effect of the tax credit on firm level innovation and analyst coverage in treated states.

In sum, the change in state R&D tax credit provides a good setting to assess the effect that firm innovation has on the learning effort of the financial sector.

### 3.2 More financial analysis leads to more innovation

The second prediction of our model is that more learning by the financial sector increases firms’ innovation effort. The ideal experiment for testing this prediction would be one in which the financial sector’s ability
to learn about firms’ innovative projects changes for exogenous reasons. The identification strategy pioneered by Hong and Kacperczyk (2010), extended by Kelly and Ljungqvist (2012), Derrien and Kecskes (2013) and others, comes close to this. These last authors exploit closures of and mergers between brokerage houses which lead to the removal or dismissal of analysts. Indeed, closures often lead to the removal of analysts who are not re-hired by a new broker, while a number of mergers lead to the dismissal of redundant analysts who follow the same stocks as analysts working for the other merging entity. Kelly and Ljungqvist (2012) and Derrien and Kecskes (2013) provide convincing evidence that the drop in analyst coverage resulting from such events is largely exogenous to firms’ policies. What matters for our purpose is that these drops reflect a significant reduction in the resources allocated to financial analysis. To assess how a decrease in analyst coverage changes firms’ innovation effort, we study the firms affected by 52 events identified by Derrien and Kecskes (2013). But unlike them, we restrict our analysis to stocks reporting strictly positive R&D expenditures in order to focus on innovative firms.

To assess how a decrease in analyst coverage changes firms’ innovation effort, we study the firms affected by the 52 events identified by Derrien and Kecskes (2013). But unlike them, we restrict our analysis to stocks reporting strictly positive R&D expenditures in order to focus on innovative firms.

The firms affected by these exogenous variations in analyst coverage are those for which an analyst who was covering the firm before the closure of his brokerage house disappears from I/B/E/S, or for which a redundant analyst who was covering a firm also covered by the other merging entity disappears from the database. In our study, we examine how the innovation effort of these firms evolves, compared to others firms, when they lose an analyst. Adopting the same specification as in our first experiment, our difference-in-differences estimator compares this change to the change experienced by control firms unaffected by the event, effectively controlling for changes (or overall trends) in firms’ innovation effort.

Overall, the broker closures and mergers events provide an excellent setting for assessing the relationship between the amount of information produced by the financial sector and firms’ innovation effort. Together, our
two experiments allow us to study the impact innovation has on financial analysis, and conversely, the impact that financial analysis has on innovation.

4 Data

We evaluate the interaction effects between analyst coverage and firms’ innovation effort on a single set of innovative manufacturing firms consistent across both shocks (R&D tax credit changes and broker events). That way, our quantitative estimate of the interaction effect is not biased by differences in firm characteristics across the two experiments. In addition, in constructing our unique sample we pay attention to have, for both experiments, treatment and control firms that are sufficiently similar. This is particularly important for the second experiment, since, as reported by Hong and Kacperczyk (2010), brokerage closures primarily affect firms that are larger than the average Compustat firm. A lack of overlap on covariates (such as size) between treatment and control groups, can lead to imprecise estimates (Crump et al., 2009). A practical solution recently suggested by Crump et al. (2009) to remedy to a potential lack of overlap is to first estimate a propensity score on all firms (i.e., estimate the probability of a firm being treated, conditional on observable characteristics), and then restrict the analysis to firms with a score between 0.1 and 0.9. We adapt this methodology to our setting and estimate the propensity score of all firms for each experiment. Our final sample firms include 1,279 innovative firms with a score on the [0.1,0.9] interval for both experiments.16

We focus on the firms that report strictly positive R&D expenditures. Firms typically report R&D expenditures in their financial statements when those expenditures are material to their business (Bound, Cummins, Griliches, Hall and Jaffe, 1984). Thus, keeping firms with strictly positive R&D expenditures ensures that our tests focus on firms for which our model is the most relevant. We exclude firms with year-on-year R&D growth of over 200% to reduce estimation noise introduced by mergers or radical strategic decisions that would have little to do with changes in analyst coverage or state R&D tax credits.

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16 The propensity score is estimated with a logit regression. The treated indicator takes the value one if the firm is treated at one point in the sample and zero otherwise. The covariates included in the logit regression are industry dummies and the time averages of the logarithm of sales and loss indicator over the pre-treatment period for firms that will eventually be treated, and over the period in which they appear in the sample for the firms that will never be treated.
We use the logarithm of R&D expenditures to measure firms’ innovation effort. To measure analyst coverage, we count the number of unique analysts making a yearly or a quarterly earnings forecast during the firm fiscal year. We then take the logarithm of that number as our measure of coverage.\textsuperscript{17} We deflate all accounting variables – taken from Compustat – using the Consumer Price Index.

Table 2 presents the summary statistics for our sample. As we require firms to be followed by at least one analyst and have positive R&D expenditures, by construction, our typical firm is large (the median amount of sales is $660m in the sample versus $63m for Compustat firms with positive R&D expenditures) and innovative.

Figure 3 shows the time series of our measures of financial analysis (number of analysts following a firm) and innovation effort (level of R&D expenditures) adjusted for both industry effects and the amount of sales made by the firm.\textsuperscript{18} The figure illustrates that, as our theory suggests, the two series are positively correlated over time.

5 Results

A visual examination of Figure 3 suggests a positive association between analyst coverage and R&D expenditures. To control for factors that could confound the relationship between those variables, Table 3 to 6 present the results of difference-in-differences estimations for states R&D tax credit (Tables 3 and 4), and for brokerage events (Tables 5 and 6).

5.1 More innovation leads to more financial analysis

We are interested in the effect that changes in R&D tax credits have on learning by the financial sector, as measured by analyst coverage. We first confirm in Table 3 that an increase in a state’s R&D tax credit leads to an increase in R&D expenditures by firms located in that state. The coefficient of interest is that on the variable TC+, which shows the total effect of a tax credit increase on the R&D of firms located in the treated state one year after the passage of the tax credit, compared to firms not experiencing a change in their state’s tax credits

\textsuperscript{17}All firms in our sample are followed by at least one analyst in all years.
\textsuperscript{18}That is, the variables plotted on the chart are the residuals of pooled regressions of each variable on industry dummies and logarithm of sales.
in that year. After a R&I tax credit is implemented, treated firms increase their R&D expenditures by 4.6% relative to control firms.\textsuperscript{19} The change takes place in the year after the tax credit implementation. It reverts partly in the following year, as the small and marginally significant coefficient on the lagged tax credit variable, TC+\textsubscript{t+1}, indicates, but the cumulative effect is an increase in R&D expenditures.\textsuperscript{20} Thus, firms increase their innovation effort in response to tax credits increases, as expected.

We turn to the first testable hypothesis, according to which an increase in the learning effort leads to an increase in the research effort. Consistent with it, Table 4 shows that, after the passage of state R&D tax credits, firms in treated states are covered by 5.6% more analysts compared to firms located in other states. Given that the average firm in the sample has 11 analysts, each firm is followed by an additional 0.6 analysts after the passage of a tax credit. Combining the numbers in Tables 3 and 4 indicates that the sensitivity of analyst coverage to R&D expenditures is about 0.056/0.046 = 1.217, i.e., a 10% increase in R&D expenditures induces a 12% increase in analyst following. The effect of control variables is consistent with previous research. In particular, the coefficient on the change in firm size is positive and significant.

5.2 More financial analysis leads to more innovation

To evaluate our second hypothesis on the effect of financial analysis on firms’ innovation effort, we use variations in analyst coverage triggered by brokerage closures and mergers. Table 6 confirms that treated firms (firms followed by analysts employed at closing or merging brokers) experience a reduction in analyst coverage in the year following a closure or merger. On average, a firm loses about 9.2% more analysts than control firms. Given that the average firm has about 11 analysts, treated firms lose on average one analyst compared to control firms. This is the magnitude we would expect, given the construction of the brokerage house experiment.

Table 6 presents the main results regarding the innovation effort. We find that treated firms’ R&D expenditures fall by 3.0%, relative to control firms, after losing analysts. Our findings confirm and strengthen those

\textsuperscript{19} Wilson (2009) finds a 1% increase in tax credit results in an increase in-state R&D of around 1.7% in the short term. Since in our sample, the average tax credit increase is 4.25%, our coefficient implies that a 1% increase in tax credit makes firms increase R&D expenditures by 1.1%.

\textsuperscript{20} A test of the hypothesis that the sum of the coefficients of year \textsubscript{t} and \textsubscript{t+1} is equal to zero is rejected at the 10% confidence level (F=3.13, p=0.0772)
of Derrien and Kecskes (2013) who report a decrease in the ratio of R&D expenditures to total assets of 0.21% in a broader sample of firms which includes firms with nonmaterial R&D, and a similar set of events. Our estimates indicate that a 10% drop in analyst coverage triggers a \( 10\% \times (-0.030)/(−0.092) = 3.26\% \) decline in R&D expenditures.\(^{21}\)

These results provide support for our second hypothesis, namely that a more intense learning effort by the financial sector spurs innovation by the corporate sector. Together, our empirical investigations support the two predictions at the core of our model: Wall Street research and Main Street innovation interact and reinforce each other.

5.3 New capital distribution and productivity dispersion

We investigate next whether these findings can be explained by the mechanism outlined in our theory and assess hypotheses 3 and 4. Hypothesis 3 suggests that an increase in investors’ learning effort leads to a more dispersed distribution of capital. To test this hypothesis, we examine how the dispersion in capital proceeds changes following an increase in state R&D tax credits. Since, in the model, an increase in innovation effort alone does not generate an increase in capital proceeds dispersion, finding an increase in proceeds after an exogenous increase in innovation is indicative of an increase in learning generated by the increase in innovation, i.e., of the feedback effect.

To test this, we collect information on all new equity issues from SDC Platinum over our sample period, and attribute each new issue to the state where the firm’s headquarters are located. Then, we adapt the methodology of Bertrand and Mullainathan (2003) for multiple treatment groups. We retain for each firm three years of observations before the tax credit change and three years after. We proceed in two stages. First,

\[^{21}\text{Using patenting as a measure of innovation output in an experiment similar to ours, He and Tian (2013) find a negative relationship between analyst coverage patenting activity. Clarke et al. (2015) show further that this relationship is driven by poor-quality innovators, i.e., firms which produce many patents but receive no citations, and that it is reversed for high-quality innovators, i.e., these firms innovate more when they are followed by more analysts. One interpretation of these findings and of those reported here and in Derrien and Kesckes (2013) is that a firm’s patenting policy responds to changes in its informational environment, such as a reduction in analyst coverage: for a given innovation effort (i.e., a given level of R&D expenditures), a firm increases its patenting activity to compensate for the loss of information that analysts no longer produce. Consistent with the existence of a trade-off between patenting and secrecy, Saidi and Zaldokas (2015) document that firms issue fewer patents, without altering their investment in innovation, when their lenders are better informed.}\]
pooling all observations (in both treated and control states), we regress firms’ new equity proceeds on state and year dummies and extract the residuals. Then, for each treated firm (from here on, control firms do not play any role), we average the residuals over the 3 years before the treatment year, and over the 3 years after. Finally, we test whether the cross-sectional standard deviation of these time-averaged residuals is equal before and after the treatment. We report the results of a parametric F-test for equality of variances in the top panel of Table 7. The hypothesis of equality of variances is rejected: after the passage of an R&D tax credit, new equity proceeds are significantly more dispersed across firms located in the treatment state, compared to other states (p<0.05). This finding provides support for the mechanism employed in the model to link innovation with learning by the financial sector.

Hypothesis 4 suggests that as firms innovate more or as financiers learn more, firms return on capital grows more dispersed. We evaluate this prediction on the innovation effort using shocks to R&D tax credits. Unfortunately, we have too few firms affected by broker shocks to estimate a meaningful change in dispersion. We examine how the dispersion in firms’ profitability - measured as the return on assets (ROA) - evolves following the passage of state R&D tax credit.22 We follow the same methodology as for hypothesis 3. As the bottom panel of Table 7 indicates, the hypothesis of equality of variances is rejected: after the passage of an R&D tax credit, the cross-sectional dispersion in ROA significantly increases in the treatment state, compared to other states (p<0.05). This finding provides support for the model, suggesting that the rise in analyst coverage that follows the passage of R&D tax credits, is related to an increase in the dispersion of firms' profits, which makes financial analysis more valuable.

5.4 Quantifying the indirect effect of learning on innovation

Our estimates also allow us to decompose the effect of an R&D tax credit into a direct effect, and an indirect effect operating through learning. The second column of Tables 6 yields an estimate of the sensitivity of R&D

22We define return on assets as the ratio of EBIT to total assets.
expenditures to analyst coverage:

\[
\Delta \ln(rd) = -0.030 \times \Delta \ln(coverage) + 0.3 \times \Delta \ln(coverage).
\]

From Table 4 (column 2), the passage of a tax credit increases analyst coverage by 5.6% on average. Hence, the indirect effect of the tax credit, operating through analysts’ response, equals \(0.3 \times 5.6\% = 1.7\%\). To put this number into perspective, we compare it to the total effect of the tax credit on \(\Delta \ln(rd)\), which equals 4.6% according to Table 3 (column 2). Thus, the indirect effect of the tax credit through analysts’ response is about one third \((36\% = 1.7\%/4.6\%)\) of the size of its total effect. For example, for a policy that triggers a 10% increase in R&D expenditures (as a total effect), about 3.6% of the increase indirectly comes from the catalyzing effect of financial analysis, as firms gain approximately one analyst \((\Delta \ln(coverage) = 10\% \times 5.6\%/4.6\% = 12\%)\).

These findings illustrate the importance of maintaining learning incentives in order to enjoy the full benefits of R&D tax credits. They also show how policies aimed at improving the functioning of financial markets, such as encouraging equity research, improving accounting standards, or reducing impediments to trading financial assets, can act as catalysts for other policies aimed at boosting firm investment.

5.5 Calibration

We calibrate the model to evaluate the importance of the interplay between learning and innovation to long term growth. We need to determine 4 parameters \((\alpha, \beta, \varepsilon_A\) and \(\varepsilon_q\)) in order to compute the speed of convergence of income to the steady-state, \(\gamma\), and its components, \(\gamma_K, \gamma_A, \gamma_q\) and \(\gamma_{Aq}\).

We start with \(\alpha\), which controls how profits are shared between firms. Only successful firms (half of all firms) earn a profit, and this profit is higher for the fraction \(q\) recognized as successful by the financier. These \(q/2\) firms account for a proportion \(F = q^{1-\alpha}/[q^{1-\alpha} + (1 - q)^{1-\alpha}]\) of aggregate profits. This proportion increases in \(\alpha\), starting from \(q\) when \(\alpha = 0\) (a fraction \(q/2\) of firms earn a fraction \(q\) of the profits) and reaching one when \(\alpha = 1\) (these firms capture all the profits). Empirically, the return on innovation is extremely skewed. For example, Scherer and Harhoff (2000) estimate that 10% of inventions capture from 48 to 93% of total returns.
in their sample. Accordingly, we set \( \alpha \) to one in order to generate the most skewed distribution of profits.

To parametrize the costs of research \( e_A \) and learning \( e_q \), we use the estimates of the sensitivities of learning to innovation and of innovation to learning, derived from our two experiments. Specifically, we assume that the economy is initially in steady-state, and that it is perturbed by a shock (changes in R&D tax credits or broker closures) in period \( T \). We interpret these shocks as a rescaling of the costs of innovating or learning (i.e., parallel shifts in their logarithm). The perturbed economy then converges toward a new steady-state. We compute in the model the change in the learning and research efforts from period \( T \) to the next period, \( T + 1 \). In period \( T \), before the shock, the economy is fully described by the equations characterizing the initial steady-state (system 16 in the appendix). Its evolution is then governed (approximately) by equations 10 to 12 under the parameters of the new steady-state. We show in the appendix how to relate \( \varepsilon_A \) and \( \varepsilon_q \) (one plus the elasticities of the cost functions evaluated at the new steady-state) to the elasticity of R&D expenditures with respect to analyst following in the broker closure experiment, and to the elasticity of analyst following with respect R&D expenditures in the R&D tax credit experiment. Solving a system of two equations then yields estimates of \( \varepsilon_A \) and \( \varepsilon_q \).

There is one remaining parameter to calibrate, \( \beta \), which measures the share of capital in total income. We consider a range of values comprising 1/3, a standard estimate assuming capital is exclusively physical, and 2/3 which corresponds to a broader definition that includes both physical and human capital (e.g., Mankiw, Romer, and Weil (1992)). Table 8 displays the results of the calibration exercise for different values of \( \beta \). The top panel shows growth rates of income per period, and the bottom panel rates per annum assuming that one period lasts 30 years.\(^{23}\)

Columns 2 and 3 of Panel A show the estimates of \( \varepsilon_A \) and \( \varepsilon_q \). When \( \beta \) equals one third, they equal, respectively, 1.73 and 0.73. They rise to 3.86 and 1.63 when \( \beta \) grows to 2/3. In the model indeed, learning and research are more responsive to one another under more slowly decreasing returns to scale (higher \( \beta \)). This has to be offset by more elastic cost functions in order to match the observed sensitivities.

\(^{23}\)The annual rate equals \((1 + \text{Per-period rate})^{1/30} - 1\).
Under $\beta = 1/3$, the speed of convergence of income is -7.5% per period or -0.26% per annum, where the minus sign indicates that income grows at a slower rate for higher levels of income (and hence that it converges to a steady-state). It is larger than the annual convergence rate reported in Barro (2015) (around 2% per year), but is reasonable once population growth (1.8% per year), which is assumed away in the model, is accounted for. In comparison, income converges at $7.8\% - 67\% = -59.2\%$ per period if agents innovate but do not learn, and at $27\% - 67\% = -40\%$ per period if they learn but do not innovate, where -67% captures the neoclassical effect of diminishing returns to capital. Obviously, the attenuation of income growth is slower when agents carry out both tasks compared to when they only undertake one. The table reveals that learning has a much larger impact than innovation on income growth ($27\%$ vs. $7.8\%$ for $\beta = 1/3$). The reason is that, in the data, analyst following is much more responsive to R&D expenditures (elasticity of 1.2 in the R&D tax credit experiment) than R&D expenditures are to analyst following (elasticity of 0.3 in the broker closure experiment).

The impact of the interaction between research and learning amounts to 24.5% per period, the difference between the actual convergence rate, -7.5%, and -32%, the rate that obtains in a fictitious economy in which agents both learn and innovate but improvements in research do not lead to improvements in learning and vice versa, except through current income. It represents about 40% of the total effect of learning and research. The relative contribution to income growth of the interaction effect is reduced for higher levels of $\beta$, but it remains large (its lower bound is 31%). Under $\beta = 2/3$ for example, its represents about one third of the total effects of learning and research.

The table shows that, under higher levels of $\beta$, the economy can escape the attraction of a steady-state and expand forever. Most interestingly, if $\beta$ is in between 0.4 and 0.5, then ongoing growth is only possible thanks to the interaction between learning and research. When $\beta$ equals 0.4 for example, income diverges at a rate of 7% per period, but it converges at a rate of -19% per period if we do not allow for the interaction.

While this calibration exercise leaves out many important realistic features of the economy (e.g., it assumes no population growth, or a 100% saving rate), it is not clear that they would affect the relative importance of
learning, research or their interaction. Therefore, we conclude that the interplay between research and learning is an important contributor to income growth.

6 Conclusion

We develop and test a model of financial development and technological progress. Its main insight is that knowledge about technologies and technological knowledge are mutually reinforcing. That is, entrepreneurs innovate more when financiers are better informed about their projects because they expect to receive more funding should their projects be successful. Conversely, financiers collect more information about projects when entrepreneurs innovate more because the opportunity cost of misinvesting, i.e. of allocating capital to unsuccessful projects and missing out on successful ones, is larger. This positive feedback promotes economic growth and leads to a variety of dynamic patterns. Its beneficial impact is permanent in some cases leading to unbounded income growth, but only transitory in others (when the marginal costs of learning and research are highly elastic and the factor share of capital is low). Poverty traps can also emerge in which only countries that are sufficiently wealthy enjoy the benefits of the feedback.

New predictions are derived from the model. The predictions are supported empirically by two quasi-natural experiments that uniquely permit to isolate the effect of innovation on learning from that of learning on innovation for a given set of firms. Moreover, we estimate that the feedback effect is about a third of the size of the total effect. For example, a 1% increase in R&D expenditures triggered by an R&D tax credit increases analyst coverage by 6%, which in turn is responsible for up to 0.36% of the total increase in R&D expenditures. An implication is that policies targeted on R&D incentives can be rendered more effective by coupling them with policies designed at increasing capital efficiency, such as reducing impediments to trading financial assets or improving accounting standards.

Overall, our model and the empirical evidence strongly support the existence of positive feedback loops between learning by the financial sector and firms’ innovation: Wall Street research and Main Street innovation interact and reinforce each other.
A Detailed Discussion of the Model’s Fit with Stylized Facts about Finance and Growth

The dynamic extension of the model is consistent by the following observations on the link between finance and growth.

1. Financial development causes growth by improving TFP.

A large literature, surveyed by Levine (1997, 2005) shows that financial development promotes economic growth. Country-level, industry-level, firm-level and event-study investigations suggest that financial intermediaries and markets have a large, causal impact on real GDP growth (e.g. Rajan and Zingales (1998), Jayaratne and Strahan (1996), Fisman and Love (2004), Levine and Zervos (1998), Beck, Levine and Loayza (2000)). Moreover, Levine and Zervos (1998) and Beck, Levine and Loayza (2000) find that their relation to TFP growth is strong whereas their link to capital growth is tenuous. Thus, it appears that financial development contributes to growth by improving TFP rather than capital accumulation.

2. Financial development stimulates R&D investments and R&D improves TFP.

Carlin and Mayer (2003) examine a sample of advanced OECD countries. They show that industries dependent on equity finance invest more in R&D and grow faster in countries with better accounting standards. They do not find a similar increase for investment in fixed assets, or for countries with a large financial sector. This suggests that finance is associated with the funding of new technologies and that informational problems are a serious impediment to providing capital. Brown, Fazzari and Petersen (2008) also establish a link between equity financing and R&D by analyzing U.S. high tech firms. They estimate that improved access to finance explains most of the 1990’s R&D boom in the U.S. Credit (Herrera and Minetti (2007)) and venture capital (Kortum and Lerner (2000), Ueda and Hirukawa (2003), Hellmann and Puri (2000)) are also essential to the funding of innovations. Moreover, there exists abundant evidence establishing that R&D is an important determinant of productivity (e.g. Griliches (1988), Coe and Helpman (1995)).

3. Financial development improves allocative efficiency and allocative efficiency improves TFP.

Countries with more developed financial sectors allocate their capital more efficiently. In a cross-country study, Wurgler (2000) documents that they increase investments more in their growing industries, and decrease investments more in their declining industries, than countries with less developed markets.24 Event-studies report similar findings. Bekker, Harvey and Lundblad (2001, 2005), Bertrand, Schoar and Thesmar (2007), Galindo, Schiantarelli and Weiss (2007), Chari and Henry (2008) show that countries that liberalize their financial sector allocate capital more efficiently. Henry (2003) and Henry and Sasson (2008) document in addition a rise in TFP. This is not surprising given that allocative efficiency is an important determinant of TFP (Caballero and Hammour (2000), Jeong and Townsend (2007), Restuccia and Rogerson (2008) and Hsieh and Klenow (2009)). For example, Hsieh and Klenow (2009) find that TFP would double in China and India if capital and labor were reallocated to equalize their marginal products across plants.

24Hartmann et al.(2007) find that the same pattern holds among OECD countries.

Rajan and Zingales (1998) and others show that the quality of information disclosure, proxied by accounting standards, enhances growth of industries dependent on external finance. Carlin and Mayer (2003) report that information disclosure is associated with more intense R&D in industries dependent on equity finance and that the relation of industry growth and R&D to information disclosure is more pronounced than to the size of the financial sector. Wurgler (2000) finds a positive cross-country relation between the efficiency of investments and the informativeness of stock prices. Herrera and Minetti (2007) show that the quality of banks’ information has a positive influence on the probability that Italian manufacturing firms innovate.\footnote{Wurgler (2000) uses a proxy for informativeness developed by Morck, Yeung and Yu (2000). They measure the extent to which stocks move together and argue that prices move in a more unsynchronized manner when they incorporate more firm-specific information. Examining the cross-section of U.S. firms, Durnev, Morck and Yeung (2004) and Chen, Goldstein and Jiang (2007) document that firms make more efficient capital budgeting decisions when their stock price is more informative. Herrera and Minetti (2007) use the duration of the credit relationship to proxy for the quality of a bank’s information about a firm.}

B Model Proofs

B.1 Proof of Proposition 1

We start with the financier’s investment decision. He allocates his wage \( w \) across the continuum of projects conceived by the entrepreneur, guided by his signals. At this stage, he takes as given the price of intermediate goods \( \rho \), the projects’ returns in case of success and failure, \( \bar{A} \) and \( A \), and the precision of his signal \( q \). We denote \( K^+ \) the amount of capital allocated to a project deemed successful by the signal, and \( K^- \) the amount allocated to a project deemed unsuccessful. There are 1/2 projects in each category. For example, projects deemed successful consist of the \( q/2 \) projects that are truly successful and correctly identified, and of the \((1 - q)/2 \) projects that are unsuccessful but incorrectly identified, leading to \( q/2 + (1 - q)/2 = 1/2 \) projects in total. The budget constraint imposes that \( K^+/2 + K^-/2 = w \). Output equals \( Y = (q/2)\bar{A}(K^+)^\alpha + [(1 - q)/2]A(K^-)^\alpha + (q/2)\bar{A}(K^-)^\alpha + [(1 - q)/2]A(K^+)^\alpha \), where the four terms represent respectively the production of successful projects correctly and incorrectly identified, and the production of unsuccessful projects correctly and incorrectly identified. A compact expression for output is \( Y = 1/2[v^+(K^+)\alpha + v^-(K^-)\alpha] \) where \( v^+ \equiv q\bar{A} + (1 - q)A \) and \( v^- \equiv (1 - q)\bar{A} + qA \). Profits follow:

\[
\pi = \rho Y = \rho/2[v^+(K^+)\alpha + v^-(K^-)\alpha] = \rho/2[v^+(K^+)\alpha + v^-((2w - K^+)\alpha]
\]

after substituting in the budget constraint. Maximizing this expression with respect to \( K^+ \) yields \( K^+ = 2w(v^+/v)^{1/\alpha} \) and \( K^- = 2w(v^-/v)^{1/\alpha} \) where \( v^{1/\alpha} \equiv (v^+)^{1/\alpha} + (v^-)^{1/\alpha} \) (provided that the signal is informative, that is \( q > 1/2 \)). The expected profit simplifies to

\[
\pi = \rho 2^{\alpha - 1} w^{\alpha} v
\]

once the optimal investment plan is set. The price of intermediate goods follows from their output:

\[
\rho = \beta(Y)^{\beta - 1} = \beta 2^{(1 - \alpha)(1 - \beta)} w^{\alpha(\beta - 1)} v^{\beta - 1}
\]
We turn to the determination of the learning and research efforts. The financier and the entrepreneur who exert efforts $\overline{A}$, $\overline{A}$ and $q$ expect a surplus of $\pi - e_q(q) - e_A(\overline{A} + \overline{A})$. They maximize this expression with respect to $\overline{A}$, $\overline{A}$ and $q$, taking the price of intermediate goods $\rho$ as given. The first-order conditions with respect to $\overline{A}$, $\overline{A}$ and $q$ are respectively:

$$\frac{\partial \pi}{\partial \overline{A}} = 2^{\alpha-1} \rho w^\alpha \left[ q \left( \frac{v^+}{v} \right)^{\frac{\alpha}{1-\alpha}} + (1 - q) \left( \frac{v^-}{v} \right)^{\frac{\alpha}{1-\alpha}} \right] = e'_A(\overline{A} + \overline{A})$$

$$\frac{\partial \pi}{\partial q} = 2^{\alpha-1} \rho w^\alpha \left[ (1 - q) \left( \frac{v^+}{v} \right)^{\frac{\alpha}{1-\alpha}} + q \left( \frac{v^-}{v} \right)^{\frac{\alpha}{1-\alpha}} \right] = e'_q(q).$$

Observe that the marginal benefit of $\overline{A}$ always exceeds that of $\overline{A}$. Formally, $\partial \pi / \partial \overline{A} > \partial \pi / \partial \overline{A}$ for all $\overline{A}$, $\overline{A}$ and $q > 1/2$, where these terms correspond to the left-hand side of the first-order conditions above with respect to $\overline{A}$ and $\overline{A}$. Since $\overline{A}$ and $\overline{A}$ are perfect substitutes in the cost of research (i.e., $e_A$ is a function of the sum $\overline{A} + \overline{A}$), the optimum with respect to $\overline{A}$ is the corner solution $\overline{A} = 0$ as long as $q > 1/2$. That is, the entrepreneur is better of concentrating her effort on improving returns in case of success provided that the financier is informed. The function $v$ equals $\overline{A} V$, and captures the impact of the research and learning efforts on output (through the terms $\overline{A}$ and $V(q)$ respectively). The first-order conditions with respect to the research and learning efforts, $\overline{A}$ and $q$, simplify to:

$$2^{\alpha-1} \rho w^\alpha \left[ q \frac{1}{1-\alpha} + (1 - q) \frac{1}{1-\alpha} \right]^{1-\alpha} = e'_A(\overline{A})$$

$$2^{\alpha-1} \rho w^\alpha \overline{A} \frac{q \frac{1}{1-\alpha} - (1 - q) \frac{1}{1-\alpha}}{q \frac{1}{1-\alpha} + (1 - q) \frac{1}{1-\alpha}} = e'_q(q).$$

In equilibrium, the price of intermediate goods $\rho$ is given by equation 14. Substituting this expression into the first-order conditions for $\overline{A}$ and $q$ yields:

$$\frac{\beta w^\alpha \beta \left[ q \frac{1}{1-\alpha} + (1 - q) \frac{1}{1-\alpha} \right]^{\beta(1-\alpha)}}{2^{\beta(1-\alpha) - 1} A^\beta} e'_A(\overline{A})$$

$$\frac{\beta w^\alpha \beta \overline{A} \beta \left[ q \frac{1}{1-\alpha} - (1 - q) \frac{1}{1-\alpha} \right]^{1-\beta(1-\alpha)}}{2^{\beta(1-\alpha)} q \frac{1}{1-\alpha} + (1 - q) \frac{1}{1-\alpha}} = e'_q(q).$$

Rearranging these expressions leads to equations 3 and 4, which characterize the equilibrium effort levels. The assumptions on the cost functions, $e_A$ and $e_q$, guarantee the existence and unicity of the equilibrium. Specifically, the equilibrium research effort $\overline{A}$ equates the marginal benefit of research in equilibrium ($\partial \pi / \partial \overline{A}$ on the left-hand side of the top equation 15) to its marginal cost ($e'_A(\overline{A})$ on the right-hand side). Given that the former is constant in $\overline{A}$ while the latter increases ($e''_A > 0$) and spans the entire real line ($e'_A(0) = 0$ and $e'_A(+\infty) = +\infty$), there exist a unique solution and it is interior. Similarly, the equilibrium learning effort $q$ is uniquely defined by the bottom equation 15 which equates the marginal benefit of research in equilibrium ($\partial \pi / \partial q$ on the left-hand side) to its marginal cost ($e'_q(q)$ on the right-hand side). Here, existence is obtained by assuming $e'_q(1/2) = e''_q(1/2) = 0$, $e'_q(1) = +\infty$ and $e''_q > 0$, and unicity by assuming $e''_q > 0$. 

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B.2 Proof of Proposition 2

The first part of the proposition follows from noting that the function \( V \) (defined in Proposition 1) is increasing in the learning effort \( q : \partial V/\partial q = V \frac{\alpha}{\alpha - 1}((v^+)^{\frac{\alpha}{\alpha - 1}} - (v^-)^{\frac{\alpha}{\alpha - 1}}) \geq 0 \) for any \( q > 1/2 \). Equation 3 then implies that the research effort \( \overline{A} \) increases with \( q \), holding wealth fixed. The second part follows from equation 4 which shows that the learning effort \( q \) increases with the research effort \( \overline{A} \), holding wealth fixed. Also note that higher wealth \( w \) stimulates learning and research because it expands revenues without affecting costs (a higher wealth implies larger investments).

B.3 Proof of Proposition 3

The distribution of capital is bimodal: half of the projects are labeled as success and receive \( K^+ \) units of capital, while the other half, labeled as failures, receive \( K^- \) units. The expressions for \( K^+ \) and \( K^- \) given in Proposition 1 imply that \( K^+/K^- = (q/(1-q))^{-1/\alpha} \). This ratio increases in \( q \) but does not depend on \( \overline{A} \). Hence, the dispersion of capital increases in \( q \), but not in \( \overline{A} \).

B.4 Proof of Proposition 4

The return on capital equals \( \overline{A}^n(K^n)^{\alpha-1} \). It can take 4 possible values: \( \overline{A}(K^+)^{\alpha-1} \) for a successful projects identified as such, which happens with a probability \( q/2 \); \( \overline{A}(K^-)^{\alpha-1} \) for a successful projects mislabeled as a failure, which happens with a probability \( (1-q)/2 \); \( \overline{A}(K^+)^{\alpha-1} = 0 \) for an unsuccessful projects mislabeled as a success, which happens with a probability \( (1-q)/2 \); and \( \overline{A}(K^-)^{\alpha-1} = 0 \) for a correctly identified unsuccessful projects, which happens with a probability \( q/2 \). It follows that \( V ar \left[ \overline{A}^n(K^n)^{\alpha-1} \right] = q(\overline{A}(K^+)^{\alpha-1})^2 + (1-q)(\overline{A}(K^-)^{\alpha-1})^2 - [q(\overline{A}(K^+)^{\alpha-1}) + (1-q)(\overline{A}(K^-)^{\alpha-1})]/2 \). Substituting in the expressions for \( K^+ \) and \( K^- \) given in Proposition 1 yields \( V ar \left[ \overline{A}^n(K^n)^{\alpha-1} \right] = \overline{A}^2 h(q)V(q)^2 \) where \( h(q) = (1/q - 1/2) + (1 + q)/(1 - q)/4 - 1/2 \). \( V ar \left[ \overline{A}^n(K^n)^{\alpha-1} \right] \) is increasing in \( q \) because both \( h \) and \( V \) are increasing in \( q \) for \( q > 1/2 \). It also increases in \( \overline{A} \).

B.5 Proof of Proposition 5

We first prove the existence of steady-states, and then describe the transition thereto. A steady-state equilibrium is characterized by the following system of equations, obtained from setting \( w_{t+1} = w_t = w^* \) in equations 8, 6 and 7:

\[
\begin{align*}
w^* &= (1 - \beta)w^*\overline{A}^{\beta-1}q^\beta \\
\beta q^*\beta w^* = \overline{A}^{1-\beta}e_A(\overline{A}) \\
\beta \overline{A}^{\beta}w^* = q^{1-\beta}e_q(q^*)
\end{align*}
\]

A trivial solution is \( w^* = \overline{A} = q^* - 1/2 = 0 \). Assuming \( w^* \), \( \overline{A} \) and \( q^* - 1/2 \) are strictly positive, we can take logs and write the system as

\[
\begin{align*}
&-\ln(w^*) + \frac{\beta}{1-\beta} \ln(q^*) + \frac{\beta}{\alpha - 1} \ln(\overline{A}) = -\frac{1}{1-\beta} \ln(1 - \beta) \\
&-\ln(w^*) - \ln(q^*) + \frac{1}{1-\beta} \ln[\overline{A}^{1-\beta}e_A(\overline{A})] = \frac{1}{1-\beta} \ln(\beta) \\
&-\ln(w^*) + \frac{1}{\beta} \ln[q^{1-\beta}e_q(q^*)] - \ln(\overline{A}) = \frac{1}{\beta} \ln(\beta)
\end{align*}
\]

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The system’s Jacobian matrix, $J$, is defined as

$$J = \begin{pmatrix} -1 & \frac{\beta}{1-\beta} \\ -1 & -1 \end{pmatrix} \begin{pmatrix} \frac{\beta}{1-\beta} \\ \frac{1}{1-\beta}(\varepsilon_A - \beta) \end{pmatrix}$$

(17)

where we use the fact that $\partial \ln[\bar{A}^{1-\beta}e_A'(\bar{A})]/\partial \ln(\bar{A}) = \varepsilon_A - \beta$ and $\partial \ln[q^{1-\beta}e_q'(q^*)]/\partial \ln(q^*) = \varepsilon_q - \beta$. The determinant of $J$ satisfies $\beta^2(1 - \beta)\varepsilon_A \varepsilon_q \det J = 1 - \beta - \beta(1/\varepsilon_A + 1/\varepsilon_q)$. Because we assume that $1/\varepsilon_A(\bar{A}) + 1/\varepsilon_q(q) - 1/\beta + 1$ never equals zero, $\det J \neq 0$ for all $\bar{A} > 0$ and $1 > q^* > 1/2$. It follows that there exists a unique non-trivial steady-state.

To study the dynamics, we log-linearize the system around its steady-state. Taylor-series expansions yield $\ln[e_q'(q_t)] \approx \ln[e_q'(q^*)] + (\varepsilon_q - 1) \ln(e_t - \ln(q^*))$ and $\ln[e_A'(\bar{A}_t)] \approx \ln[e_A'(\bar{A})] + (\varepsilon_A - 1) \ln(\bar{A}_t - \ln(\bar{A}))$. We substitute these expressions into equations 4 and 3 after taking logs and using conditions 16 characterizing a steady-state and obtain:

$$\ln(\bar{A}_t/\bar{A}) \approx \frac{\beta}{\varepsilon_A - \beta} [\ln(q_t/q^*) + \ln(w_t/w^*)],$$

(18)

and

$$\ln(q_t/q^*) \approx \frac{\beta}{\varepsilon_q - \beta} [\ln(\bar{A}_t/\bar{A}) + \ln(w_t/w^*)].$$

(19)

Solving for $\ln(q_t/q^*)$ and $\ln(\bar{A}_t/\bar{A})$ yields

$$\ln(q_t/q^*) \approx \frac{\gamma + 1}{\varepsilon_q} \ln(w_t/w^*) \quad \text{and} \quad \ln(\bar{A}_t/\bar{A}) \approx \frac{\gamma + 1}{\varepsilon_A} \ln(w_t/w^*).$$

(20)

Finally, we take the log of equation 5 and use conditions 16 to write:

$$\ln(w_{t+1}/w^*) \approx \beta \ln(q_t/q^*) + \beta \ln(\bar{A}_t/\bar{A}) + \beta \ln(w_t/w^*).$$

(21)

Substituting the expressions for $\ln(\bar{A}_t/\bar{A})$ and $\ln(q_t/q^*)$ back into this equation leads to

$$\ln(w_{t+1}/w^*) \approx (\gamma + 1) \ln(w_t/w^*),$$

(22)

where $\gamma$ is defined in Proposition 2. Income grows if $\gamma > -1$ (i.e. $1/\varepsilon_A + 1/\varepsilon_q < 1/\beta$), at a declining rate if $\gamma < 0$ and at an expanding rate if $\gamma > 0$. The former occurs if $1/\varepsilon_A + 1/\varepsilon_q < 1/\beta - 1$ and the latter if $1/\varepsilon_A + 1/\varepsilon_q > 1/\beta - 1$. If instead $\gamma < -1$ (i.e. $1/\varepsilon_A + 1/\varepsilon_q > 1/\beta$), then income oscillates. The cycles are unstable because $\gamma < -1$ implies that $\gamma < -2$.

### B.6 Proof of Corollary 6

We show how to compute $\bar{\tau}$, the speed of convergence of income in an economy with no interplay between learning and research. This economy is governed by the following system of equations:

$$\begin{cases} 
\beta q^\beta w^\beta_t = \bar{A}^{1-\beta}e_A'(\bar{A}_t) \\
\beta A^\beta w^\beta_t = q_t^{1-\beta}e_q'(q_t) \\
w_{t+1} = (1 - \beta) \ast w^\beta_t \ast q^\beta_t \ast \bar{A}^\beta_t.
\end{cases}$$

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Notice that $q$ and $\overline{A}$ are arbitrary constants in, respectively, the first and second equations (they have no time subscript).\textsuperscript{26} Thus, $q_t$ and $\overline{A}_t$ both grow with income but they do not influence each other directly. Computations similar to those performed in the proof of Proposition 5 yield $\gamma = \frac{\beta e_\gamma e_\delta - \beta}{(\epsilon_A - \beta)(\epsilon_q - \beta)}$. The impact on income of the interplay between learning and research is captured by the growth rate differential between the two economies: $\gamma - \gamma = \frac{\beta^2 (\gamma + 1)}{(\epsilon_A - \beta)(\epsilon_q - \beta)}$. This expression is positive if $\gamma + 1 > 0$, i.e., if income grows.

\section*{B.7 Model Calibration}

We show how to relate the parameters of the cost functions, specifically the elasticities $\varepsilon_A$ and $\varepsilon_q$, to the estimates derived from the broker closure and the R&D tax credit experiments. We assume that the economy is initially in a steady-state, and that it is perturbed in period $T$ by a shock to the cost of either learning or research. Specifically, we assume that these functions are scaled by positive multiplicative parameters, $c_A$ and $c_q$, as follows: $c_A * e_A(\overline{A} + A)$ and $c_q * e_q(q)$. An increase in R&D tax credits is interpreted as a decline in $c_A$, i.e., as a reduction in the cost of research (with no effect on the cost of learning). Conversely, broker closures are interpreted as an increase in $c_q$, i.e., as an increase in the cost of learning (with no effect on the cost of research). Once perturbed, the economy converges towards a new steady-state (denoted $\ast$). In period $T$, before the shock, it is fully described by the system of equations 16 characterizing the initial steady-state. Its evolution is then governed (approximately) by equations 10 to equations 12 under the parameters of the new steady-state.

We compute below the elasticity of R&D expenditures to analyst following in the broker closure experiment from period $T$ (the initial steady-state) to period $T + 1$ (the first period under the new dynamics), $\ln(\overline{A}_{T+1}/\overline{A}_T)/\ln(q_{T+1}/q_T)$. We assume that the cost of learning changes ($c_q$ rises), while the cost of research does not ($c_A$ is constant). The change in the learning intensity then triggers a change in the research intensity, expressed as $\ln(\overline{A}_{T+1}/\overline{A}_T) = \ln(\overline{A}_{T+1}/\overline{A}_T) - \ln(\overline{A}_{T+1}/\overline{A}_T^\ast)$. We start by evaluating the second term. After eliminating income from the second equation of the system 16 thanks to the first equation, and taking logs, we obtain the following expression for the initial steady-state (and a similar one for the final steady-state with $\ast$ replacing $T$):

$$\left(\frac{1}{\beta} - \frac{1}{1 - \beta}\right) \ln(\overline{A}_T) + \frac{1}{\beta} \ln(e_A(\overline{A}_T)) = \ln \left(\beta^{1/\beta}(1 - \beta)^{1/(1 - \beta)}\right) + \frac{1}{1 - \beta} \ln(q_T).$$

Substituting in the log-linearized expression for $\ln[e_A(\overline{A}_T)]$ around the final steady-state, $\ln[e_A(\overline{A})] + (\varepsilon_A - 1) * \ln(\overline{A}_{T+1}/\overline{A}_T)$, yields:

$$\left(\frac{1}{\beta} - \frac{1}{1 - \beta}\right) \ln(\overline{A}_T) + \frac{1}{\beta} \ln[e_A(\overline{A})] + \frac{1}{\beta} (\varepsilon_A - 1) * \ln(\overline{A}_{T+1}/\overline{A}_T) = \ln \left(\beta^{1/\beta}(1 - \beta)^{1/(1 - \beta)}\right) + \frac{1}{1 - \beta} \ln(q_T).$$

Subtracting the equation 23 written for the final steady-state, leads to

$$\ln(\overline{A}_{T+1}/\overline{A}) = \beta / [(1 - \beta)(\varepsilon_A - \beta)] \ln(q_T/q^\ast).$$

\textsuperscript{26}These constants determine the steady-state level of income but have no bearing on its growth rate in the vicinity of the steady-state.
We turn to evaluating \( \ln(\bar{A}_{T+1}/\bar{A}) \). Equation 20 implies that

\[
\ln(\bar{A}_{T+1}/\bar{A}) \approx \frac{\gamma + 1}{\varepsilon_{q}} \ln(w_{T+1}/w^*) \approx \frac{(\gamma + 1)^2}{\varepsilon_{A}} \ln(w_{T}/w^*),
\]

where the first equality is implied by equation 20 and the second by equation 12. Taking logs of the first equation of the system 16 for the initial and final steady-states, and taking the difference, yields \( \ln(w_{T}/w^*) \approx \frac{\beta}{1-\beta} \ln(q_{T}/q^*) + \ln(\bar{A}_{T}/\bar{A}) \)). Equation 24 then implies \( \ln(w_{T}/w^*) \approx \beta \varepsilon_{A}/[(1-\beta)\varepsilon_{A} - \beta] \ln(q_{T}/q^*) \).\(^{27}\) Substituting this expression into the formula for \( \ln(\bar{A}_{T+1}/\bar{A}) \) yields

\[
\ln(\bar{A}_{T+1}/\bar{A}) \approx \frac{\beta(\gamma + 1)^2}{(1-\beta)\varepsilon_{A} - \beta} \ln(q_{T}/q^*).
\]

Together equations 24 and 25 imply

\[
\ln(\bar{A}_{T+1}/\bar{A}) \approx \frac{\beta((\gamma + 1)^2 - 1)}{1-\beta)\varepsilon_{A} - \beta} \ln(q_{T}/q^*).
\]

We proceed in a similar way to evaluate \( \ln(q_{T+1}/q_{T}) \):

\[
\ln(q_{T+1}/q_{T}) = \ln(q_{T+1}/q^*) - \ln(q_{T}/q^*)
\approx \frac{\gamma + 1}{\varepsilon_{q}} \ln(w_{T+1}/w^*) - \ln(q_{T}/q^*)
\approx \frac{(\gamma + 1)^2}{\varepsilon_{q}} \ln(w_{T}/w^*) - \ln(q_{T}/q^*)
\approx \frac{\beta \varepsilon_{A}(\gamma + 1)^2}{\varepsilon_{q}[(1-\beta)\varepsilon_{A} - \beta]} \ln(q_{T}/q^*) - \ln(q_{T}/q^*)
\approx \left( \frac{\beta \varepsilon_{A}(\gamma + 1)^2}{\varepsilon_{q}[(1-\beta)\varepsilon_{A} - \beta]} - 1 \right) \ln(q_{T}/q^*).\]

As a result, the elasticity of R&D expenditures to analyst following in the broker closure experiment is given by:

\[
\frac{\ln(\bar{A}_{T+1}/\bar{A})}{\ln(q_{T+1}/q_{T})} \approx \frac{(\gamma + 1)^2 - 1}{\varepsilon_{A}(\gamma + 1)^2/\varepsilon_{q} - (1/\beta - 1)\varepsilon_{A} + 1}. \tag{26}
\]

We obtain a symmetric formula for the elasticity of analyst following to R&D expenditures in the R&D tax credit experiment:

\[
\frac{\ln(q_{T+1}/q_{T})}{\ln(\bar{A}_{T+1}/\bar{A})} \approx \frac{(\gamma + 1)^2 - 1}{\varepsilon_{q}(\gamma + 1)^2/\varepsilon_{A} - (1/\beta - 1)\varepsilon_{q} + 1}. \tag{27}
\]

Equations 26 and 27 (given equation 12 defining \( \gamma \)) form a system of two equations in two unknowns, \( \varepsilon_{A} \) and \( \varepsilon_{q} \). Note that we assume here that \( \varepsilon_{A} \) and \( \varepsilon_{q} \) do not change much across the two experiments so that they can be treated as the same unknowns in these equations.

\(^{27}\) We cannot exploit here equation 20 relating deviations of \( w \) and \( q \) from steady-state because the cost of learning is not held constant in this experiment (this equation derives from the first-order condition for \( q \), displayed in the third equation of the system 16).
C Model Extension: Controlling Projects’ Probability of Success

We solve here a version of the model in which the entrepreneur controls projects’ probability of success rather than their return. Projects’ returns are exogenously set to 1 in case of success and 0 in case of failure. Creating projects with a success probability \( p \) requires a research effort \( e_p(p) \) where \( e_p \) is continuous, increasing, convex, \( e_p(0) = e'_p(0) = 0, \ e'_p(1) = +\infty \) and \( \varepsilon_p(p) = 1 + p e''_p(p)/e'_p(p) > 0. \)

The financier allocates \( K^+ \) units of capital to a project deemed successful by his signal, and \( K^- \) to a project deemed unsuccessful. The former consist of successful projects correctly identified – there are \( p_t q_t \) of them, and unsuccessful projects incorrectly identified – in number \( (1-p_t)(1-q_t) \), leading to \( p_t q_t + (1-p_t)(1-q_t) \) projects in total. Similarly, the number of projects deemed unsuccessful equals \( (1-p_t)q_t + p_t(1-q_t) \). The budget constraint imposes that \( [p_t q_t + (1-p_t)(1-q_t)] K^+ + [(1-q_t)q_t] K^- = w_t. \) Output equals \( Y_t = p_t q_t K^+ + p_t(1-q_t) K^- \), where the first term represents the production of correctly identified successful projects, and the second term that of incorrectly identified successful projects. Substituting in the budget constraint and maximizing this expression leads to \( K^+ = w_t/[p_t q_t + (1-p_t)(1-q_t)], \ K^- = 0, \ Y_{t+1} = w_t w_t \) and \( \pi(p_t, q_t, w_t) = \rho_{t+1} w_t w_t \) where \( \omega_t \equiv p_t q_t/[p_t q_t + (1-p_t)(1-q_t)] \). We note that these expressions match those obtained in the main model if \( p_t = 1/2 \) and \( A_t = 1 \) (equations 13 and 14).

Maximizing the surplus, \( \pi(p_t, q_t, w_t) = e_q(q_t) - e_p(p_t) \), with respect to \( p_t \) and \( q_t \), taking the price of intermediate goods \( \rho_{t+1} \) as given yields the first-order conditions:

\[
\rho_{t+1} w_t (1 - \omega_t)^2 / (1 - p_t)^2 = e'_q(q_t) \quad \text{and} \quad \rho_{t+1} w_t (1 - \omega_t)^2 / (1 - q_t)^2 = e'_p(p_t).
\]

Finally, substituting \( \rho_{t+1} = \beta(\omega_t w_t)^{\beta-1} \) into these equations leads to the conditions characterizing the equilibrium:

\[
\frac{\beta w_t^\beta (1 - \omega_t)^2}{\omega_t^{1-\beta}(1 - p_t)^2} = e'_q(q_t) \quad \text{and} \quad \frac{\beta w_t^\beta (1 - \omega_t)^2}{\omega_t^{1-\beta}(1 - q_t)^2} = e'_p(p_t).
\]

We assume from now on that \( p_t \) is small, i.e. projects are very risky (this happens in particular if \( e_p \) is very strongly convex). The existence of an interior solution to equations 28 is guaranteed for any \( \rho_{t+1} \) if \( e_q \) is sufficiently convex, namely if \( \varepsilon_q(q) > (1 + q)/(1 - q) \) for \( 1 > q > 1/2 \). Equations 29 simplify to

\[
\beta w_t^\beta p_t = q_t^{1-\beta}(1 - q_t)^{1+\beta} e'_q(q_t) \quad \text{and} \quad \beta w_t^\beta q_t = p_t^{1-\beta}(1 - q_t)^{1+\beta} e'_p(p_t).
\]

The equilibrium properties of learning and research are the same as in the main model (Corollary 2): the financier learns more when the entrepreneur carries out more research (\( q_t \) increases with \( p_t \) holding \( w_t \) fixed), while the entrepreneur carries out more research when the financier learns more (\( p_t \) increases with \( q_t \) holding \( w_t \) fixed).

The dynamic system is characterized by equations 29 and 5, and the initial income \( w_0 \). It admits two steady-states, one of which is zero. The transition to the steady-states \( \hat{w}^* > 0 \) is governed by the following equation:

\[
\ln\left(\frac{w_{t+1}}{\hat{w}^*}\right) \approx (\gamma + 1) \ln\left(\frac{w_t}{\hat{w}^*}\right), \quad \text{where} \quad \frac{1}{\gamma + 1} = \frac{1}{\beta} - \frac{1}{\varepsilon_p} - \frac{1}{\varepsilon_q - (1 + \varepsilon_q)q^*}.
\]
If $1/\varepsilon_p^* - 1/\varepsilon_q - (1 + \varepsilon_q)q^* < 1/\beta$, then income grows (note that the second term is positive since $\varepsilon_q > (1 + q^*)/(1 - q^*)$ by assumption). Its growth rate declines if $\hat{\gamma} < 0$ ($\tilde{\omega}^*$ is a stable steady-state) and expands if $\hat{\gamma} > 0$ (unbounded growth). The former occurs if $1/\varepsilon_p^* - 1/\varepsilon_q - (1 + \varepsilon_q)q^* < 1/\beta - 1$ and the latter if $1/\varepsilon_p^* - 1/\varepsilon_q - (1 + \varepsilon_q)q^* > 1/\beta - 1$. These dynamic patterns are similar to those derived in the main model.

References


Bloom, Nicholas, Schankerman, Mark, and John Van Reenen, 2013, ”Identifying technology spillovers and product market rivalry”, *Econometrica* 81, 1347-1393.

Brown, James, Steven M. Fazzari, and Bruce B. Petersen, 2009 “Financing Innovation and Growth”, *Journal of Finance, 64*(1), 151-185.


40


Figure 1. Timing.

<table>
<thead>
<tr>
<th>Generation t</th>
<th>Young</th>
<th>Old</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Entrepreneur and financier choose:</strong></td>
<td><strong>Financier:</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Earns wage $w_t$</td>
<td>• Intermediate goods are produced</td>
</tr>
<tr>
<td></td>
<td>• Observes signal $S_t$</td>
<td>• Final goods are produced</td>
</tr>
<tr>
<td></td>
<td>• Invests $K_t^a$ across projects</td>
<td>• Agents consume</td>
</tr>
<tr>
<td></td>
<td>Research effort $A_t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Learning effort $q_t$</td>
<td></td>
</tr>
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</table>

<table>
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<tr>
<th>Generation $t+1$</th>
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<th>Old</th>
</tr>
</thead>
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<td><strong>Financier:</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Earns wage $w_{t+1}$</td>
<td>• Intermediate goods are produced</td>
</tr>
<tr>
<td></td>
<td>• Observes signal $S_{t+1}$</td>
<td>• Final goods are produced</td>
</tr>
<tr>
<td></td>
<td>• Invests $K_{t+1}^a$ across projects</td>
<td>• Agents consume</td>
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<td></td>
<td>Research effort $A_{t+1}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Learning effort $q_{t+1}$</td>
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</table>
Figure 2. The Learning Effort.

The picture displays the marginal benefit of learning (solid curve) and its marginal cost (dotted and dashed curves) as a function of the learning effort $q_t$. The dotted curve assumes that $e'_q(q) = [\{(q - 1/2)(1 - q)\}^{0.4}]$ and the dashed curve that $e'_q(q) = [\{(q - 1/2)(1 - q)\}^{0.4} + 1$. The optimal learning effort lies at the intersection of the solid and dotted curves in the first case, and in the corner ($q_t = 1/2$) in the second case. The other parameters are $\beta = 1/2$ and $\omega_t = \lambda_t = 1$. 

![Learning Effort Graph](image-url)
Figure 3. Coverage and Innovation Measures.

The figure shows the average number of analyst covering sample firms and the amount of R&D expenditures, both expressed in logarithm and adjusted for industry and size effects between 1993 and 2006. The series are constructed by taking the average of the residuals of pooled regressions of each variable on industry (sic2) dummies and the logarithm of sales. The correlation between the two series is 0.56.
Table 1. R&D Tax Credits Rate Changes Implemented by US States between 1993 and 2006.

Data on states R&D tax credit is taken from Daniel Wilson’s website (http://www.frbsf.org/economic-research/economists/daniel-wilson/). In this table, given our focus on high-tech firms, we report the statutory tax credit for the highest tier of R&D spending, though for most states the tax credit rate does not vary with the level of R&D spending. Our regressions are based on the direction of the change in tax credit only, not the actual level.

<table>
<thead>
<tr>
<th>State</th>
<th>Year</th>
<th>Tax Credit</th>
<th>Direction of Tax Credit Rate Change</th>
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<td>AZ</td>
<td>1994</td>
<td>20.0%</td>
<td>+</td>
</tr>
<tr>
<td>AZ</td>
<td>2001</td>
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<td>-</td>
</tr>
<tr>
<td>CA</td>
<td>1997</td>
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<td>+</td>
</tr>
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<td>CA</td>
<td>1999</td>
<td>12.0%</td>
<td>+</td>
</tr>
<tr>
<td>CA</td>
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<td>15.0%</td>
<td>+</td>
</tr>
<tr>
<td>CT</td>
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<td>+</td>
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<td>DE</td>
<td>2000</td>
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<td>+</td>
</tr>
<tr>
<td>GA</td>
<td>1998</td>
<td>10.0%</td>
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<td>HI</td>
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<tr>
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<td>IN</td>
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<td>LA</td>
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<tr>
<td>WV</td>
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<td>-</td>
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</table>

Table 2. Summary Statistics.

This table presents the summary statistics on the full sample. The sample includes US firms reporting strictly positive R&D expenditures. The statistics are computed on one observation per firm (the time average of the variable). The last column reports the same statistics from the Compustat sample of firms with strictly positive R&D expenditures. RoA denotes the return on asset and is defined as the ratio of EBIT to total assets.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>S.D.</th>
<th>25th</th>
<th>50th</th>
<th>75th</th>
<th>N</th>
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<tr>
<td>Coverage</td>
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<td>4.00</td>
<td>8.13</td>
<td>14.50</td>
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<td>R&amp;D ($m)</td>
<td>197.74</td>
<td>626.76</td>
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<td>29.50</td>
<td>91.02</td>
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<td>R&amp;D/assets</td>
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<td>0.06</td>
<td>0.01</td>
<td>0.04</td>
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<td>Sales ($m)</td>
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<tr>
<td>Capx/assets</td>
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<td>1.279</td>
<td>0.04</td>
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Table 3. State R&D Tax Credits: Effect on Firm R&D Expenditures.

The table presents the results of the difference-in-difference estimation for the effect of R&D tax credit implementation on firms’ R&D expenditures. TC\(_{t+1}\) is a dummy variable which equals one in the year following the passage or increase in an R&D tax credit in the state in which a firm is headquartered. Loss is a dummy that equals one if the firm reports negative earnings before interests and taxes. The regressions are estimated in first difference, which control for firms’ time invariant characteristics (firms fixed effects). All regressions include year dummies to control for aggregate shocks in each year. Standard errors are clustered at the firm level. T-stats are in brackets. ***, **, * denote significance at 1%, 5%, and 10%, respectively.

<table>
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<tr>
<th></th>
<th>(\Delta\ln(R&amp;D))</th>
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<th>(\Delta\ln(R&amp;D))</th>
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<td>0.046***</td>
<td>0.053***</td>
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<tr>
<td></td>
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<td>[0.014]</td>
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<tr>
<td>TC(_{t+1})</td>
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<tr>
<td>(\Delta\ln(\text{sales}_{t-2}))</td>
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<td>0.039</td>
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Table 4. State R&D Tax Credits: Effect on Analyst Coverage.

The table presents the results of the difference-in-difference estimation for the effect of R&D tax credit implementation on firms’ coverage by financial analysts. TC\(_{t}\) is a dummy variable which equals one in the year following the passage or increase in an R&D tax credit in the state in which a firm is headquartered. Loss is a dummy that equals one if the firm reports negative earnings before interests and taxes. The regressions are estimated in first difference, which control for firms’ time invariant characteristics (firms fixed effects). All regressions include year dummies to control for aggregate shocks in each year. Standard errors are clustered at the firm level. T-stats are in brackets. ***, **, * denote significance at 1%, 5%, and 10%, respectively.

<table>
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</thead>
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<td>(3)</td>
</tr>
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</tr>
<tr>
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<td>TC(_{t})</td>
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</tr>
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<td>TC(_{t+1})</td>
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<td></td>
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<td></td>
<td>[0.026]</td>
<td>[0.026]</td>
<td></td>
</tr>
<tr>
<td>(\Delta\text{loss}_{t-2})</td>
<td>-0.022</td>
<td>-0.03</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.027]</td>
<td>[0.027]</td>
<td></td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>N</td>
<td>7955</td>
<td>5725</td>
<td>4808</td>
</tr>
<tr>
<td>R(^2)</td>
<td>0.01</td>
<td>0.013</td>
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</table>
Table 5. Brokerage closures: Effect on Analyst Coverage.

The table presents the results of the difference-in-difference estimation for the effect of R&D tax credit implementation on firms’ R&D expenditures. AN_1 is a dummy variable that equals one in the year following the loss an analyst due to a brokerage house merge or closure. Loss is a dummy that equals one if the firm reports negative earnings before interests and taxes. The regressions are estimated in first difference, which control for firms’ time invariant characteristics (firms fixed effects). All regressions include year dummies to control for aggregate shocks in each year. Standard errors are clustered at the firm level. T-stats are in brackets. ***, **, * denote significance at 1%, 5%, and 10%, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Δln(coverage)</th>
<th>Δln(coverage)</th>
<th>Δln(coverage)</th>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
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<tr>
<td>AN_{t-1}</td>
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<td>[0.015]</td>
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</tr>
<tr>
<td>AN_1</td>
<td>-0.117***</td>
<td>-0.092***</td>
<td>-0.090***</td>
</tr>
<tr>
<td></td>
<td>[0.017]</td>
<td>[0.018]</td>
<td>[0.019]</td>
</tr>
<tr>
<td>AN_{t+1}</td>
<td></td>
<td></td>
<td>-0.016</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[0.017]</td>
</tr>
<tr>
<td>Δln(sales_{t-2})</td>
<td></td>
<td>0.097***</td>
<td>0.107***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.026]</td>
<td>[0.026]</td>
</tr>
<tr>
<td>Δloss_{t-2}</td>
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<td>-0.026</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.027]</td>
<td>[0.027]</td>
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</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm FE</td>
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<td>Yes</td>
<td>Yes</td>
</tr>
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<td>4,808</td>
</tr>
<tr>
<td>R2</td>
<td>0.012</td>
<td>0.014</td>
<td>0.015</td>
</tr>
</tbody>
</table>

Table 6. Brokerage closures: Effect on R&D Expenditures.

The table presents the results of the difference-in-difference estimation for the effect of R&D tax credit implementation on firms’ coverage by financial analysts. AN_1 is a dummy variable that equals one in the year following the loss an analyst due to a brokerage house merge or closure. Loss is a dummy that equals one if the firm reports negative earnings before interests and taxes. The regressions are estimated in first difference, which control for firms’ time invariant characteristics (firms fixed effects). All regressions include year dummies to control for aggregate shocks in each year. Standard errors are clustered at the firm level. T-stats are in brackets. ***, **, * denote significance at 1%, 5%, and 10%, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Δln(R&amp;D)</th>
<th>Δln(R&amp;D)</th>
<th>Δln(R&amp;D)</th>
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</tr>
<tr>
<td>AN_1</td>
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<td>-0.030**</td>
<td>-0.041***</td>
</tr>
<tr>
<td></td>
<td>[0.014]</td>
<td>[0.013]</td>
<td>[0.015]</td>
</tr>
<tr>
<td>AN_{t+1}</td>
<td></td>
<td></td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[0.013]</td>
</tr>
<tr>
<td>Δln(sales_{t-2})</td>
<td></td>
<td>0.142***</td>
<td>0.127***</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>[0.021]</td>
</tr>
<tr>
<td>Δloss_{t-2}</td>
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<td>-0.047**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.017]</td>
<td>[0.019]</td>
<td></td>
</tr>
<tr>
<td>Year FE</td>
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<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Firm FE</td>
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<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>N</td>
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<td>5,725</td>
<td>4,808</td>
</tr>
<tr>
<td>R2</td>
<td>0.02</td>
<td>0.037</td>
<td>0.038</td>
</tr>
</tbody>
</table>
Table 7. State R&D Tax Credits: Effect on the Distribution of New Equity Proceeds and on the Dispersion of RoA.

The table tests whether the cross-section standard deviation of firm RoA in the state increases after the passage of an R&D tax credit. RoA denotes the return on asset and is defined as the ratio of EBIT to total assets. The test methodology is adapted from Bertrand and Mullainathan (2003). We retain all firms that are never treated as well as three observations before the tax credit change and three observations after the change for treated firms. In a first stage, we pool all observations and regress firms’ RoA on state and year dummies. We extract the residuals. Then, for each treated firm, we aggregate the residuals in two observations: we average the residuals before the treatment year and after the treatment year. The table reports the result of the test of equality of cross-sectional standard deviation of time-averaged residuals before and after the treatment year.

<table>
<thead>
<tr>
<th>Ln(new issue proceeds)</th>
<th>Mean</th>
<th>S.D.</th>
<th>F-statistic for Variance Test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before (3 year average)</td>
<td>0.049</td>
<td>0.041</td>
<td>0.79** (0.029)</td>
</tr>
<tr>
<td>After (3 year average)</td>
<td>-0.104</td>
<td>0.069</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RoA</th>
<th>Mean</th>
<th>S.D.</th>
<th>F-statistic for Variance Test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before (3 year average)</td>
<td>0.015</td>
<td>0.005</td>
<td>0.28*** (0.000)</td>
</tr>
<tr>
<td>After (3 year average)</td>
<td>-0.012</td>
<td>0.009</td>
<td></td>
</tr>
</tbody>
</table>
Table 8. Model Calibration.

This table displays the results of the model calibration. Column 1 displays $\beta$, the share of capital in total income. Columns 2 and 3 display the parameters of the costs of research and learning, $\varepsilon_A$ and $\varepsilon_q$. They are derived from the estimates of the elasticity of R&D expenditures with respect to analyst following in the broker closure experiment ($\approx 1.2$), and from the estimates of the elasticity of analyst following with respect R&D expenditures in the R&D tax credit experiment ($\approx 0.3$), assuming that the economy, perturbed by a shock (changes in R&D tax credits or broker closures) transitions from one steady-state to another. In column 4, $\gamma$ measures the speed of convergence of income to its steady-state. Columns 5 to 7 display the first three components of $\gamma$, i.e., the contributions to income growth of, respectively, diminishing returns to capital, $\gamma_K$, research, $\gamma_A$, and learning, $\gamma_q$. Column 8 displays $\bar{y}$, the growth rate of income in an economy with no interplay between learning and research. Column 9 displays the final component of $\gamma$, namely the contribution to income growth of the interaction between research and learning, $\gamma_{AQ}$ (defined as $\gamma - \bar{y}$). Column 10 displays $\gamma_{AQ}$ in proportion of the total contribution of research and learning. Returns to scale in the intermediate goods sector are assumed constant ($\alpha = 1$). The top panel shows growth rates of income per period, and the bottom panel rates per annum assuming that one period lasts 30 years so the annual rate equals $(1 + \text{per period rate})^{\frac{1}{30}} - 1$.

<table>
<thead>
<tr>
<th>Panel A: Rates per period</th>
<th>(1) $\beta$</th>
<th>(2) $\varepsilon_A$</th>
<th>(3) $\varepsilon_q$</th>
<th>(4) $\gamma$</th>
<th>(5) $\gamma_K$</th>
<th>(6) $\gamma_A$</th>
<th>(7) $\gamma_q$</th>
<th>(8) $\bar{y}$</th>
<th>(9) $\gamma_{AQ}$</th>
<th>(10) $\frac{\gamma_{AQ}}{\gamma_A + \gamma_q + \gamma_{AQ}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.47</td>
<td>0.20</td>
<td>-0.64</td>
<td>-0.90</td>
<td>0.03</td>
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</tr>
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</tr>
<tr>
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<td>-0.38</td>
<td>0.23</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td><strong>0.33</strong></td>
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<td><strong>0.73</strong></td>
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<td><strong>-0.67</strong></td>
<td><strong>0.08</strong></td>
<td><strong>0.27</strong></td>
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</tr>
<tr>
<td>0.40</td>
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<td>0.91</td>
<td>0.07</td>
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<td>0.09</td>
<td>0.31</td>
<td>-0.19</td>
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<td>0.39</td>
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<td><strong>0.14</strong></td>
<td><strong>0.45</strong></td>
<td><strong>0.24</strong></td>
<td><strong>0.31</strong></td>
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<table>
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<tr>
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<th>$\gamma$</th>
<th>$\gamma_K$</th>
<th>$\gamma_A$</th>
<th>$\gamma_q$</th>
<th>$\bar{y}$</th>
<th>$\gamma_{AQ}$</th>
<th>$\frac{\gamma_{AQ}}{\gamma_A + \gamma_q + \gamma_{AQ}}$</th>
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<td>-4.79</td>
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