Firm Dynamics During an Improvement in Financial Development*

James A. Costantini

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Abstract

A better set of financial institutions enables firm access to external finance with less consequent distortion of firm policies, such as investment, entry and exit policies. At the limit, there is no distortion. In contrast, there are many different ways that a poor set of financial institutions may distort firm policies. I consider how the transition towards better financial development depends on the type of poor financial development initially present. I develop a model in which firm dynamics, including entry, investment and exit, are affected by financial development. I contrast the transition to a financial system with no distortions from two initial states, one with restricted use of external finance by firms (e.g., credit constraints) and the other with lax use of external finance by firms (e.g., during a bubble). I highlight substantial differences in firm dynamics, including patterns of entry and price levels, across these two transitions towards better financial development.

JEL Codes: G30, L11, O16

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1 Introduction

The effect of financial institutions on firm dynamics and industry growth has received growing attention. In particular, the effect of a poor set of financial institutions on firm size and growth, and the effect on firms of an improvement in financial institutions. One question this raises is what constitutes a poor set of financial institutions. A straightforward starting point is to consider what a good set of financial institutions are: this will be an institutional arrangement that results in firm policy decisions not being distorted by the issues relating to access to external finance. For instance, in the typical industrial organization models with no financial sector the assumption is just this: the firm has access to external funds needed to fund valuable projects.

A more complicated consideration is what constitutes poor financial development. A major focus has been on issues that limit provision of external finance to firms. The basis underlying problems emphasized are weak external investor rights (e.g., as in Hart and Moore (1994)) or information asymmetry (e.g., as in Stiglitz and Weiss (1981)). In particular, one stream of literature has focused on how credit constraints are consistent with empirical patterns of firm growth and size. Theoretically, credit constraints are able to explain the within-industry patterns of how firm growth and exit vary with firm age and size, as well as the evolution of firm financial structure (Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2002)). Limited access to external finance constrains the growth of firms that would prefer to grow at a rate requiring more financial resources than the cash flow generated by the firm’s current activities. In the theoretical models firms that grow to a sufficient size typically cease to have dependence on external finance, as the funds generated from current internal activities are sufficient to fund future activities. Also, there is empirical evidence that is consistent with smaller, younger firms having their growth limited by poor access to external finance (Beck et. al. (2005), and Cabral and Mata (2003)). Also, estimates of costs to access external finance indicate substantial costs, and that these costs are more substantial for smaller firms (Hennessy and Whited (2007)).

Motivated by these same basic issues of property rights and information sharing, another stream of literature has provided empirical evidence on the extent to which cross-country differences in development of financial institutions limits industry growth (Rajan and Zingales (1998), Beck et. al. (2005), Fisman and Love (2004)).

However, poor financial development may distort financing to firms not only by limiting funding but by providing too much financing. That is, the terms offered to firms may be too lax inducing
the firm to access more external finance than the firm would if there were a good set of financial institutions. One situation in which this occurs is during speculative periods, for instance, as in Japan in the late 1980s, during the Asia crisis of the late 1990s, during the internet boom in the late 1990s, and recently in the U.S. credit market (Kroszner et al. (2007), Kaminsky and Reinhart (1999)). Another situation is where, potentially for just certain sectors, governments provide subsidies and other guarantees that lower the cost of external finance (Gual (1999)).

This raises the issue of how different is the transition from a poor to a good financial system, given that there are many different ways in which a poor financial system may distort provision of external finance to firms. In this paper I focus on how different are firm dynamics during the transition from a situation with restricted access to external finance to a situation with undistorted access to external finance, as compared to the transition from a situation with lax access to external finance to a situation with undistorted access to external finance. In effect, how do the starting points affect the transition. This issue is complementary to the literature that has tended to focus on just the transition from restricted to undistorted.

My contribution is to develop a model in which firm dynamics are affected by the degree of financial development and that allows for different types of poor financial development, in particular both restricted and lax access to external finance. In the model, the degree of financial development affects the cost of external finance. This shapes firm policies, as firms jointly decide the investment, to increase firm capital stock, and financing, including dividend payout, retention of earnings in the firm, and access to external funding. Hence, the model connects institutional cost of external finance to internal firm financial and investment decisions. This connection is also inter-temporally consistent, reflecting the firm’s expectations of the future evolution of firm characteristics, such as productivity, and costs to access external finance. Also, the model is one of heterogeneous firms within an industry, and hence each firm’s policy decision are affected by the policy choices of other firms in the industry.

I find the transition from restricted to undistorted access to external finance is very different to the transition from lax to undistorted access to external finance. Major differences in firm policies are evident in how firm entry and exit decisions change over time: in particular, the extent to which entry and exit is affected in anticipation of or after a change in financial development. Also, the evolution of average industry price and the number of firms over the short-term period immediately around the change in cost of external finance is not necessarily indicative of the long run change. These effects highlight the empirical challenge that the effects of a change in
financial development may not only be concurrent with the change in financial development. Also, the effect of an improvement in financial development is highly heterogeneous across firms in the industry: this provides support for arguments of how political economy considerations may shape the development of financial institutions.

Thus the transition to an improved degree of financial development involves complex dynamic effects due to the firm simultaneously deciding financing and investment policies at each point in time that are consistent with the firm’s future expectations of, for instance, costs to access external finance. To address this situation, I build a model with heterogeneous firms that incorporates both idiosyncratic firm uncertainty (future productivity is stochastic) and forward looking investment decisions subject to sunk costs and access to external finance. The simultaneous consideration of external financing, liquidity, and investment policies is consistent with recent literature (Almeida, Campello and Weisbach (2004)) building on earlier literature on the link between financial constraints and investment (Fazzari, Hubbard and Petersen (1988), Kaplan and Zingales (1997)). The importance of considering the firm’s policies in a dynamic setting have been highlighted, as firms policies reflect future expectations of investment opportunities and costs to access external finance (Titman and Tsyplakov (2007), and Almeida, Campello and Weisbach (2006)).

In my model, all of the forward looking firm decisions concerning entry, exit and incorporate a sunk cost component. I focus on the interaction between the firm decisions to invest in capital stock, access external finance, pay out dividends and/or retain profits as cash within the firm: in effect, the nexus between real investment decisions and the key financing decisions of the firm. The benefit of investment is the accumulation of capital stock, with the actual realized capital stock stochastic. Given firm heterogeneity, this leads to a sorting of firms based on the dependence on external finance and extent of investment. A change in financial development leads to a change in this sorting of firms, and hence I focus on how this sorting changes during the transition to better financial development.

I characterize both the stationary equilibrium with stable cost to access external finance, as well as the equilibrium transition path from a stationary equilibrium to another stationary equilibrium along any arbitrary path of financial development that changes the cost to access external finance. In particular, I compare the transition between the following stationary states: from restricted to undistorted access to external finance; and from lax to undistorted access to external finance. The transition dynamics are affected by firms factoring into current policy decisions the future evolution of cost of access to external finance and the current and future policy decisions of other firms. As
the firm policy decisions are affected by sunk costs (irreversibility), although the end point of both transitions is the same the difference in starting points deeply affects the transition dynamics.

I rely on numerical methods to solve for these equilibria. I develop a general computational algorithm that can be used to solve a wide set of related dynamic industry evolution models.\footnote{These methods have also been concurrently used in Costantini (2006) to study the effects of credit constraints on industrial evolution and in Costantini and Melitz (2007) to study the effect of trade opening on industrial evolution. Similar methods applied to a continuous innovation decision in a general equilibrium setting have also recently been developed by Atkeson and Burstein (2006). The computational methods I use in the current paper apply to a monopolistically competitive sector with a large number of competing firms (where the mass of firms evolves endogenously). Hence, these methods are radically different from the seminal contribution to the computation of such equilibria with a small number of firms under oligopoly in Pakes and McGuire (1994), following the development of the theoretical version of the model in Erikson and Pakes (1995).} I describe this algorithm in detail in the appendix.

2 Model Setup

As highlighted above, I develop the model to analyze the evolution of an industry comprised of heterogeneous firms in response to an improvement in financial development. The firms in the industry are distinguished by their productivity, capital stock and cash. Firm investment in capital stock plus current depreciated capital stock determines the firm’s target capital stock, with the actual capital stock achieved dependent on a stochastic shock. I focus on the interdependent firm policies of investment, to increase capital stock, and financing, which includes use of external finance, payment of dividends and holding of retained earnings as cash. Firm productivity is stochastic. Clearly, these policies are affected by financial development that affects the cost of external finance. I focus on how firm policy choices differ based on different starting points to the process of financial development. I analyze this model in a partial equilibrium setting with respect to the industry: I assume a demand system for the industry as a whole, and a perfectly elastic labor supply to the industry at the economy wide wage.

The core elements of the model are based on Hopenhayn (1992), with the addition of investment to accumulate capital stock subject to financing decisions. The link between the firm investment and firm financing is related to the approach in Cooley and Quadrini (2001). I then computationally solve this extended model for its stationary equilibrium along with any transition paths between two stationary states. The computational model is related to the one in Costantini (2006) and Costantini and Melitz (2007). Across these models the basic algorithm for iterating towards an industry equilibrium in each stationary state and in the transition between stationary states is the same. The difference is the particular set of firm policies focused on and interaction with
institutional characteristics.\footnote{In Costantini (2006) the focus is similarly on financial development and industry dynamics but in a simpler set up, as financial development is either undistorted or no external finance is available (i.e., the limit of the restricted case), and only the stationary states are considered, not the transition between stationary states. In Costantini and Melitz (2007) the focus is on trade and firm dynamics, with the firm dynamics affected by two firm policy decisions not part of the current model, whether to enter the export market and whether to undertake a one-off innovation to improve productivity, and there is no accumulation of capital stock or financing decisions.} I next describe each part of the model, the equilibrium, and how I calibrate the model based on the empirical literature.

**Demand**

Consumer preferences for the differentiated varieties in the industry are C.E.S. with elasticity $\sigma > 1$. There is a continuum of varieties $\omega \in \Omega$. Let $P_t = \left( \int_{\omega \in \Omega} p_t(\omega)^{1-\sigma} \right)^{1/(1-\sigma)}$ be the C.E.S. price index for the aggregated differentiated good $Q_t = \left( \int_{\omega \in \Omega} q_t(\omega)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$ at time $t$, where $p_t(\omega)$ and $q_t(\omega)$ are the price and quantity consumed of the individual varieties $\omega$. Total industry revenues are given by $R = QP$.

**Production**

Each variety is produced by a firm with productivity $v$, capital stock $z$, and labor $l$. There are no frictions in the labor market, with the cost of a unit of labor, $w$, supplied inelastically by each of the $L$ consumers normalized to unity. I normalize the price of capital to unity by appropriate choice of units, and capital is assumed to be available from a competitive market. Hence, total industry revenues $R = wL$.

Firms produce with a Cobb-Douglas technology, with weights $\alpha_v$ on productivity, $\alpha_z$ on capital and $\alpha_l$ on labor, along with an overhead per-period fixed cost $F$ (measured in labor units). Given the demand system and a continuum of competing firms, all firms set a constant markup $\sigma/(\sigma - 1)$ over marginal cost. In each period, the firm, conditional on its productivity and capital stock, optimizes production by hiring the required labor.\footnote{The relative timing of capital and labor decisions follows Olley and Pakes (1996).} The per-period profit is:

$$
\pi_t(v, z) = RP_t^{\sigma-1}p_t^{-\sigma} - \left( \frac{R}{P_t} \right)^{(1/(1-\alpha_v-\alpha_z))(\alpha_v \alpha_z^{\alpha_z} \alpha_z^{\alpha_z})/(1-\alpha_v-\alpha_z) - F. \tag{1}
$$

The firm’s productivity evolves stochastically in each time period with a known martingale process. The firm’s capital stock is more involved, as resulting from both the firm’s choice of how much to invest, subject to available financing, and the stochastic evolution of current capital stock: 
the next section describes this process more fully.

**Capital Stock Evolution**

The firms’ capital stock evolves within each time period as follows. First, the firms’ current capital stock depreciates by a factor $\delta_z$. Next, the firm decides how much to invest in capital stock.

The firm’s choice of how much to invest is financed by accumulated internal funds and access to external finance. In each period, the firm has internally generated financial resources based on accumulated cash $c_t$ plus current profits $\pi_t$. The accumulated cash resources earn interest at the rate of $\beta$, which is the discount rate, from one period to the next.

If the firm has positive internal financial resources, the firm has a choice of how much to invest $I_t$ to increase capital stock, to retain as cash within the firm for future use, and/or pay out as dividends $d_t$. Cash retained and dividends are constrained to be not more than internal financial resources. However, investment may be more than current internal financial resources: in this case, the firm accesses external finance, $e_t$, to finance the difference between the investment and internal financial resources, and the firm’s cash retained for future use and dividend are constrained to zero. In effect, the assumption is for a pecking order use of funding, with first use of internal funds and then use of external funds.

If the firm has negative internal financial resources, the firm must fund the shortfall by access to external finance, and cash retained and dividends are constrained to be zero.\(^4\) In this case if the firm chooses to invest the firm accesses external funds to cover the shortfall plus the desired level of investment.

The firm’s target future level of capital stock, $z_{t+1}^T$, is thus given by $z_{t+1}^T = I_t + (1 - \delta_z)z_t$, where $(1 - \delta_z)z_t$ is the current capital stock remaining after depreciation. The actual level of next period capital stock, $z_{t+1}$, is based on an idiosyncratic shock based on a martingale process with mean at the target capital stock $z_{t+1}^T$.\(^5\)

**External Finance**

Firms have access to external finance in the form of equity finance with a cost wedge $\phi$: specifically, the firm’s inflow of external finance $\phi_t e_t$. This is represented as a negative dividend flow $d_t = \phi_t e_t < 0$. Thus, for $\phi_t = 1$ there are no distortions to the cost of external finance, as in a standard

\(^4\)Note that if external finance were not available the firm would be obliged to exit: this is equivalent to the extreme case of the cost of external finance being infinite.

\(^5\)I introduce this martingale process only to generate a smoother firm distribution across capital stock.
industrial organization model with no financial frictions: I refer to this as the undistorted case, as external finance does not affect firm decisions. The case of $\phi_t > 1$ is when firm’s access to external finance is costly: I refer to this as the restricted case. This is the typical case considered, as it represents a case of financial frictions in the provision of finance. As $\phi_t$ tends to infinity this represents a shut-down of the provision of external finance, as the cost becomes infinite. As discussed above, the frictions could arise either because of information asymmetries, as in Stiglitz and Weiss (1981), and/or limited rights of providers of external finance, as in Hart and Moore (1984). The case of $0 < \phi_t < 1$ is when firm’s access to external finance is subsidized: I refer to this as the lax case. This could be due to government subsidies or because investors provided finance at lower cost than they should as, for instance, during a bubble, as during the Asia crisis, the internet boom, or the current credit crisis in the U.S..

This set up for the firm’s access to external finance aims to be simple yet capture the essence of different financial scenarios. In particular, the three cases allow for comparison of the transition from restricted and lax states to the undistorted state: that is, of different paths for the improvement in financial institutions towards a non-distorted provision of external finance.

Nonetheless, as is clear from the examples illustrating the restricted and lax cases, there would be value in considering separately equity and debt, as well as different types of underlying problems. Clearly, this would substantially complicate the model. For instance, the introduction of debt would require interest rates on the debt to be set endogenously to reflect the situation of the firm borrowing and the amount borrowed. More generally, there are many different ways in which the financial sector could be distorted, both in the provision of external equity as well as debt. The aim in this paper is to highlight the difference to the transition to a better financial system dependent on the starting point. Hence, the value of considering these simple scenarios of different types of distortions.

Hence, with the current set up dividends are given by:
\[ d_t(v_t, z_t, c_t) = \phi_t[c_t + \pi_t(v_t, z_t) - I_t - c_{t+1}] \]  

(2)

where

\[
\begin{align*}
\text{if } [c_t + \pi_t(v_t, z_t) - I_t - c_{t+1}] & \geq 0 \text{ then } \phi = 1 \\
\text{if } [c_t + \pi_t(v_t, z_t) - I_t - c_{t+1}] & = e_t < 0 \text{ and } \\
& \begin{cases} 
\text{Lax state then } 0 < \phi_t > 1 \\
\text{Undistorted state then } \phi_t = 1 \\
\text{Restricted state then } \phi_t > 1 
\end{cases}
\end{align*}
\]

Value Functions and Firm Policy Decisions

I next discuss the firm policy decisions. Within each time period, the timing of events is as follows, as illustrated in Figure 1. First, firms decide whether to continue in the industry or exit. This is based on the maximization of firm value \( V_t(v, z, c) \), with subscripts on the arguments dropped where this does not cause confusion. The firm compares the value of continuing, \( V^C_t(v, z, c) \), to the value of exit \( V^L_t(v, z, c) \):

\[ V_t(v, z, c) = \max \left[ V^C_t(v, z, c), V^L_t(v, z, c) \right]. \]  

(3)

As discussed above, continuing firms maximize their value by optimally choosing investment and dividends. Firms discount next period profits at the exogenous rate \( \beta \), and internalize the exogenous probability \( \delta \) of a death-inducing shock (which is independent of productivity \( v \), capital \( z \), or cash \( c \)).\(^6\) The firm policy choices must satisfy the Bellman equation:

\[ V^C_t(v_t, z_t, c_t) = \max_{i_t, d_t} \left\{ d_t(v_t, z_t, c_t) + \beta (1 - \delta) \int_{v', z'} V_{t+1}(v', z', c') dG[v' | v_t] dG[z' | v_t, I_t, d_t] \right\} \]

s.t.

\[ d_t(v_t, z_t, c_t) = \phi_t[c_t + \pi_t(v_t, z_t) - I_t - c_{t+1}] \]

where

\[
\begin{align*}
\text{if } [c_t + \pi_t(v_t, z_t) - I_t - c_{t+1}] & \geq 0 \text{ then } \phi = 1 \\
\text{if } [c_t + \pi_t(v_t, z_t) - I_t - c_{t+1}] & = e_t < 0 \text{ and } \\
& \begin{cases} 
\text{Lax state then } 0 < \phi_t > 1 \\
\text{Undistorted state then } \phi_t = 1 \\
\text{Restricted state then } \phi_t > 1 
\end{cases}
\end{align*}
\]

\(^6\)Thus, there is both endogenous exit (due to a bad productivity shock) and exogenous exit due to the death shock.
Also, the value of exit is \( V_t^L(v, z, c) = \alpha_L z + c \), with \( \alpha_L \) a parameter for the proportion of capital sold when the firm liquidates (with the remaining \( 1 - \alpha_L \) proportion of capital scrapped).

The firm policies are thus whether to exit and, if continuing, what investment to make and dividends to pay. Hence, the set of optimal firm policies arising from (4) vary across the set of states \((v, z, c)\). The optimal policies may be summarized by partitioning the set of states \((v, z, c)\) into regions each with similar type of optimal policy. A first distinction is between the exit region and the continuation region. The exit region comprises for each capital stock and cash combination \((z, c)\) the set of low enough productivity levels \(v\) that make exit optimal, with the upper bound being the cutoff exit productivity. Note that considering a given level of cash, \(c\), the exit boundary given by the cutoff exit productivity may vary with capital stock depending on the cost to access external finance.

If access to external finance is undistorted, then the exit boundary should vary little with capital stock. A firm contemplating whether to increase capital stock will be indifferent to whether this is funded by internal or external funds (as these cost the same). Nonetheless, capital stock is subject to sunk costs. Capital stock may not be sold off by the firm but just decreased through gradual depreciation and at exit only part of the value of the capital stock is recovered. Hence, there is likely to be some variation in productivity exit boundary across capital stock levels at a given level of cash.

The exit region of the restricted and lax cases differ. In the lax case, a firm with low level of capital stock will choose to continue when the firm is at productivity levels that would trigger exit were access to external finance restricted. The difference across scenarios is explained by considering the three components of firm value affecting this decision: the value of exit, the value due to current profits and the value in future time periods. The value of exit does not depend on cost of external finance. The value due to current profits is similar as at low levels of capital stock firms have negative profits, with the magnitude largely determined by the fixed cost. Hence, in the lax case, the value in future time periods is higher. In part this is due to the lower cost of external finance incurred to accumulate capital. In addition, in the lax case, relative to the restricted case, a firm is less likely to exit while attempting to grow, as the exit boundary is at lower productivity levels, thus increasing the value of continuation. That is, in the lax case the option to grow to larger capital stock levels is more valuable than in the restricted case.

The set of continuation states may be subdivided into regions based on the type of firm policy. For example, firms that access external funds both to fund deficit and to grow capital stock: hence,
these are firms with negative profitability and for which it is optimal to invest to grow capital stock (as opposed to just investing to maintain stable level of capital stock, not investing and allowing capital stock to depreciate, or exiting). A second example of a region is for firms that are profitable but with optimal investment levels above current profits thus requiring external funding. A third example of a region is for firms for which optimal investment just replaces depreciated capital stock to maintain stable size, and with firm profits high enough (as firm productivity and/or capital stock is high enough) that current profits fund investment and enable dividend payout. These regions highlight how within the model set up the optimal firm policies reflect the basic evolution of firm dynamics. Smaller firms require external funds to survive and grow. If the firm reaches sufficient size and productivity, external financial dependence ceases and the firm pays out dividends. During the firm’s growth a sufficiently bad evolution of firm productivity results in firm exit.

Changes in the costs of external finance affects the size and shape of each of these regions, thus affecting firm dynamics while preserving the basic underlying process of small firms aiming to grow to large firms. The prior discussion highlighted the changes in the exit region with changes in access to external finance. These and other changes to the regions due to changes in cost to access to external finance will be discussed below in more detail with the simulation results.

**Entrants**

At the start of each period, new entrants can potentially enter the industry. An entrant pays a sunk cost of entry, \( S \), then realizes its initial productivity draw from a known invariant distribution \( G_E(v, z, c) \), and pays the value of the initial capital stock and cash. Entry is not otherwise restricted. Entrants arrive into the industry with cash set to zero, and a range of initial productivity and capital stock levels. Thereon, entrants are indistinguishable from incumbent firms with the same productivity, capital stock and cash. A prospective entrant therefore faces a net value of entry

\[
V_t^E = \int_{v,z,c} [V_t(v, z, c) - (z + c)] dG_E(v, z, c) - S
\]

When this value is negative, entry is unprofitable.

**3 Equilibrium**

Let \( \mu_{v,z,c,t} \) represent the measure function for producing firms over states \( (v, z, c) \) in period \( t \). This function summarizes all information on the distribution of producing firms across productivity
levels, as well as the total mass of producing firms in state \((v, z, c)\), \(M_{v, z, c, t} = \mu_{v, z, c, t}(Y)\). A dynamic equilibrium is characterized by a time path for the price index \(\{P_t\}\), the measure of firms in each state, \(\{\mu_{v, z, c, t}\}\), and the mass of entrants \(\{M_{E, t}\}\). Note that a choice of \(\{P_t\}\) uniquely determines the time path for \(\{V^C_t(v, z, c)\}\) and thus determines all the optimal choices for any firm, given its productivity \(v\), capital stock \(z\) and cash \(c\). An equilibrium \(\{P_t\}, \{\mu_{v, z, c, t}\}, \text{ and } \{M_{E, t}\}\) must then satisfy the following three conditions:

**Firm Value Maximization** All firms’ choices for exit/continuation, and, if continuing, for investment and dividends, conditional on \(v\), \(z\) and \(c\), must satisfy (3) and (4). In the aggregate, this means that \(\mu_{v, z, c, t}\) is entirely determined by \(\mu_{v, z, c, t-1}\) and the choices for \(\{P_t\}\) and \(\{M_{E, t}\}\). Starting with a mass and distribution of firms at time \(t - 1\), a share \(\delta\) of firms receive the exogenous death shock. The remaining \((1 - \delta)\) share of firms update capital stock and cash, based on choice of investment and dividends and the realization of the capital stock shock, and also update productivity based on the realization of the productivity shock. To these firms are added the \(M_{E, t}\) new entrants, with a distribution determined by \(G_E(v, z, c)\). All firms then make their endogenous exit decisions. The remaining firms result in a distribution and mass of firms for every state. In equilibrium this must match the chosen \(\mu_{v, z, c, t}\).

**Free Entry** In equilibrium, the net value of entry \(V^E_t\) must be non-positive, since there is an unbounded pool of prospective entrants and entry is not limited beyond the sunk entry cost and cost of initial capital stock and cash. Furthermore, entry must be zero whenever \(V^E_t\) is negative.

**Aggregate Industry Accounting** The mass and distribution of firms over productivity levels (aggregating over states) implies a mass and distribution of prices (applying the profit maximizing markup rule to firm marginal cost). Aggregating these prices into the C.E.S. price index must yield the chosen \(P_t\) in every period.

**Stationary Equilibrium**

A time invariant level of external finance cost \(\phi\) leads to a stationary equilibrium with a time invariant price index \(P\), measure of firms \(\mu_{v, z, c}\), and mass of entrants \(M_E\). In such a stationary equilibrium, entry must be positive since there is always an exogenous component to exit. Thus \(V^E_t\) must be zero in this equilibrium. Although an equal mass of firms enter and exit, their distributions over productivity, capital stock and cash will not generally match. This is due to the
productivity transition dynamics among incumbent firms and the investment and financing choices of firms. Jointly, these productivity, capital stock and cash transitions, along with the distribution of entrants and exiting firms, lead to a stationary distribution of firms for every state.

**Equilibrium Along Transition to Improved Financial Development**

I compare two scenarios that entail a transition towards an improved access to external finance from different starting points. The two scenarios are a transition from the restricted to undistorted scenarios (i.e., \( \phi \) decreasing from above one down to one), and from lax to undistorted scenarios (i.e., \( \phi \) increasing from below one up to one). The scenarios have period 1 as a stationary state, in which firms expect the cost of external finance and other parameters to remain stable over time. Depending on the scenario, this is either the restricted or lax case. At the end of period 1, firms are informed that starting from period 13 the cost to access external finance will change to the unconstrained scenario: that is, there is a step change in the cost to access external finance from period 12 to period 13. As there is a pre-announcement of the change in cost of external finance firms have the opportunity to shape their policy decisions in anticipation of the forthcoming changes in costs of external finance. From period 13 onwards the cost of external finance is the undistorted case. Over subsequent periods the equilibrium gradually converges towards the stationary state equilibrium of the undistorted case. In particular, I consider a sufficiently long time period such that by the final period, the industry is arbitrarily close to its stationary equilibrium consistent with the undistorted cost of external finance.

Thus, a summary description of the total long-run change in the industry is provided by a comparison of the stationary states generated by the initial and final set of parameters. The equilibrium path for the price index \( P_t \), measure of firms \( \mu_{v,z,c,t} \), and entrants \( M_{E,t} \) will thus begin at their initial stationary levels until a change in costs of external finance is announced, then follow a transition path until they reach the new stationary state levels, and remain constant thereafter. During the transition, as opposed to the stationary states, the net value of entry may be negative resulting in periods of zero entry.

**Calibration**

I search for the equilibrium paths of \( \{P_t\} \), \( \{\mu_{v,z,c,t}\} \), and \( \{M_{E,t}\} \) using numerical methods. The appendix provides a description of the algorithm used. In essence: I first compute the values of \( P \), \( \mu_{v,z,c} \), and \( M_E \) in the initial and final stationary equilibria. The algorithm then iterates over
candidate equilibrium paths for \( \{P_t\} \) and \( \{M_{E,t}\} \). The choice for \( \{P_t\} \) determines all of the policy choices for any incumbent firm (this is the crucial benefit of abstracting from strategic interactions in the monopolistic competition equilibrium). Since \( \mu_{v,z,c} \) in the initial stationary state is known, I can thus compute \( \{\mu_{v,z,c,t}\} \) based on those policy choices, and the choice for the number of entrants. In turn, I can then compute a new price index \( \{P_t\} \) based on the distribution and mass of firms (which implies a distribution of prices). I iterate until this new price path \( \{P_t\} \) matches the prior choice of the candidate \( \{P_t\} \).

I next describe how I set the parameters of the model to run the model simulations. The model is calibrated to reflect the typical patterns of firm dynamics within industries, in particular: Bartelsman et. al. (2000); Cooley and Quadrini (2001); and Olley and Pakes (1996).

I first describe the grid over time periods, productivity levels and capital stock on which to run the model (Table 1). I set each time period to correspond to one month. This is relatively short thus smoothing out the dynamic processes. I set the total number of time periods to 100 (i.e., around 8 years) as this is long enough to ensure that by the final period the industry has converged close to the stationary equilibrium corresponding to the final set of parameters. Note that I do not impose this final stationary state as the end point: rather, I allow the industry to evolve towards it.

I have the cost of external finance varying over time with the following set up. The change in cost of external finance is announced at the end of period 1, and the change occurs in period 13. That is, there is a step-change in the cost of external finance.

I set the productivity range to \( v = [0.2, 20] \), capital stock range to \( z = [0.2, 20] \), and the cash range to \( c = [0, 20] \). This grid size is exogenous to any firm decisions. The grid is set wide enough such that the exit cutoffs are sufficiently above the lower bound. I set the number of grid points to \( 15 \times 15 \times 5 = 1125 \); high enough that there are sufficient grid points to reduce any effects from the discreteness of the grid. For instance, a finer grid allows for the productivity cutoffs to more smoothly adjust over time.

I next discuss the demand and production parameters (see Table 2 for details). The main demand parameters is the elasticity of substitution between varieties, which I set to \( \sigma = 4 \). The weights of the Cobb-Douglas production function are: productivity \( \alpha_v = 1/6 \), capital stock \( \alpha_z = 3/6 \), and labour \( \alpha_l = 2/6 \), with an overhead cost \( F = 9 \). Throughout the calibration I set the capital stock variables such that capital stock has a high share in production but depreciates fast. This is in part to capture a broad definition of capital, for instance including working capital.
I set the range of costs of external finance variable to $\phi = 5$ for the restricted case, $\phi = 0.2$ for the lax case, and $\phi = 1$ for the undistorted case. The restricted and lax states are set at fairly extreme values to highlight the effects.

Next I discuss my choices for the productivity transitions (see Table 3). First the death shock: I set this at 15% per year, which is higher than the firm level exit rates observed empirically (of around 3-7% per year).

I set the stochastic productivity transition based on a lognormal distribution. For each firm, the draw is from a distribution with mean corresponding to the current firm’s productivity. The standard deviation is the same across all firms (with truncation of extreme changes in productivity, in part to avoid accumulation of firms at the edge of the productivity grid). Thus each firm has the same probability of experiencing a similar percent increase or decline in productivity. I set the stochastic evolution of capital stock based on a lognormal distribution, with mean corresponding to the firm’s target capital stock and with truncation of extreme outcomes.

Finally, I specify the distribution of potential entrants over productivity levels as lognormal with $\log(3)$ mean and 2 standard deviation, and over capital stock as lognormal with $\log(0.3)$ mean and 2 standard deviation, and initial cash to zero. The endogenous exit productivity cutoffs will always be above the .2 productivity lower bound so that some entrants with low productivity draws around this cutoff choose to immediately exit and not produce. Overall, in the simulations, entrants enter with an average productivity lower than that of incumbent firms. Thus, the simulations replicate the robust empirical findings that recent entrants are on average smaller, and exhibit higher exit rates than incumbent firms. The entry sunk cost is set to $S = 60$, which is equivalent to a per-period interest charge of 3% of fixed costs (at the 5% annual discount rate $\beta$).

4 Simulated Results

I first describe the numerical properties of the three stationary states: the undistorted, restricted, and lax cases, respectively with $\phi = 1$, $\phi = 5$, and $\phi = 0.2$. In each case, in equilibrium the specific firm policies depends on the firm’s productivity, capital stock and cash. Within the continuation region, the firm’s investment and dividend polices change with changes in the state variables. This gives rise to regions of the state space within which firms pursue similar types of

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Note that, although the productivity transition has no effect on the firm’s expected productivity, this is not true for the effect on firm’s expected profit. The profit function rises steeply with productivity - hence in expectation, profits rise with a productivity shock. The death shock in part compensates for this effect (otherwise no firm would ever want to exit). Another modeling alternative would be to specify a mean reverting productivity transition.
policies. In particular, four main regions comprise the continuation space within the simulation results, characterized by whether or not external funds are accessed, the use of external funds, and whether or not the firm is seeking to grow. One region has firms that access to external funds to fund deficit and to grow capital stock (this is region 1 in Figure 3). These are smaller firms with negative profitability that want to grow. For instance, most entrants are in this region. At the other extreme, another region is with firms that do not use external funds, just internal funds, and aim to maintain stable capital stock. This is for firms that have grown to high levels of capital stock and high productivity generating high current profits that provide enough funds to replace depreciating stock and payout dividends (this is region 4 in Figure 3). In terms of firm dynamics, these two regions are the typical start for small entrants and the final region for firms that have successfully grown. Along the growth path of firms, the firms may enter other regions (including the exit region). For instance, another region is with firms that are currently profitable and aim to grow capital stock, typically requiring access to external finance (this is region 2 in Figure 3). Another region has firms that access external funds to fund deficit, as profitability is negative, but with investment only aiming to maintain stable capital stock. These are firms typically close to the exit boundary for which aggressive growth in capital stock is not warranted as productivity is low (this is region 3 in Figure 3).

The relative location of these regions in state space does not change, as this is largely driven by the basic underlying firm dynamics of small firms that enter the industry and then grow through a process of additions to capital stock, in part determined by the firm and in part stochastic, and stochastic evolution of productivity. This results in a range of firm dynamics, including exit and growth to become large, productive firms.

Nonetheless, comparing across the external finance scenarios, the size and shape of these regions changes. As the cost of external finance increases, there is an expansion of the set of states in which it is optimal for firms to invest to maintain capital stock and payout dividends out of profits, with no access to external finance (this is region 4). Also, as the cost of external finance increases, the exit boundary shifts. At low levels of capital stock, the higher the cost of external finance, the greater the range of productivity levels falling into the exit regions. At high levels of capital stock, the narrower the range of productivity levels falling into the exit region.

This is reflected in the distribution of firms: most of the difference across scenarios is in the distribution of firms across capital stock, less so across productivity levels (Figure 4). In particular, for the distribution across capital stock, the lax case has more small, more large and less medium
size firms as compared to the restricted case. That is, the lax case has a distribution across capital stock that is more skewed towards the extremes, as compared to the lax case, and hence the cumulative distributions for the restricted and lax cases cross at intermediate levels of capital stock. The undistorted case is intermediate to the restricted and lax cases. The distribution of firms across productivity levels is more similar across scenarios as a fundamental driver is the evolution of productivity shocks, which does not change across scenarios.

Hence, in equilibrium, the distribution of firms in the lax case, relative to the restricted case, is over a broader set of productivity levels, as the exit region is smaller, with a greater proportion of low productivity, low capital stock firms and a greater proportion of high productivity, high capital stock firms: this is summarized in Figure 5. Hence, in the lax case, relative to the restricted case, a greater proportion of new entrants continue in the industry as opposed to exiting. Also, these continuing firms face a lower chance of subsequent exit as the exit region is smaller. These changes in dynamics raise the expected value of entry. However, in equilibrium the expected value of entry is the same as there is common sunk cost of entry across scenarios. Hence, in equilibrium the overall price level is lower in the lax case: consequently the equilibrium number of firms is larger in the lax case than in the restricted case.

The comparison of the stationary states corresponding to the restricted, lax and undistorted scenarios highlights that in considering the transition from the restricted to undistorted scenarios, and from the lax to undistorted scenarios, key aspects relate to the evolution of the exit boundary, change in number of firms over time, and path of the aggregate price level. In the stationary states the number of entrants is always positive (as there is forced exit through the death shock). However, the extent of changes across scenarios in the exit boundary and number of firms indicate potentially interesting changes in number of entrants over time, including periods of zero entry: this is indeed the case, as is discussed in the next section focused on the transition over time.

**Transition from Restricted to Undistorted Scenarios: Reduction in Cost to Access External Finance**

In this section I describe the transition from the restricted scenario to the undistorted scenario. In the next section I describe the transition from the lax scenario to the undistorted scenario, and then compare the two transition paths. In the restricted case $\phi = 5$, representing a state of poor financial development in that access to external finance is much more costly than use of internal funds. The transition is to the undistorted state $\phi = 1$, in which there is no cost to external finance.
and hence access to funding does not distort firm decisions. The evolution of the external finance cost $\phi$ is level at $\phi = 5$ for periods 1 to 12, and then $\phi = 1$, from period 13 on, with the change in $\phi$ announced at the end of period 1: that is, this is an anticipated, abrupt lowering of the cost of external finance. As time periods correspond to one month, the anticipation period corresponds to one year.

As described above, the model numerically solves for firm policies for each month. To illustrate the transition I show in Figure 7 the exit region, summary firm size distribution and total number of firms for selected time periods before the change in $\phi$ (periods 1, 6 and 12) and after the change in $\phi$ (periods 13 and 24). Also, I show the stationary state towards which the equilibrium converges in the long run (labelled as period 101). In addition, I present in Figure 8 the evolution of the price index $P_t$ and number of entrants $M_{E,t}$ over all time periods.

In the long-run the number of firms rises, however initially, up to period 12, the number of firms decreases and then, from period 13 onwards, rises. Similarly, the price index overshoots, rising until period 12 and then in period 13 dropping abruptly to the long run average. The exit region changes somewhat before period 12, and then rapidly changes in period 13 to be very similar to the long-run exit region. Also, entry is zero up to period 12, then there is a spike in entry in period 13 followed by a gradual reduction in entry towards the long-run level.

Initially incumbent firms face, over the long run, a decrease in the cost of external finance and a decrease in prices and hence some firms exit: the exit region expands. Consequently, the value of entry declines, as the likelihood of continuation decreases. Indeed, the decline in the continuation value for entrants is sufficient to drive expected value from entry to below zero, leading to zero entry. Thus by period 12 there are fewer firms than in period 1, and much less than the number of firms in the new long-run stationary equilibrium. Once $\phi$ falls, in period 13, there is sufficient entry to drive the price index down close to the long-run level. This level of entry is above the long-run level of entry and hence the total number of firms rises. Over subsequent periods this pattern continues with entry and total number of firms converging towards their long-run levels.

**Transition from Lax to Undistorted Scenarios: Increase in Cost to Access External Finance**

In this section I describe the transition from the lax scenario to the undistorted scenario, and then compare the two transition paths. In the lax case $\phi = 0.2$, representing a state of poor financial development in that access to external finance is subsidized relative to the cost of internal funds.
The transition is to the undistorted state $\phi = 1$, in which there is no cost to external finance and hence access to funding does not distort firm decisions. The evolution of the external finance cost $\phi$ is level at $\phi = 0.2$ for periods 1 to 12, and then $\phi = 1$, from period 13 on, with the change in $\phi$ announced at the end of period 1: that is, this is an anticipated, abrupt rise in the cost of external finance. As time periods correspond to one month, the anticipation period corresponds to one year.

As above, to illustrate the transition I show in Figure 6 the exit region, summary firm size distribution and total number of firms for selected time periods before the change in $\phi$ (periods 1, 6 and 12) and after the change in $\phi$ (periods 13, 24). Also, I show the stationary state towards which the equilibrium converges in the long run (labelled as period 101). In addition, I present in Figure 8 the evolution of the price index $P_t$ and number of entrants $M_{E,t}$ over all time periods.

The transition from lax to undistorted is characterized by a very different pattern as compared to the transition from restricted to undistorted. In the transition from lax to undistorted there is gradual change in exit cutoff and number of firms, with the changes starting post announcement and before the rise in cost of external finance $\phi$, see Figure 6. In contrast, the price index jumps up mostly from period 12 to 13 when $\phi$ changes, with some gradual rise in the price index before and after the jump. Entry gradually declines up to period 12, and is then zero until period 22, before rising gradually towards the long-run final stationary state level. The period of zero entry is evident in Figure 6 in the period 13 picture in the white area in which firms have low capital stock and high productivity.

In this transition, the long run shift is towards an industry with higher average price level and fewer firms. The rise in price levels and rise in cost to access external finance induces incumbent firms to remain in the industry and entrants to continue to enter (before the rise in $\phi$). Nonetheless, some firms right on the exit boundary choose to exit in anticipation of the change in cost of external finance. These are firms sufficiently far from the exit boundary that will prevail after the drop in $\phi$ that these firms do not find it optimal to stay in the industry in the hope an improvement in productivity (and investment in capital stock would not necessarily shift them closer to the new long-run continuation region). Hence, the exit boundary shifts ahead of the rise in $\phi$. Despite these firms exiting, more firms remain in the industry as of period 13 relative to the number in the new long run stationary state. The over-hang of firms just after the drop in $\phi$ results in a price level lower than in the long-run. This lower price level reduces the value of entry sufficiently that entry drops to zero. Entry starts again once the the number of firms has reduced enough (due to
ongoing exit) to result in a sufficient rise in the price index.

In summary, two key differences across the transition scenarios are that for a transition from lax to undistorted the price index and number of firms adjusts monotonically, whereas for the transition from restricted to undistorted there is overshooting of the price index. Also, the pattern of entry is very different, in particular zero entry occurs before the drop in cost of external finance, for the transition from restricted to undistorted, whereas zero entry occurs after the rise in cost of external finance, for the transition from lax to undistorted.

5 Conclusion

In this paper, I build a dynamic model of firm-level adjustment to financial development that jointly addresses firm’s decisions investment and financing decisions. I analyze the equilibrium transition from two different initial stationary states of poor financial development, to a final state of good financial development. I represent good financial development as there being no cost to access external finance: hence, firm’s policies are not distorted by financing decisions. I consider two types of poor financial development in which firm policies are distorted. In one case, access to external finance is costly, thus use is restricted. In the other case, external finance is lower cost than in the undistorted case: there is lax provision of finance. I find the transition from restricted to undistorted access to external finance is very different to the transition from lax to undistorted access to external finance. Major differences in firm policies are evident in how firm entry and exit decisions change over time: in particular, the extent to which entry and exit is affected in anticipation of or after a change in financial development. Also, the evolution of average industry price and the number of firms over the short-term period immediately around the change in cost of external finance is not necessarily indicative of the long run change. These effects highlight the importance of considering dynamic effects in assessing the effect of changes in financial constraints, as the effects of a change in financial development may not be concurrent with the change in financial development. More generally the results indicate the substantial effect of changes in institutional development on firm dynamics, in this case improvement in financial institutions.

References


Table 1: Calibration: Model Timing and Productivity Grid

<table>
<thead>
<tr>
<th>Variable</th>
<th>Empirical evidence*</th>
<th>Explanation of model calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time path of trade policy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duration of time periods</td>
<td>Most empirical data is annual data</td>
<td>Set each time period to correspond to one month, so time period short relative to typical empirical unit of analysis. Set total number of time periods such that the industry has converged close to the final stationary state.</td>
</tr>
<tr>
<td><strong>Timing of policy change</strong></td>
<td>Typical timeframes involve a lead time, with anticipation of policy change, and time to implement dependent on particular policy change</td>
<td>Set policy change to start after 12 periods (i.e., one year) after announcement, with change taking effect rapidly over one period (i.e., one month)</td>
</tr>
<tr>
<td>Discount rate β</td>
<td>5% per year, thus 0.4% per month (i.e., per time period)</td>
<td></td>
</tr>
<tr>
<td><strong>State space grid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity v</td>
<td>Relative size of largest to smallest firms often over 100x [B et al]</td>
<td>Set v range to [0.2,20] to allow a sufficiently broad range of firm sizes.</td>
</tr>
<tr>
<td>Capital stock z</td>
<td>Relative size of largest to smallest firms often over 100x [B et al]</td>
<td>Set z range to [0.2,20] to allow a sufficiently broad range of firm sizes.</td>
</tr>
<tr>
<td>Cash c</td>
<td></td>
<td>Set v range to [0.2,20] to allow a sufficiently broad range of firm sizes.</td>
</tr>
<tr>
<td><strong>Normalization</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage per period w</td>
<td></td>
<td>Normalize monthly wage to one</td>
</tr>
<tr>
<td><strong>Scaling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of workers L</td>
<td>Choice of L scales size of market: Set L=15,000,000.</td>
<td></td>
</tr>
</tbody>
</table>

* The reference to the empirical literature is as follows: [B et al] Bartelsman (2003)
Table 2: Calibration: Demand, Production and Finance Costs

<table>
<thead>
<tr>
<th>Variable</th>
<th>Empirical evidence*</th>
<th>Explanation of model calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td>$\sigma$</td>
<td>Set to 4</td>
</tr>
<tr>
<td>Production</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weights</td>
<td>$\alpha_v$, $\alpha_z$, $\alpha_l$</td>
<td>Set weights in Cobb-Douglas production function as $\alpha_v=1/6$, $\alpha_z=3/6$, and $\alpha_l=2/6$</td>
</tr>
<tr>
<td>Fixed costs</td>
<td>$F$</td>
<td>Set $F=9$ so that for firms on average fixed labor cost is around 20% of total labor cost</td>
</tr>
<tr>
<td>Finance costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost wedge for external finance</td>
<td>$\varphi$</td>
<td>Set for undistorted scenario $\varphi=1$, for other scenarios wide range: restricted scenario $\varphi=5$, and for lax scenario $\varphi=0.2$</td>
</tr>
</tbody>
</table>

Table 3: Calibration: Evolution of Productivity, Capital Stock, and Entry

<table>
<thead>
<tr>
<th>Variable</th>
<th>Empirical evidence*</th>
<th>Explanation of model calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Productivity transitions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Death shock δ</td>
<td>Exit rate ~3-7% per year [B et al]</td>
<td>Set to 15% per year (with additional exit from firm productivity dropping below exit productivity cutoff), thus 1.4% per month (i.e., per time period).</td>
</tr>
<tr>
<td>Transition for firm productivity</td>
<td>Productivity evolves according to truncated lognormal evolution with mean log((\sigma)) and 1 standard deviation (hence, mean zero change in productivity). Also, truncate increase/decrease to future productivity to within [0.5x, 2x] current v.</td>
<td></td>
</tr>
<tr>
<td><strong>Capital stock transition</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transition for firm target capital stock</td>
<td>Capital stock evolves according to truncated lognormal evolution with mean log(target z) and 0.5 standard deviation (hence, mean zero change in capital stock relative to target capital stock. Also, truncate increase/decrease to capital stock to within [0.5x, 2x] target z.</td>
<td></td>
</tr>
<tr>
<td>Depreciation δ of capital stock</td>
<td>7% per year used by [C and Q] and 4% for buildings and 12% for equipment per year used by [O and P]</td>
<td>Set δ = 5% per month, so around 46% per year</td>
</tr>
<tr>
<td>Liquidation α of capital stock</td>
<td></td>
<td>Set α = -0.5.</td>
</tr>
<tr>
<td><strong>Entrants</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entrant size</td>
<td>Entrants smaller than incumbents on average. Also, around 50% of entrants survive to 7 years, with 20% failed in year 1 and around 10% failed thereafter [B et al]</td>
<td>Set entrants as distributed with independent draws on productivity and capital stock. Productivity draw lognormal, with mean log(3) and std dev=2. Capital stock draw lognormal, with mean log(0.3) and std dev=2. Cash zero. This results in entrants with relative to incumbents, lower average productivity and capital stock, and higher exit rates.</td>
</tr>
<tr>
<td>Entry sunk cost S</td>
<td></td>
<td>Set S=60, which corresponds to a monthly interest charge of 0.2 (i.e., around 3% of per period fixed costs).</td>
</tr>
</tbody>
</table>

Figure 1: Timing and description of events within time periods

Figure 2: Evolution of Cost of External Finance, Phi. The change in phi is announced at the end of period 1, with the change taking effect as of period 13.
Figure 3: Stationary States: Firm policy regions for stationary states with lax, undistorted, and restricted access to external finance. Firm policies shown at cash \( c = 0 \) across range of productivity \( v \) and capital stock \( z \).
Figure 4: Stationary states: Cumulative distribution of firms across productivity and capital stock. Horizontal axis are grid point labels.
Figure 5: Stationary states: Number of firms and distribution of firms across productivity and capital stock for lax, undistorted and restricted financing scenarios. Darker colour indicates higher proportion of firms, and white region is exit region.
Figure 6: Evolution from lax to undistorted stationary state: Number of firms and distribution of firms across productivity and capital stock for selected time periods. Darker colour indicates higher proportion of firms, and white region is exit region.
Figure 7: Evolution from restricted to undistorted stationary state: Number of firms and distribution of firms across productivity and capital stock for selected time periods. Darker colour indicates higher proportion of firms, and white region is exit region.
Figure 8: Evolution of price index and entry for transition of financing scenarios from lax to undistorted, and from restricted to undistorted.
Appendix

A Model Algorithm

Following we describe the algorithm for numerically solving the model, focusing on the equilibrium conditions required and the sequence of calculations performed. The demand structure leads to monopolistic competition. In particular, this means that each firm in each time period \( t \) need only know industry aggregate outcomes for industry price \( P \) from time \( t \) onwards, \( \{P_t, ..., P_T\} \), to determine its specific policies conditional on its current productivity \( v \), capital stock \( z \), and cash \( c \). Firm policy choices are whether to \{Continue, Exit\}, and, if continuing, how much to invest, and whether to access external finance, pay out dividends or retain cash.

The algorithm comprise three steps. Step 1 is to set parameters. Step 2 is to compute the firm policies and firm-size distribution \( \mu_{v,z,c,1} \) corresponding to the initial parameter values, the initial stationary state equilibrium at \( t = 1 \). Within Step 2, there is an iteration over the aggregate price for the stationary state \( P_1 \). Step 3 computes the firm policies and firm-size distribution for the evolution from the initial stationary state through to period \( T \). Within the Step 3, there is an iteration over the price path \( \{P_2, ..., P_T\} \).

1) Set initial parameters, including for industry characteristics and grid structure.

2) \( P_1 \) iteration:

- Choose candidate value for \( P_1 \).

- Firm Value and Policy Iteration:
  - Compute profit \( \pi(v, z) \) at each productivity \( v \) and capital stock \( z \), based on the specific demand system and production function chosen.
  - Pick a candidate value function \( V_1(v, z, c) \).
  - Determine \{Continuation/Exit\} and choice of policy for investment and financing at each \( \{v, z, c\} \).
  - The set of firm policies over continuation and choice of policy state imply a next iteration value for the value function, \( V'_1(v, z, c) \), based on computing the value of continuing and comparing to the value of exit.
  - Check whether new \( V'_1(v, z, c) \) is sufficiently close to \( V_1(v, z, c) \).
* If not, continue iteration with \( V'_1(v, z, c) \).

* If close enough, return to \( P_1 \) iteration.

- Check the value of entry. As seek equilibria with positive entry the condition should be close to zero. Compute firm-size distribution \( \mu_{v,z,c} \).
  
  - If close enough to zero, \( P_1 \) iteration is complete.
  
  - If not, then adjust candidate \( P_1 \) accordingly: if condition is positive lower \( P_1 \), if negative raise \( P_1 \).

3) \( \{P_2, \ldots, P_T\} \) iteration:

- Choose candidate value for \( \{P_2, \ldots, P_T\} \).
  
  - Compute price corresponding to stationary state at final parameter values.
  
  - Set initial guess for \( \{P_2, \ldots, P_T\} \) based on prices corresponding to initial and final parameter values.

- Firm value and policy iteration
  
  - Firm Value and Policy Iteration for \( t = T \):
    
    * Compute profit \( \pi_T(v, z) \) at each productivity \( v \) and capital stock \( z \), based on the specific demand system and production function chosen.
    
    * Pick a candidate value function \( V_T(v, z, c) \).
    
    * Determine \( \{\text{Continuation/Exit}\} \) and choice of policy for investment and financing at each \( \{v, z, c\} \).
    
    * The set of firm policies over continuation and choice of policy state imply a next iteration value for the value function, \( V'_T(v, z, c) \), based on computing the value of continuing and comparing to the value of exit.
    
    * Check whether new \( V'_T(v, z, c) \) is sufficiently close to \( V_T(v, z, c) \).
      
      - If close enough, return to \( \{P_2, \ldots, P_T\} \) iteration.
      
      - If not, continue iteration with \( V'_T(v, z, c) \).

  - Firm Value and Policy Iteration for \( t = \{2, \ldots, T - 1\} \):
* Compute profit \( \pi_t(v, c) \) at each productivity \( v \) and capital stock \( z \), based on the specific demand system and production function chosen.

* Iterate back to compute \( V_{T-1}(v, z, c) \) based on \( \pi_t(v, z) \) and \( V'_T(v, z, c) \), and period \( T \) policies, based on computing the value of continuing and comparing to the value of exit. Hence, determine period \( T-1 \) policies \( \{Continuation/Exit\} \) and choice of policy for investment and financing at each \( \{v, z, c\} \).

* Iterate back to period \( t = 2 \).

- Compute value of entry in each time period \( t = \{2, ..., T - 1\} \).

- Compute the size-distribution of firms \( \mu_{v,z,c} = \{\mu_{v,z,c,2}, ..., \mu_{v,z,c,T}\} \) consistent with the computed firm policies.
  - Compute \( \mu_{v,z,c,2} \) based on \( \mu_{v,z,c,1} \) and firm policies computed for \( t = 2 \).
  - Determine number of entrants:
    * If value of entry negative for \( t = 2 \), set entry to zero.
    * If value of entry is non-negative, set entry such that:
      - Case 1: If the distribution of incumbents implies a price below \( P_2 \) then entry is zero, as adding entrants would further distance the firm distribution from the current value of price path
      - Case 2: If the distribution of incumbents implies a price above \( P_2 \), then add entrants until the firm distribution (including entrants) implies a price equal to \( P_2 \)
  - Iterate forward to compute \( \mu_{v,z,c} = \{\mu_{v,z,c,3}, ..., \mu_{v,z,c,T}\} \).

- Check whether price path \( \{P_2, ..., P_T\} \) is close enough to an equilibrium:
  - Objective function to assess equilibrium comprised of two parts:
    * The first part measures the distance between the price path and firm distribution: \( (P_{\text{max}} - P) \)
    * The second part measures an equivalent gap based on the value of entry: \( (P_{\text{fe}} - P) \).
      - This is zero if value of entry is negative (to capture instances when this value is close to zero but negative, we consider this to be zero if value of entry/sunk cost of entry is larger than \(-10^{-4}\)).
· This is negative if the value of entry is positive. We calculate $P_{fe}$ as what the price in the time period in question would need to change to in order to close part of the gap in value of free entry. Hence, if value of entry is positive the price change is negative so as to lower profitability and thus lower the value of entry. The adjustment is moderated by the extent to which price adjustments for future periods (which have been determined as the algorithm work backs through time periods) are for increases or decreases in prices.

* The objective function is then the Euclidian distance of these two measures: $\left( (P_{max} - P)^2 + (P_{fe} - P)^2 \right)^{\frac{1}{2}}$

- If objective function not sufficiently small, construct new candidate price path.

* The suggested price adjustment is the average of $(P_{max} - P)$ and $(P_{fe} - P)$.

* The actual price adjustment is only part of the suggested price adjustment, to reduce the risk of cycling over successive iterations of the price path.