CREDIT RISK AND THE DEPOSIT INSURANCE PREMIUM, a Note

by Jean Dermine and Fatma Lajeri

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Abstract

Previous research on market-based evaluation of deposit insurance premia has modeled the bank as a corporate firm with risky assets and insured liabilities. No attempt was made to analyze explicitly the risk characteristics of bank assets. The purpose of this note is to model bank lending explicitly and calculate loan-risk sensitive insurance premia. The lending function of banks creates the need to model equity as a 'capped' call option. A simulation exercise shows that market-based estimates of deposit insurance premium which ignore the cap lead to significant underestimation.

Running title : Credit Risk and Deposit Insurance Premium
JEL Classification Codes : G21, G28
Keywords : banking, banking regulation, deposit insurance.
The seminal work by Black and Scholes (1973) on the valuation of options has led to four applications in the banking literature: Pricing of deposit insurance (Merton (1977,1978), Pennacchi (1987), Allen and Saunders (1993)), design of capital requirement (Pyle (1986), Ronn and Verma (1986)), risk premium evaluation for bank subordinated debt (Gorton and Santomero,1990), and determination of examination schedules (Kuester and O'Brien, 1991). Applied papers by Marcus and Shaked (1984), Ronn and Verma (1987, 1989), Giammarino, Schwartz and Zechner (1989), and Cordell and King (1992) elicit from observable market data the necessary parameters to evaluate deposit insurance premia. In these papers on market-based estimates, the role of banks as financial intermediaries is not modeled explicitly. The bank is considered as a standard corporate firm with insured liabilities and risky assets that follow a Wiener process. Equity, subordinated debt, deposits or deposit insurance liability are priced as contingent claims on the assets of the bank. Equity is equivalent to a call option, while the deposit insurance liability is modeled as a put option. Three exceptions in this literature are McCulloch (1985), Crouhy and Galai (1986, 1991), and Duan, Moreau and Sealey (1995) who model explicitly the maturity transformation role of banks and who calculate interest-sensitive risk premia. The purpose of this note is to call the attention to the fact that credit risk affects the distribution of bank asset returns so that the standard Merton methodology that has been used to provide market-based estimation of deposit insurance premia needs to be adapted. To the best of the authors’ knowledge, this is the first paper to introduce lending risk explicitly in the literature on market-based estimation of deposit insurance premia.

The results are as follows. The lending function of banks creates specific risk characteristics and the necessity to model the equity of a bank as a ‘capped’ call option. Empirical estimates of insurance premia that are based on a 'naked' call assumption underestimate significantly the fair value of deposit insurance.

The paper is organized as follows. The theoretical valuation model is developed in Section One. A numerical analysis of the effect of misspecification on the market-based estimation of insurance premia follows in Section Two. Section Three concludes that market-based estimates of deposit insurance premia are highly sensitive to the assumption made about the probability distribution of bank
SECTION ONE : THE VALUATION MODEL

The standard contingent claim approach is applied to a bank-borrowing firm situation. A firm is funding at time 0 an asset (market value=$A_f$) with a bank loan ($L_f$) and equity ($E_f$). The bank funds the loan with insured deposits ($D_b$) and equity ($E_b$). The promised payments on the zero coupon loan and deposits, $L$ and $D$ respectively, are set in a perfectly competitive market. Loans and deposits have a maturity $T$. The deposit insurance premium paid by the bank at time 0 is denoted by $P$. The balance sheets of the firm and of the bank at time 0 are as follows:

\[
\begin{array}{ccc}
A_f & L_f & L_f \\
E_f & P & D_b \\
\end{array}
\]

Borrowing Firm                                Bank

The market value of the asset of the firm varies continuously over the time interval $(0,T)$ according to the stochastic process

\[
dA_t = \mu A_t dt + \sigma A_t dW_t,
\]

where $\mu$ is the instantaneous expected rate of return on the asset, $\sigma$ is the instantaneous standard deviation of the return and $W_t$ is a Wiener process. The stochastic process implies that the value of the asset $A_t$ will follow a lognormal distribution.

As is well known from option theory (Black and Scholes, 1973), the market value of the equity of the bank ($MV_E$) is a call option on the asset of the bank, that is

\[
MV_E = \text{Call} (\text{Value of loan, } D) \\
= \text{Call} (L - \text{Put}(A, L), D),
\]

i.e., the ability to buy the asset of the bank at an exercise price $D$. Given the limited liability of the firm, the value of the loan at maturity is the promised payment on the loan ($L$) reduced by a put option given
to the borrower who can sell his end-of-period asset A at a price L (that is, the bank takes over the assets A of the firm when it defaults). Applying the Put-Call-Parity theorem to the value of the asset of the bank \( L_f \) and denoting by \( r \) the instantaneous risk free rate, one obtains:

\[
L_f = e^{-rT}L - \text{Put}(A, L) = \text{Call}(L - \text{Put}(A, L), D) + e^{-rT}D - \text{Put}(L - \text{Put}(A, L), D) \quad (1).
\]

The asset of the bank is equal to the equity (the call), plus the discounted value of the exercise price (D) minus the liability of the deposit insurer (the put). This allows to write the market value of the equity of the bank \( MV_E \) as follows:

\[
MV_E = \text{Call} = (e^{-rT}L - \text{Put}(A, L)) - e^{-rT}D + \text{Put}(L - \text{Put}(A, L), D)
= L_f - D_b + \text{Put}(A, D) \quad (2).
\]

The simplification of the insurance liability occurs because the put on the asset of the bank will only be exercised when the bank defaults, that is when the borrower defaults and hands his assets A to the bank.

Graphically, the value of the equity of the bank can be represented as follows:

Insert Figure One

The value of the equity of the bank is bounded upward by a cap, the promised loan payment (L) net of the deposit (D). It is bounded downward by the put received from the deposit insurer. The upward sloping segment represents the borrower's default case with the bank holding the end-of-period asset A. An alternative way to interpret the risk characteristics is to observe that the deposits and equity of the bank are equivalent to a senior claim and subordinated debt on the assets of the borrower. The depositors (or deposit insurer) are protected by a double cushion coming from the equity of the borrower and of the bank. The bank shareholders received a fixed payment when the borrower is solvent, and hold the borrower's asset when he defaults.

In this model, the liability of the deposit insurer is modeled as a put on the asset of the borrower,
Deposit Insurance Liability = $\text{Put}_{\text{ins}} = \text{Put}(A, D)$  (3).

Applying the Risk Neutral Valuation methodology and denoting by $N(.)$ the cumulative normal distribution, one obtains standard valuation formula 4:

\[
MV_E^A_{\frac{\ln(A_f D)}{\sigma^2 T}} - \exp\left[\frac{\ln(A_f L)}{\sigma^2 T}ight] \cdot A_f N(\frac{A_f}{\sigma^2 T}) - \exp\left[\frac{\ln(A_f L)}{\sigma^2 T}ight] \cdot A_f N(\frac{A_f}{\sigma^2 T})
\]

Equation (4) can be interpreted as the value of a call on the asset $A$ of the borrower at an exercise price $D$, net of a call given to the borrower on the same asset at an exercise price $L$. The last two terms represent the loss of value resulting from the cap. It is a decreasing value of the loan repayment, going to zero as $L$ goes to infinity. The case analyzed in the literature (the 'naked' call) is a limit case of the 'capped' valuation.

The liability of the deposit insurer is valued as follows:

\[
\text{PUT}_{\text{ins}} = e^{\sigma T D (\frac{\ln(\frac{A_f}{\sigma^2 T})}{2})} - \exp\left[\frac{\ln(A_f L)}{\sigma^2 T}ight] \cdot A_f N(\frac{A_f}{\sigma^2 T})
\]

The valuation of the insurance liability is similar to the put option discussed in the literature, except that the underlying asset is the one of the borrower, not the one of the bank.

Moreover, the variance of the bank equity return is given by
\[
\delta_{MV}^{2} \left[ N\left( \frac{\ln \frac{A^f}{L^2} - \delta^2 D}{\delta \sqrt{T}} \right) - \delta N\left( \frac{\ln \frac{A^f}{L^2} - \delta^2 L}{\delta \sqrt{T}} \right) \right] \ln \frac{A^f}{MV^E} x \delta^2 A \tag{6}
\]

The variance of equity returns differs significantly from the one of a naked call. It can be seen that it is an increasing function of \( L \), equal to the variance of a 'naked' call as \( L \) goes to infinity.

The explicit analysis of bank lending and credit risk has two implications. Firstly, the equity of the bank is equivalent to a 'capped' call. In 'good' economic states, the value of equity cannot exceed the promised payment on the loan net of the deposit repayment. Secondly, the liability of the deposit insurer is directly related to the riskiness of the borrower's asset, the maturity of the deposit and a leverage defined as the deposits to borrower's asset ratio.

The model presented in the paper considers the case of one bank and one borrower with assets that follow a lognormal process. More complex cases could be analyzed. For instance, one could consider the case of bank A lending to bank B that lends to a borrowing firm.

<table>
<thead>
<tr>
<th>Loan(_b)</th>
<th>Deposits(_a)</th>
<th>Loan(_t)</th>
<th>Loan(_b)</th>
<th>Asset(_f)</th>
<th>Loan(_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity(_a)</td>
<td>Equity(_b)</td>
<td>Equity(_t)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Bank A**

**Bank B**

**Borrowing Firm**

In that case too, the relevant risk for the depositors or deposit insurer of bank A will be driven by the value of the borrowing firm's asset. For low values, both the borrowing firm and bank B default so that bank A takes over the asset of the firm. Over some threshold value for the borrower's asset, bank B is able to service the loan and the asset of bank A is capped by that loan repayment. If the mathematics would be slightly different from the one presented in the paper, the conclusions resulting from the cap would hold completely\(^6\).

In the next section, we assess the impact of misspecification on the *market-based estimates* of deposit insurance premia.
SECTION TWO: MISSPECIFICATION AND INSURANCE PREMIUM ESTIMATES

It has been shown that the lending function of banks creates a specific risk structure and the necessity to model the equity of banks as 'capped' call options. Ronn-Verma (1986) have used a 'naked' call option to estimate fair insurance premia. One can wonder whether a cap would alter the estimation of deposit insurance premium significantly. A numerical exercise is designed to test the robustness of deposit insurance premium estimates.

Starting from a set of assumptions on the value of the asset of a borrower, the variance of its return, the size of the loan and the bank equity-to-asset ratio, we calculate the market value of the bank equity, the standard deviation of its return and the insurance premium which are consistent with equations (4) to (6).

In a second step, we apply the Ronn-Verma methodology\(^7\) to the 'observed' computed values obtained for the bank equity and the variance of its return. A 'naked' call methodology is applied in a 'capped' world to assess the extent of the bias.

In the first case reported in Table One, we consider an asset of 100 with variance of 0.1 financed with a loan of 70. We let the bank equity-to-asset ratio fluctuate between 1 % and 10 %. It is assumed that a competitive risk free interest rate of 7 % is paid on bank deposits and that the loan is priced in a perfectly competitive market, that is the terms of the loan are such that its market value is equal to its book value \(L\). In line with the literature, we assume a short maturity of one year for deposits.

Insert Table One.

Setting initially the deposit insurance premium at zero, we find as expected that the \((q)\) ratio of
the market value of the equity of the bank to its book value increases sharply with leverage as the bank extracts a rent from the deposit insurance agency. The ‘q’ ratio jumps from 1.11 to 3.27 as the equity-to-asset ratio decreases from 10 % to 1 %. The third column of Table One reports the fair insurance premium which ranges from 1.26 % to 2.29 % of deposits for a bank leverage range from 10 % to 1 %. The last three columns report the ‘Ronn-Verma’ estimates for the market value of the bank asset, its variance and the fair insurance premium. Two observations stand out. There is a systematic overvaluation of the asset of the bank and an undervaluation of the insurance premium. The Ronn-Verma estimates are zero for all cases, while the true insurance premia calculated with the capped methodology range from 1.26 % to 2.29 % of deposits.

In Table Two, we consider a higher degree of leverage for the borrower (90 %) and various degrees of bank leverage. In this case, there is again an overvaluation of the bank assets and a large undervaluation of the deposit insurance premium. As in Table One, the undervaluation is higher for lower equity-to-asset ratios.

In Table Three, we consider a constant leverage for the bank (8%), but the leverage of the borrower is increased from 30 % to 90 %. The bank ‘q’ ratio increases from 1 to 1.67. There is an overvaluation of the bank’s assets for highly leveraged firms, and a systematic undervaluation of the deposit insurance premium.

In the three cases reported, the Ronn-Verma estimates of insurance premia are lower than their true value. The economic intuition that explains the bias is as follows. Assume an underlying economic process in which the value of bank equity is capped, a ‘true’ variance of asset return and an observed market value of bank equity. If one was using a ‘naked’ call with the ‘true’ variance to value the bank equity, it would lead to a higher figure than the observed one because of the unbounded asset assumption. Therefore, the ‘naked’ call valuation procedure used by Ronn-Verma is forced to reduce the variance of the asset so as to match the calculated value of the ‘naked’ call with the observed value of bank equity. The low variance estimate is a direct implication of the cap. This reduces the probability of default and the estimate of deposit insurance premia.

The existence of the bias has been tested for variances of asset return ranging from 0.05 to 0.20, and for risk free interest rates of 3 percent and 12 percent. In all these cases, the Ronn-Verma
methodology leads to a underestimation of the deposit insurance premia. But as Tables Four and Five illustrate, the relative size of the bias is inversely related to the variance of asset return. For instance, a fair deposit insurance premium reported in Table Four in the case of a leverage of 8% and a variance of asset return of 0.15 is 3.04%, an estimate 6.5 times larger than the Ronn-Verma estimate. In the case of a variance of asset return of 0.20 reported in Table Five, the fair insurance premium of 4.71% is 1.5 times larger than the Ronn-Verma estimate. Finally, the estimation of q-value and deposit insurance premium are insensitive to the level of interest rates because the return on the asset of the firm, on the bank deposits and on the loan adjust fully to an increase of interest rate in this perfectly competitive market model.

Insert Tables Four and Five

SECTION THREE : CONCLUSIONS

The objective of the paper is to show that bank lending and credit risk create a specific stochastic process for the asset of a bank. Equity is equivalent to a 'capped' call and the leverage relevant for the insurer is the deposits to borrower’s asset ratio. It has been shown that 'naked call' evaluations of insurance premia could be unreliable estimates when credit risk is the significant risk factor. The underestimation bias is inversely related to the variance of the borrower’s asset return.

A corollary of the paper is that regulatory capital or deposit insurance premia should be related to the loan-to-value ratio policy of the bank. Indeed, depositors are protected not only by the equity of the bank but also by the equity of borrower. The recent BIS proposals (BIS, 1999) for a revision of international capital guidelines go into that direction. If accepted, they would take into account the effective riskiness of loans and the loan-to-value ratio.

Two words of caution should apply. The first is that a one borrower-one bank case has been analyzed. The issue is whether a multi-borrower case would lead to a lognormal distribution of the value of a loan portfolio. Indeed, one could argue that the assumption of a normal distribution for the return
on a loan portfolio is justified by a Central Limit Theorem applied to a sum of independent returns. However, recent simulation-based research on loan portfolio diversification\textsuperscript{11}, such as Lucas (1995), McAllister and Mingo (1996), CreditMetrics (1997) or Zhou (1997), do report that significant correlation across defaults leads to a highly skewed distribution of the market value of a loan portfolio. There is a large probability of almost no change in the value of a loan portfolio and a left tail distribution with lower values. The economic interpretation of the cap is that few loan defaults occur in periods of economic expansion so that only the principal and interest are recovered, while the left tail of the distribution with lower values is explained by credit losses at the time of recession. The call expressed in this note for attention to the ‘capped’ value of loans appears warranted by these simulation-based results.

A second limitation of the model has been the exclusive emphasis on credit risk. Additional activities such as investment banking services, trading or equity holdings could lead to a distribution closer to that of a standard lognormal. The issue is therefore an empirical one as to whether credit risk on the loan portfolio is the dominating force -making the cap relevant- or whether these additional activities justify the use of a standard lognormal distribution.
REFERENCES


Figure One: The Market Value of the Bank's Equity
Table One: Robustness with respect to Bank Leverage

<table>
<thead>
<tr>
<th>BANK EQUITY (% of Loan)</th>
<th>q RATIO&lt;sup&gt;a&lt;/sup&gt; (MV&lt;sub&gt;1&lt;/sub&gt;/E&lt;sub&gt;b&lt;/sub&gt;)</th>
<th>PREMIUM&lt;sup&gt;a&lt;/sup&gt; (% of DEPOSITS)</th>
<th>RONN-VERMA&lt;sup&gt;b&lt;/sup&gt;</th>
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<sup>a</sup>The market value and insurance premium have been computed with equations (4) and (5) using the following parameters:
Asset = 100, Loan = 70, Variance = 0.1, Interest = 0.07, Maturity = 1 year.

<sup>b</sup>The Ronn-Verma methodology discussed in footnote 7 is applied to the market value and variance computed with equations (4) and (6).
## Table Two: Robustness with respect to Bank Leverage

<table>
<thead>
<tr>
<th>BANK EQUITY (% of Loan)</th>
<th>q RATIO(^a) (MV(_b)/E(_b))</th>
<th>PREMIUM(^a) (% of DEPOSITS)</th>
<th>ASSET</th>
<th>VARIANCE OF ASSET</th>
<th>PREMIUM (% of DEPOSITS)</th>
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</thead>
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</tr>
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</table>

a. The market value and insurance premium have been computed with equations (4) and (5) using the following parameters: Asset=100, Loan=90, Variance=0.1, Interest Rate=0.07, Maturity=1 year.

b. The Ronn-Verma methodology discussed in footnote 7 is applied to the market value and variance computed with equations (4) and (6).
## Table Three: Robustness with respect to Borrower Leverage

<table>
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<tr>
<th>LOAN (% of Asset)</th>
<th>q RATIO&lt;sup&gt;a&lt;/sup&gt; (MV&lt;sub&gt;P&lt;/sub&gt;/E&lt;sub&gt;P&lt;/sub&gt;)</th>
<th>PREMIUM&lt;sup&gt;a&lt;/sup&gt; (% of DEPOSITS)</th>
<th>RONN-VERMA&lt;sup&gt;b&lt;/sup&gt;</th>
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<sup>a</sup>The market value and insurance premium have been computed with equations (4) and (5) using the following parameters: Asset=100, Equity=8 %, Variance=0.1, Interest Rate=0.07, Maturity=1 year.

<sup>b</sup>The Ronn-Verma methodology discussed in footnote 7 is applied to the market value and variance computed with equations (4) and (6).
Table Four: Robustness with respect to Variance
(Variance of Asset Return = 0.15)

<table>
<thead>
<tr>
<th>BANK EQUITY (% of Loan)</th>
<th>q RATIO(^a) (MV(_b)/E(_b))</th>
<th>PREMIUM(^a) (% of DEPOSITS)</th>
<th>ASSET</th>
<th>VARIANCE OF ASSET</th>
<th>PREMIUM (% OF DEPOSITS)</th>
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</table>

\(a\). The market value and insurance premium have been computed with equations (4) and (5) using the following parameters: Asset = 100, Loan = 70, Variance = 0.15, Interest = 0.07, Maturity = 1 year.

\(b\). The Ronn-Verma methodology discussed in footnote è is applied to the market value and variance computed with equations (4) and (6).
Table Five: Robustness with respect to Variance
(Variance of Asset Return = 0.2)

<table>
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<th>BANK EQUITY (% of Loan)</th>
<th>q RATIO(^a) (MV(_b)/E(_b))</th>
<th>PREMIUM(^a) (% of DEPOSITS)</th>
<th>ASSET</th>
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<th>PREMIUM (OF DEPOSITS)</th>
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\(a\). The market value and insurance premium have been computed with equations (4) and (5) using the following parameters:
Asset=100, Loan=70, Variance = 0.2, Interest = 0.07, Maturity = 1 year.

\(b\). The Ronn-Verma methodology discussed in footnote 7 is applied to the market value and variance computed with equations (4) and (6).
1. The option valuation framework has long been used to value corporate credit risk (for instance Black and Cox, 1976, CreditMetrics, 1997, or Crouhy, Galai, and Mark, 1998), but the implication for the estimation of deposit insurance premium has gone unnoticed. The purists will argue that the option methodology relies on perfect market assumptions which make bank lending redundant. An answer is that, in the absence of a more general model, the option framework does allow to discuss relevant insights.

2. As the focus of the paper is on modeling bank lending explicitly, we do not incorporate the fact that the deposit insurer (the writer of the put) can choose the exercise (closure) date. As discussed in Allen-Saunders (1993), the choice of the closure date creates to a callable put.

3. The assumption of equal maturity for deposits and loans permits this simplification. Otherwise, the value of bank assets, at the deposit maturity date, would be equal to
\[ e^{-r(T_L - T_D)} L - \text{Put}(A,L), T_L \text{ and } T_D \text{ denoting the maturity of the loan and of the deposits.} \]

4. A detailed application of the Risk Neutral Valuation method to subordinated debt is available in Black and Cox (1976), and Smith (1980).

5. This follows from equation (2) and the expression for the random return on a put option (P) on an asset (A):

\[ r_{put}' = \frac{\alpha A}{P} \sqrt{r_A} \]

where \( \alpha \) is the standard delta on a put option and \( r_A \) is the random return on the underlying asset (Galai and Masulis, 1976).

6. Other cases could include different stochastic processes for the value of the borrower’s asset with for instance a cap on the value of the asset of the borrower (consumer). If the valuation results would be different, we offer the conjecture that the insight of the paper would remain valid. As is discussed below, an estimation procedure for deposit insurance premium that is based on a ‘naked’ call assumption will underestimate the insurance premium because it ignores the implication of the cap on the asset of the bank. The promised repayment on the loan limits the upside gain on the bank’s asset whatever the stochastic process assumed for the borrower’s asset.

7. Ronn and Verma (1986) solve for the value of bank asset (V) and its variance a system of two non-linear equations.

\[ MV_E' \text{ Call}(V,D)' VN(d_1) \delta De^{\delta T}N(d_2) \]
\[ \delta'_{MV_E' \text{ V N}(d_1) x \delta V} \]

where \( d_1' = ((\ln V/D) \sqrt{T} / \delta \sqrt{2T}) / \delta \sqrt{T} \)
\[ d_2 = \delta \sqrt{T} \]

The value and variance of the assets are used to calculate the risk insurance premium. The software
Mathematica is used for the numerical simulation.

8. A discussion on the sharing of the insurance rent is available in Ronn and Verma (1986).

9. The example could also be used to demonstrate the ineffectiveness of the standard capital regulation, as an increase in bank capital can easily be defeated by an increase in the leverage of the corporate borrower. For instance, it can easily be shown that an increase in bank equity from 8 to 10 % (a 25 % relative increase) necessitates an increase in the loan from 70 to 71.55 (a relative increase of 5 % in the borrower equity-to-asset ratio) to keep the insurance-based subsidy constant.

10. For the coherence of the simulation exercise, it is implicitly assumed that the deposit insurance premium is funded with additional equity so as to keep the loan and deposit constant. In any practical application, this assumption would need to be made more realistic in order to calculate insurance premia more precisely.

11. Simulations are used because of the absence of a closed form solution for the distribution of the value of a loan portfolio.