Firm Operating and Financial Responses to a Financial Crisis: "Maximising Value" while "Managing for Cash" During a Credit Crunch

James A. Costantini
INSEAD
Draft working paper
April 2, 2009

Abstract

A consequence of a financial crisis is to raise the cost of external finance to firms. I consider how firm operating and financial policies respond, from onset of the crisis through to a return to pre-crisis conditions. I develop a model with heterogeneous firms in a competitive set up, with firm policy choices of: labor hiring; investment in capital stock; and cash holding. I find the transition through the crisis back to pre-crisis conditions is faster for firm cash holding and labor hiring than for investment, which declines more slowly but takes longer to recover. In response to the crisis firms rapidly raise cash, especially less productive firms close to exit. Firms with limited initial financial resources cut labor and investment. However, other firms do not cut labor and increase investment. As cash balances increase, labor hiring picks up as firms may use cash balances to finance short-term payroll needs. Investment takes longer to recover as firms hold cash rather than fund long-term investment. I highlight how with an increase in cost of external finance the firm’s policy choices become more interdependent: hence the managerial challenge of managing for value while reacting to a financial shock. I contrast the different responses to a financial crisis and a demand shock, during which firms reduce cash and most firms reduce investment, not just firms with limited financial resources.

JEL Codes: G30, L11, O16

Keywords: Firm and industry dynamics; Financial constraints; Corporate liquidity

*I am grateful for feedback and suggestions from Massimo Massa, Timothy Van Zandt, Ilian Mihov, Pushan Dutt, and Hong Zhang.
1 Introduction

Financial crises present particular challenges for firms, as well as for society in general. For firms, one challenge is that a financial crisis is often part of a broader change in macro-economic conditions that potentially affect the firm. A second challenge is that a financial crisis may manifest itself to a firm in many ways as firms vary in their use of financial instruments, crises may affect some parts of the financial system more than other parts, and institutional arrangement vary substantially across countries.

This paper is focused on a third challenge: the repercussions to the firm of the rise in the cost of external finance due to the crisis. This substantially increases the wedge between the cost of internal and external funds. However, the effect is likely heterogeneous across firms in turn suggesting that crisis periods substantially shape competitive dynamics within the industry. Indeed, there is evidence of heterogeneity in the effect of crises across firms. For instance the effect of the Great Depression was not uniform across firms within industries both in terms of entry, exit, labor hiring and investment (Bresnahan and Raff (1991), Harber (1992)) and financial policies, as some firms accumulated large financial resources whereas other firms struggled to access external funding (Hunter (1982)). Recent evidence from the global credit crisis of 2008 includes sharp differences across firms in the effect of the crisis on firm financial and real policies, depending on the extent of the firm’s financial constraints (Campello, Graham and Harvey (2009)).

A rise in the cost of external finance has an effect on the policy choices of the firm: in particular, the firm’s choice of financial and operational policies becomes much more intertwined absent a well-functioning financial system. For instance, the full cost of an investment includes the extent of external funding used, in turn dependent on other policy choices, for instance the quantity of labor hired for production and thus working capital requirements: thus the full costs of a particular policy are contingent on the combined set of policy choices of the firm.

In turn this highlights a challenge within the firm of how to manage through a financial crisis. Periods of uncertainty and rapid change are particularly challenging for managers: for example, strategies need adapting (e.g., Park and Mezias (2005)) and risk of exit increases (Anderson and Tushman (2001)). In the context of a financial crisis this management pressure is often characterized so as to highlight the need to “manage for cash”. Yet, pursuing this blindly would mean little regard for the more usual emphasis to “manage for value”. Nor would this reflect the differences across firms and thus the extent to which firms should adapt their policies. Thus, the managerial
decision making process is particularly complex during a crisis and likely prone to distortions in
decision making. In this paper I do not focus on the potential distortions to the firm’s decision 
making process in managing through a financial crisis, though recognizing this would be a fruitful 
line of enquiry as deviations from optimal decision making could well feature prominently in a crisis 
setting. I do consider the complexity of the managerial task.

I take as a starting point that firms aim to maximize firm value while factoring in the current 
and future financing costs in their evaluation of alternative firm policies. Specifically, I develop 
a model to consider the firm’s challenge of adjusting operating and financial policies during a 
financial crisis in a competitive context with heterogeneous firms. Thus within the model there 
is rich variation in policy response across firms. Through aggregation of the heterogeneous firm 
responses the model allows consideration of the aggregate response of the industry to the onset and 
evolution of the financial crisis, for instance, in terms of aggregate real and financial measures such 
as investment and use of external finance. So as to facilitate a focus on the firm policy choices I 
keep the characterization of the financial crisis and provision of external finance relatively simple.

Financial crises have occurred in rich and poor countries, recently and in the past (Reinhart 
and Rogoff (2008)). Often the financial crisis is part of a broader crisis. For instance, developing 
countries struck by the twin-crises of banking and balance of payments (Kaminsky and Reinhart 
(1999)). Also, some crises have been more broad and severe, whereas other crises more narrowly a 
financial crisis. A major area of focus has been the link between financial crises and more general 
economic slowdowns, recessions and depressions. In particular, the role of the credit channel in 
exacerbating recessions (Bernanke and Gertler (1989), Bernanke and Gertler (1995)). The credit 
channel operates through the differential effect across firms of the restriction of credit by financial 
intermediaries due to a more general economic slowdown. In this paper, I abstract from some of 
this complexity by taking the perspective of firms within an industry for which the financial shock 
may be taken as exogenous and potentially varying across firms. Also, to highlight the effects of 
the financial shock I do not simultaneously affect demand: I compare a pure financial shock to a 
pure demand shock. In reality, firms often experience a combination of these shocks and the model 
set-up permits consideration of more complex sets of combined shocks.

Financial crises come in many flavors. For instance, the resolution of a crisis reflects the partic-
ular institutional arrangements that may vary across countries as well as over time. Renegotiation 
of financial claims is affected by the extent to which financial systems operate through arms-length 
markets versus relationships, and bankruptcy law, which varies substantially across countries (La
Porta, Lopez-de-Silanes, Shleifer and Vishny (1998)). This variation is also evident in the range of issues highlighted by theoretical models with financial frictions, for instance, with emphasis on weak external investor rights (e.g., as in Hart and Moore (1994)) or information asymmetry (e.g., as in Stiglitz and Weiss (1981)). Hence there are a variety of potential distortions to firm’s access to external finance, with the specific distortions varying by type of financial instrument. Thus, introducing alternative sources of external finance requires specification of the financial frictions for each source, for instance for each of debt and equity (e.g., Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2002), Cooley and Quadrini (2001)). An alternative approach has considered just one type of external finance, accessed with a cost wedge, potentially zero, relative to internal funds (e.g., Froot, Scharfstein and Stein (1993), Almeida, Campello and Weisbach (2006)). This is of value when the focus is less on changes in capital structure or particular financial instruments, but more on the link between financial conditions and firm operating decisions.

I take this alternative approach. However, even a simple cost wedge for external finance substantially complicates the firm’s policy choice. In short, with financial frictions Modigliani and Miller (1958) conditions do not hold: maximizing the value of the firm needs to account for the links between financial and operational decisions. Also, operating decisions that with no financial frictions may be considered independently need to be considered jointly in the presence of financial frictions.

Theoretical and empirical contributions on the effect of financial frictions on firms have frequently emphasized how the firm’s external financial dependence evolves over time, as the firm grows, and across heterogeneous firms at a point in time. Theoretically, credit constraints are able to explain the within-industry patterns of how firm growth and exit vary with firm age and size, as well as the evolution of firm financial structure (e.g., Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2002)). Limited access to external finance constrains the growth of firms that would prefer to grow at a rate requiring more financial resources than the cash flow generated by the firm’s current activities. In the theoretical models firms that grow to a sufficient size typically cease to have dependence on external finance, as the funds generated from current internal activities are sufficient to fund future activities. Also, there is empirical evidence that is consistent with smaller, younger firms having their growth limited by poor access to external finance (Beck, Demirgüç-Kunt and Maksimovic (2005), and Cabral and Mata (2003)), and estimates suggest substantial costs to access external finance, with higher costs for smaller firms (Hennessy and Whited (2007)).
Hence I develop a model of heterogeneous firms that are potentially subject to financial frictions, specifically a cost wedge to access external finance that varies with firm capital stock. The model has firms characterized by three state variables: productivity, capital stock, and cash holdings. The basic structure of the model is one of heterogeneous firms with exogenously varying productivity that evolve over time, with entry and exit in equilibrium, and labor hired in each period for production. The firms are in a monopolistic competitive set up.

To this set up I add a second firm characteristic, capital stock, and allow firms to invest to grow capital stock. I also add a financial asset, cash, which firms may use within a period to fund operations, or carry-over to the next period, potentially adding to current cash from profits not distributed. Hence, an additional policy choice of the firm is financial, specifically dividend payout. Thus the firm policy choices within each period are: labor to hire for production; investment to increase capital stock; and dividends (considered as a dividend payout if positive, and inflow of external finance if negative). In addition, the firm decides whether to continue or exit, and there is an ongoing entry process.

In the context of this model I consider the effect of a financial shock. The shock is modeled as a substantial increase in the cost wedge to access external finance, with a larger increase for firms with smaller capital stock, that then gradually reduces back to pre-crisis levels. Hence, the model connects institutional cost of external finance to internal firm financial and investment decisions. The dynamic set up of the model includes the firm evolution, and industry evolution obtained by aggregation of the firms, over the course of the crisis: the immediate effect at the onset of the crisis, the subsequent periods during the crisis, and the gradual return to pre-crisis conditions. Thus the model captures both short-run responses and long-run transitions. Throughout the crisis firm policies are consistent with competing firm policies, and thus the aggregate industry equilibrium, both within a time period and inter-temporally.

I find the response of firms differs across firms and over time as the crisis unfolds. The firm’s initial response to the crisis is to rapidly raise cash. Also, firms with limited initial financial resources also cut labor and investment. However, other firms do not cut labor and increase investment. The transition through the crisis back to pre-crisis conditions, in the aggregate across firms, is faster for firm cash holding and labor hiring than for investment. The slower return of investment to pre-crisis levels is also reflected in entry and exit patterns. There is an initial rise in exit and drop in entry. Thereafter, during the crisis entry and exit shift to lower levels than pre-crisis levels, with exit out-pacing entry. Gradually, entry and exit rise, with entry rising faster
so as to overtake exit, leading to an increase in the total number of firms back to pre-crisis levels.

Thus the model enables a view of how the shock affects labor hiring, investment and financing decisions, and how this varies across firms within the industry. The financial shock increases the cost wedge, increasing the interdependence across the firm’s policy choices. With no financial frictions to access external finance, in each time period the firm hires labor for production, taking the current firm’s productivity and capital stock as given. In particular, choice of labor to hire is independent of the choice of investment to grow capital stock. However, with financial frictions labor hiring and investment need to be considered jointly, as each policy choice has financing implications in terms of use of the firm’s cash and access to external finance. Also, these decisions are now linked inter-temporally, as any cash used in the current period is not available in the future. In contrast, with no financial frictions firm cash accumulation is irrelevant, as there is no difference to accessing funds from internal or external sources. Hence, the value of a model set up in which firm policies include operating and financial aspects, and inter-temporal issues.

The simultaneous consideration of external financing, liquidity, and investment policies is consistent with recent literature (Almeida, Campello and Weisbach (2004)) building on earlier literature on the link between financial constraints and investment (Fazzari, Hubbard and Petersen (1988), Kaplan and Zingales (1997)). The importance of considering the firm’s policies in a dynamic setting have been highlighted, as firms policies reflect future expectations of investment opportunities and costs to access external finance (Titman and Tsyplakov (2007), and Almeida, Campello and Weisbach (2006)). The heterogeneity in response across firms and the simultaneous effect on both financial and operating policies is in line with recent evidence of the 2008 financial crisis (Campello, Graham and Harvey (2009), Ivashina and Sharfstein (2008)).

I characterize the stationary equilibrium with invariant cost to access external finance and the equilibrium transition path from a stationary equilibrium to another stationary equilibrium: the consideration of the equilibrium transition path is the main contribution of the model. The model enables consideration along any arbitrary path of costs to access external finance: I focus on the transition through a financial shock with eventual return to pre-crisis conditions. The transition dynamics are affected by firms factoring into current policy decisions the future evolution of cost of access to external finance and the current and future policy decisions of other firms.

I rely on numerical methods to solve for these equilibria. I develop a general computational algorithm that can be used to solve a wide set of related dynamic industry evolution models.\footnote{These methods have also been concurrently used to study the effects of credit constraints on industrial evolution}
present numerical solutions of a response to a financial shock, and contrast this with three other scenarios. First, I compare to a demand shock, meant to represent a recession. The response to a financial crisis is very different to the response to a demand shock, during which firm cash balances decrease and most firms reduce investment, not just firms with limited financial resources. Second, I compare the response when firms do not have a financial asset, cash. The substantial difference in response if only operational policies are allowed shows that in the base case operational and financial policies are jointly determined and affecting each other substantially. Third, I compare the effect of a uniform financial shock, as opposed to the base case which has a stronger shock for smaller capital stock firms. Despite the uniform shock, the underlying firm heterogeneity means that the shock nonetheless has a non-uniform effect across firms.

In the rest of the paper I first describe the model set up and then the equilibrium conditions. Then I present the numerical solutions, first describing the calibration for the numerical solutions, next the properties of the baseline scenario, and then the comparison to the alternative scenarios. The appendix describes in detail the numerical algorithm.

2 Model Setup

As highlighted above, I develop the model to analyze the evolution of an industry comprised of heterogeneous firms in response to a shock to the cost of external finance. The firms in the industry are distinguished by their productivity, capital stock and cash, respectively \((v_t, z_t, c_t)\). The timing of events within the model are as follows, as illustrated in Figure 1. At the start of each period new entrants pay a sunk cost of entry and thereon indistinguishable from incumbent firms surviving from the prior period with the same productivity, capital stock and cash. Firms then decide whether to continue or exit.

The value of continuation depends on the firm’s policy choices: operational choices of labor to hire for production and investment to grow capital stock; and financial choice of dividends. These policy choices are considered jointly by the firm. The labor choices determines current production and hence profits realized during the period. The labor and investment choices together determine

and the effect of trade opening on industrial evolution (Costantini (2006) and Costantini and Melitz (2007)). Similar methods applied to a continuous innovation decision in a general equilibrium setting have also recently been developed by Atkeson and Burstein (2006). The computational methods I use in the current paper apply to a monopolistically competitive sector with a large number of competing firms (where the mass of firms evolves endogenously). Hence, these methods are radically different from the seminal contribution to the computation of such equilibria with a small number of firms under oligopoly in Pakes and McGuire (1994), following the development of the theoretical version of the model in Erikson and Pakes (1995).
the firm’s need for external finance to fund working capital, if these expenditures exceed the firm’s cash at the start of the period. The dividend could be positive, indicating a payout from the firm, or negative, indicating an inflow of external finance. Dividends are paid out once current period profits are realized. The dividend choice also determines the cash holding of the firm. Thus at the end of the period the firm’s cash flow, which I refer to as the cash flow from financing, may be positive or negative, depending on whether initial cash plus profits minus investment exceeds or not retained cash for next period. If negative this increases the firm’s external finance needs beyond those required to fund working capital. At the end of the period the uncertainty is resolved for the transition to the next period. There is the death shock, and productivity and capital stock evolutions.

I analyze this model in a partial equilibrium setting with respect to the industry: I assume a demand system for the industry as a whole, and a perfectly elastic labor supply to the industry at the economy wide wage. I next describe in detail each part of the model, the equilibrium, and how I calibrate the model to generate the numerical solutions.

**Demand**

Consumer preferences for the differentiated varieties in the industry are C.E.S. with elasticity $\sigma > 1$. There is a continuum of varieties $\omega \in \Omega$. Let $P_t = \left( \int_{\omega \in \Omega} p_t(\omega)^{1-\sigma} \right)^{1/(1-\sigma)}$ be the C.E.S. price index for the aggregated differentiated good $Q_t \equiv \left( \int_{\omega \in \Omega} q_t(\omega)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$ at time $t$, where $p_t(\omega)$ and $q_t(\omega)$ are the price and quantity consumed of the individual varieties $\omega$. Total industry revenues are given by $R_t = Q_t P_t$.

**Production: Choice of labor**

Each variety is produced by a firm with productivity $v$ and capital stock $z$. The firm hires labor in a labor market with no frictions, with the cost of a unit of labor, $w$, normalized to unity. Labor is supplied inelastically by each of the $L_t$ consumers. I normalize the price of capital to ten by appropriate choice of units, and capital is assumed to be available from a competitive market. Hence, total industry revenues $R_t = w L_t$.

Firms produce with a Cobb-Douglas technology, with weights $\alpha_v$ on productivity, $\alpha_z$ on capital and $(1-\alpha_v-\alpha_z)$ on labor, along with an overhead per-period fixed cost $F$ (measured in labor units). Given the demand system and a continuum of competing firms, all firms set price $p_t(v_t, z_t, c_t)$ a constant markup $\sigma/(\sigma-1)$ over marginal cost. In each period, the firm takes its productivity
and capital stock, as well as current cash, as given when optimizing hiring of production labor, \(l_t(v_t, z_t, c_t)\).\(^2\) The per-period gross profit from production, \(\pi_t(v_t, z_t, c_t)\), is:

\[
\pi_t(v_t, z_t, c_t) = R_tP_t^{\sigma-1}p_t(v_t, z_t, c_t)^{1-\sigma} - l_t(v_t, z_t, c_t) - F
\]  

(1)

Note that current profits depend on choice of labor (which in turn depends on the state variables): for later convenience I highlight this by the following notation: \(\pi_t(v_t, z_t, c_t) = \pi_t(v_t, z_t, c_t; l_t)\).

**Evolution of productivity and capital stock**

The firm’s productivity evolves stochastically in each time period with a known martingale process. The firm’s capital stock is more involved, as resulting from both the firm’s choice of how much to invest and the stochastic evolution of capital stock, evolving within each time period as follows. At the end of each period, the firms’ current capital stock depreciates by a factor \(\delta_z\) with \((1-\delta_z)z_t\) the capital stock remaining after depreciation. At the start of each period, the firm decides how much to invest, \(I_t(v_t, z_t, c_t)\), in capital stock, with the realized increase in capital stock affected by a multiplicate stochastic shock \(\varepsilon_z\) from a known distribution.\(^3\) Thus next period capital stock is given by:

\[
z_{t+1} = (1 - \delta_z)z_t + I_t(v_t, z_t, c_t)(1 + \varepsilon_z)
\]  

(2)

**External finance**

The firm’s use of external finance is comprised of the external finance required to fund the firm’s policy choices scaled by a cost wedge \(\phi_t(v_t, z_t, c_t)\). When \(\phi_t(v_t, z_t, c_t) > 1\) the cost wedge reflects the deadweight costs of a firm accessing external funds, relative to internal funds. Firm’s need for external finance arises for potentially two reasons. The first reason is within a time period during which the firm may need external finance to fund operations. The second reason is at the end of the period at which point the firm may need external finance based on desired cash level for next period.

The firm may need external finance to fund production: in essence, financing of working capital. At the start of the period, the firm has cash on hand (restricted to be non-negative), hires labor

\(^2\)The relative timing of capital and labor decisions follows Olley and Pakes (1996).

\(^3\)I introduce this martingale process only to generate a smoother firm distribution across capital stock.
and invests in capital stock. Only at the end of the period are revenues realized. Thus the firm’s net cash flow from operations, $CF^O_t(v_t, z_t, c_t)$, is:

$$CF^O_t(v_t, z_t, c_t) = c_t - l_t(v_t, z_t, c_t) - F - I_t(v_t, z_t, c_t)$$ (3)

If $CF^O_t(v_t, z_t, c_t)$ is negative, the firm has need for external finance so as to fund the start of period operational choices.

At the end of the period, once revenues are realized, the firm may retain some cash for the next period, $c_{t+1}$. The net cash flow from financing, $CF^F_t(v_t, z_t, c_t)$, is defined as:

$$CF^F_t(v_t, z_t, c_t) = c_t + \pi_t(v_t, z_t, c_t) - I_t(v_t, z_t, c_t) - c_{t+1}$$ (4)

If $CF^F_t(v_t, z_t, c_t)$ is negative the firm needs external finance.

Thus the firm’s total access to external finance, $EF^F_t(v_t, z_t, c_t)$, is

$$EF^F_t(v_t, z_t, c_t) = -\{\min(CF^O_t(v_t, z_t, c_t), 0) + \min(CF^F_t(v_t, z_t, c_t), 0)\}$$

Note that each component may lead to the need for external finance. Hence the total cost wedge the firm pays for for access to external finance, $\eta_t(v_t, z_t, c_t)$, is:

$$\eta_t(v_t, z_t, c_t) = (\phi_t(v_t, z_t, c_t) - 1)EF^F_t(v_t, z_t, c_t)$$ (5)

To highlight which policy choice affect which variables, note that choice of labor and investment affect each of $CF^O_t$ and $CF^F_t$, and hence $EF^F_t$ and $\eta_t$. Using similar notation as above, $CF^O_t(v_t, z_t, c_t) = CF^O_t(v_t, z_t, c_t; l_t, I_t)$ and similarly for the other variables.

The firm dividends reflect the choice of operational policy choices of labor and investment, and choice of cash to retain in the firm$^4$. The dividends, $d_t$, represent the net flow of funds out of the firm. If positive, this represents a dividend payment. If negative, this reflects the firm accessing to external finance at the end of the periods: that is, negative cash flow from financing. Thus

$^4$I set up the model to make dividends the choice of the firm, with cash holdings a consequence. This keeps policy variables separate from state variables. However, in presenting the firms need for external finance I discuss first the cash the firm decides to retain, with dividends a consequence: this is typically easier to follow.
dividends are given by:

\[ c_{t+1} = c_t + \pi_t(v_t, z_t, c_t) - I_t - \eta_t(v_t, z_t, c_t) - d_t \]

\[ = c_t + \pi_t(v_t, z_t, c_t) - I_t - (\phi_t(v_t, z_t, c_t) - 1)EF^F_t(v_t, z_t, c_t) - d_t \]

(6)

Hence, if the firm is does not require external finance, \( EF^F_t(v_t, z_t, c_t) = 0 \), then the above simplifies to \( c_{t+1} = c_t + \pi_t(v_t, z_t, c_t) - I_t - d_t \).

Thus, for \( \phi_t = 1 \) there are no distortions to the cost of external finance, as in a standard industrial organization model with no financial frictions. In this case external finance does not affect firm labor hiring and investment decisions. The case of \( \phi_t > 1 \) is when firm’s access to external finance is costly. This is the typical case considered, as it represents a case of financial frictions in the provision of finance.\(^5\) As \( \phi_t \) tends to infinity this represents a shut-down of the provision of external finance, as the cost becomes infinite. As discussed above, the frictions could arise either because of information asymmetries, as in Stiglitz and Weiss (1981), and/or limited rights of providers of external finance, as in Hart and Moore (1984).

The external finance cost wedge depends on the both the characteristics of the providers of finance as well as of the firms. Hence, in general, \( \phi_t \) would be dependent on the amount of external finance demanded by a firm. In this case, I simplify by setting \( \phi_t(v, z, c) = \phi_t(z) \). Thus \( \phi_t \) varies with firm capital stock: in particular, \( \phi_t \) does not vary, conditional on the firm’s capital stock, with quantity of external finance used by the firm.

This specification avoids the full complexity of making \( \phi_t \) endogenous. In particular, to make the cost wedge endogenous requires specification of the financial instruments available to the firm (e.g., debt, equity) and the particular distortions the firm faces in accessing these sources of external finance. For instance, the introduction of debt would require interest rates on the debt to be set endogenously to reflect the situation of the firm borrowing and the amount borrowed. More generally, there are many different ways in which the financial sector could be distorted, both in the provision of external equity as well as debt. The set up for the firm’s access to external finance aims to be simple yet capture the essence of different financial scenarios. This approach of a more general specification that may encompass a variety of potential distortions is one approach used in the literature to model frictions to access external finance (e.g., Almeida, Campello and Weisbach (2006)).

\(^5\) The case of \( 0 < \phi_t < 1 \) is when firm’s access to external finance is subsidized (e.g., due to government subsidies) or because investors provided finance at lower cost than they should as, for instance, during a bubble.
Value Functions and Firm Policy Decisions

I next discuss how the firm policy decisions are embedded in the firm value functions. In addition to the operational and financial policies discussed above, firms decide whether to continue in the industry or exit. This is based on the maximization of firm value $V_t(v_t, z_t, c_t)$, comparing the value of continuing, $V^C_t(v_t, z_t, c_t)$, to the value of exit $V^L(v, z, c)$:

$$V_t(v_t, z_t, c_t) = \max \left[ V^C_t(v_t, z_t, c_t), V^L(v, z, c) \right]. \quad (7)$$

The value of exit is $V^L(v, z, c) = Lz + c$, with $L$ a parameter for the proportion of capital sold when the firm liquidates (with the remaining $1 - L$ proportion of capital scrapped). There is no time subscript as the value of exit, conditional on $(v, z, c)$, is constant over time.

As discussed above, continuing firms maximize their value by optimally choosing labor, investment and dividends. Firms discount next period profits at the exogenous rate $\beta$, and internalize the exogenous probability $\delta$ of a death-inducing shock (which is independent of productivity $v$, capital $z$, or cash $c$).\(^6\) The firm policy choices must satisfy the Bellman equation:

$$V^C_t(v_t, z_t, c_t) = \max_{l_t, I_t, d_t} \left\{ \begin{array}{c} d_t(v_t, z_t, c_t) \\ + \beta (1 - \delta) \int_{v', z'} V^C_{t+1}(v', z', c_{t+1}) dG [v' | v_t] dG [z' | z_t, I_t] \end{array} \right\} \quad (8)$$

s.t.

$$c_{t+1} = c_t + \pi_t(v_t, z_t, l_t) - I_t - \eta_t(v_t, z_t, c_t) - d_t \quad (9)$$

Three aspects of the above set up merit highlighting, relating to: the effect of the cost wedge of external finance on the link between operational and financial policies; the role of cash; and heterogeneity in use of external finance across firms.

Cost of external finance

If there is no cost wedge for external finance in the current and future periods for all firms, $\phi_t(v, z, c) = \phi = 1 \forall \{t, v, z, c\}$, then the above set up simplifies substantially to a standard IO model with no financing frictions.

With no financing frictions there is no need for the firm to hold cash as internal and external funding has the same cost, including inter-temporally. Thus dividend policy is just the reflection of

\(^6\)Thus, there is both endogenous exit (due to a bad productivity shock) and exogenous exit due to the death shock.
labour and investment choices: negative if the firm accesses external financing, positive otherwise. In addition, importantly, the choices of labor and investment are separable. In particular, the choice of labor is conditional just on the firm’s capital stock and productivity: indeed, labor choice is optimized by maximizing per-period profit $\pi_t$. The choice of investment $I_t$ may be considered separately, as not dependent on current profits and hence not on choice of labor. Thus the firm’s policy choice problem is much simplified, with two policy choices that may be considered independently: first, labor is chosen, then investment, and these choices result in a specific dividend payout.

Specifically, if $\phi_t(v, z, c) = \phi = 1 \ \forall \ \{t, v, z, c\}$, then $\eta_t(v, z, c) = 0$, and the firm holds zero cash. Thus equation 8 simplifies to:

\[
V^C_t(v_t, z_t, 0) = \max_{l_t, I_t, d_t} \left\{ d_t + (1 - \delta) \int_z V^C_{t+1}(v_t', z_t', 0) dG[v_t' | v_t] dG[z_t' | z_t, I_t] \right\} \\
\text{where } d_t = \pi_t(v_t, z_t, l_t) - I_t
\]

Consequently, the choice of dividends is fully determined by the choice of labor and investment, and the choice of optimal labor and optimal investment may be separated:

\[
V^C_t(v_t, z_t, 0) = \max_{l_t} \{ \pi_t(v_t, z_t, l_t) \} \\
+ \max_{l_t} \left\{ -I_t + (1 - \delta) \int_z V^C_{t+1}(v_t', z_t', 0) dG[v_t' | v_t] dG[z_t' | z_t, I_t] \right\}
\]

In contrast, with financial frictions, $\phi_t(v, z, c) > 1$ for some $\{t, v, z, c\}$, the choice of labor and investment within a time period are interdependent, and the choice of dividend policy is an additional policy choice as the firm may hold positive cash balances. For instance, in selecting the optimal choice of labor a firm with negative working capital ($CF^O_t < 0$) would incur the cost of external financing to hire labor. Thus the marginal cost of additional labor includes the incremental cost due to the additional external finance. Hence, labor choice may be different to when there are no financial frictions. In addition, labor choice affects use of the firm’s cash and thus the internal funds available to fund investment. Consequently, the full cost of investment, including the cost of incremental external finance, is potentially affected by the choice of labor: and vice versa. Also, hiring of labor may use cash that could otherwise be maintained for future periods, thus affecting future period financing costs. That is, there is also an inter-temporal link to policy choices that
is absent with no frictions. Similarly for the choice of investment, as the full cost of investment includes the cost of external finance and current investment affects future capital stock and thus future profitability. In summary, in the presence of financial frictions, the firm’s operational and financial policies become intertwined, both within each time period and across time periods. Thus the complexity of "managing for value" while "managing for cash".

No Cash holding, with financial frictions

As discussed in the preceding section, with no financial frictions the firm’s cash holding is not relevant as there is no difference in cash from internal funds and from external sources. However, if there are financial frictions a setup that does not have cash, or some form of similar financial stock, as a firm characteristic substantially changes the potential choices of the firm. Without cash holding, the firm may only reduce cost of external finance for production through reducing labor hired and/or cutting investment, not by, for instance, accumulating cash balances. In particular, in the above set up, with no cash then dividends are fully determined by choices of labor and investment, and thus equation 8 reduces to:

$$\begin{align*}
V_t^C(v_t, z_t) &= \max_{l_t, I_t} \left\{ \pi_t(v_t, z_t, l_t) - \eta_t(v_t, z_t) \\
&\quad + \beta (1 - \delta) \int_{v', z'} V_{t+1}(v', z', c_{t+1}) dG[v' | v_t] dG[z' | z_t, I_t] \right\}
\end{align*}$$

Note that in this case the choice of labor and investment are not separable, as both affect the cost to the firm of accessing external finance, $\eta_t(v_t, z_t)$. Thus allowing for cash holdings, or some similar financial asset, give the firm greater flexibility to address shocks: without cash a greater margin for adjustment is borne by labor and investment choices.

Heterogeneity in use of external finance

The model allows for substantial heterogeneity in firm use of external finance, even if the cost wedge of external finance is uniform across firms. For instance, a firm with large enough cash balances would not require external finance, whereas a small growing firm would likely invest beyond current profits and cash holdings and thus be dependent on external finance. Thus in an industry setup with a mix of entrants and incumbents, smaller and larger firms, more or less productive firms, there will be substantial heterogeneity in use of external finance.

Hence, heterogeneity in the cost wedge is an additional source of heterogeneity in the use of external finance, beyond that due to the underlying firm characteristics and industry dynamics.
Thus even a shock to the cost of external finance that is uniform across firms will lead to heterogeneous response across firms. Also, given firm heterogeneity, the response to a financial shock could well be substantially different to the response to other shocks, for instance a demand shock, if for no other reason that the other shock is unlikely to have a similar pattern across firms. As the response of firms would differ, so may the industry aggregate across firms.

**Entrants**

At the start of each period, new entrants can potentially enter the industry. An entrant pays a sunk cost of entry, $S$, then realizes its initial productivity draw from a known invariant distribution $G_E(v, z, c)$, and pays the value of the initial capital stock and cash. Entry is not otherwise restricted. Entrants arrive into the industry with cash set to zero, and a range of initial productivity and capital stock levels. Thereon, entrants are indistinguishable from incumbent firms with the same productivity, capital stock and cash. When the value of entry is negative, entry is unprofitable, with a prospective entrant facing a net value of entry of:

$$V_t^E = \int_{v,z,c} [V_t(v, z, c) - z - c]dG_E(v, z, c) - S$$

### 3 Equilibrium

Let $\mu_{v,z,c,t}$ represent the measure function for producing firms over states $(v, z, c)$ in period $t$. This function summarizes all information on the distribution of producing firms across productivity levels, as well as the total mass of producing firms in state $(v, z, c)$, $M_{v,z,c,t} = \mu_{v,z,c,t}(\Upsilon)$. A dynamic equilibrium is characterized by a time path for the price index $\{P_t\}$, the measure of firms in each state, $\{\mu_{v,z,c,t}\}$, and the mass of entrants $\{M_{E,t}\}$. Note that a choice of $\{P_t\}$ uniquely determines the time path for $\{V_t^C(v, z, c)\}$ and thus determines all the optimal choices for any firm, given its productivity $v$, capital stock $z$ and cash $c$. An equilibrium $\{P_t\}$, $\{\mu_{v,z,c,t}\}$, and $\{M_{E,t}\}$ must then satisfy the following three conditions:

**Firm Value Maximization** All firms’ choices for exit/continuation, and, if continuing, for labor, investment and dividends, conditional on $v$, $z$ and $c$, must satisfy (7) and (8). In the aggregate, this means that $\mu_{v,z,c,t}$ is entirely determined by $\mu_{v,z,c,t-1}$ and the choices for $\{P_t\}$ and $\{M_{E,t}\}$. Starting with a mass and distribution of firms at time $t - 1$, a share $\delta$ of firms receive the exogenous death shock. The remaining $(1 - \delta)$ share of firms update capital stock and cash,
based on choice of labor, investment and dividends. Firm productivity is updated based on the realization of the productivity shock. To these firms are added the $M_{E,t}$ new entrants, with a distribution determined by $G_E(v, z, c)$. All firms then make their endogenous exit decisions. The remaining firms result in a distribution and mass of firms for every state. In equilibrium this must match the chosen $\mu_{v, z, c, t}$.

**Free Entry** In equilibrium, the net value of entry $V_t^{E}$ must be non-positive, since there is an unbounded pool of prospective entrants and entry is not limited beyond the sunk entry cost and cost of initial capital stock and cash. Furthermore, entry must be zero whenever $V_t^{E}$ is negative.

**Aggregate Industry Accounting** The mass and distribution of firms over productivity levels (aggregating over states) implies a mass and distribution of prices (applying the profit maximizing markup rule to firm marginal cost). Aggregating these prices into the C.E.S. price index must yield the chosen $P_t$ in every period.

**Stationary equilibrium**

A time invariant level of external finance cost $\phi$ leads to a stationary equilibrium with a time invariant price index $P$, measure of firms $\mu_{v, z, c}$, and mass of entrants $M_E$. In such a stationary equilibrium, entry must be positive since there is always an exogenous component to exit. Thus $V_t^{E}$ must be zero in this equilibrium. Although an equal mass of firms enter and exit, their distributions over productivity, capital stock and cash will not generally match in equilibrium. This is due to the productivity transition dynamics among incumbent firms and the labor, investment and financing choices of firms. Jointly, these productivity, capital stock and cash transitions, along with the distribution of entrants and exiting firms, lead to a stationary distribution of firms for every state.

**Equilibrium during financial shock**

I compare three scenarios that entail a response to a financial shock, and one scenario with response to a demand shock. The scenarios have period $t = 1$ as a stationary state, in which firms expect the cost of external finance and other parameters to remain stable over time. At the end of period $t = 1$, firms are informed of the time path of the shock. The finance shock involves a step-change in the cost of external finance, with a larger increase for firms with a low capital stock. The cost of external finance remains at these higher levels for some time, before gradually reducing to
pre-shock levels. The demand shock has a similar path, though inverted: there is an immediate drop in aggregate demand, which is sustained for some time before gradually rising to pre-shock levels.

In the model, firms know the future time path of all aggregate variables, in particular of the financial or other shock: there is no aggregate uncertainty. Nonetheless, the dynamic response of firms is complex. The firm’s response to the shock reflects both near-term changes in conditions (e.g., increase in cost of external finance) as well as the future, eventual, convergence back towards the pre-shock stationary state. Also, the response of firms is heterogeneous, reflecting both the differences in firm characteristics and the competitive dynamics within the industry, for instance the choice of some firms to exit.

The equilibrium conditions hold throughout the response to the shock. The equilibrium path for the price index \( P_t \), measure of firms \( \mu_{v,z,c,t} \), and entrants \( M_{E,t} \) will thus begin at their initial stationary levels until a change in costs of external finance is announced, then follow a transition path through the shock and with gradual return to the prior stationary state levels, and remain constant thereafter. Thus the model is of an industry that transitions through a disruptive shock, during which the dynamics present in the stationary equilibrium change substantially. For instance, during the transition, as opposed to the stationary states, the net value of entry may be negative resulting in periods of zero entry. Also, the transition back towards the stationary equilibrium is gradual and, typically, continues past the point in time at which parameter values have returned to pre-shock levels. The main reason for this is due to the presence of sunk costs that lead to hysteresis effects.

4 Simulated Results

I search for the equilibrium paths of \( \{P_t\} \), \( \{\mu_{v,z,c,t}\} \), and \( \{M_{E,t}\} \) using numerical methods. The appendix provides a description of the algorithm used. In essence: I first compute the values of \( P, \mu_{v,z,c} \), and \( M_E \) in the initial and final stationary equilibria. The algorithm then iterates over candidate equilibrium paths for \( \{P_t\} \) and \( \{M_{E,t}\} \). The choice for \( \{P_t\} \) determines all of the policy choices for any incumbent firm (this is the crucial benefit of abstracting from strategic interactions in the monopolistic competition equilibrium): that is knowing \( \{P_t\} \) each firm may optimize \( (l_t, I_t, d_t) \) conditional on its characteristics \((v, z, c)\). Since \( \mu_{v,z,c} \) in the initial stationary state is known, I can thus compute \( \{\mu_{v,z,c,t}\} \) based on those policy choices, and the choice for the number of entrants. In turn, I can then compute a new price index \( \{P_t\} \) based on the distribution and mass of firms
(which implies a distribution of prices). I iterate until this new price path \( \{P_t\} \) matches the prior choice of the candidate \( \{P_t\} \).

The equilibrium, after the shock, gradually converges back towards the pre-crisis stationary state equilibrium. I consider a sufficiently long time path such that by the final period the industry has converged back, arbitrarily close, to the pre-crisis stationary equilibrium.

**Calibration**

I next describe how I set the parameters of the model to run the model simulations. The model is calibrated to reflect the typical patterns of firm dynamics within industries, in particular: Bartelsman, Scarpetta and Schivardi (2000); Cooley and Quadrini (2001); and Olley and Pakes (1996).

I first describe the grid over time periods, productivity levels and capital stock on which to run the model (Table 1). I set each time period to correspond to one quarter. This is relatively short thus smoothing out the dynamic processes. I set the total number of time periods to 100 (i.e., around 25 years) as this is long enough to ensure that by the final period the industry has converged close to the stationary equilibrium corresponding to the final set of parameters. Note that I do not impose this final stationary state as the end point: rather, I allow the industry to evolve towards it.

I set the productivity range to \( v = [0.5, 10] \), capital stock range to \( z = [0.8, 50] \), and the cash range to \( c = [0, 140] \). This grid size is exogenous to any firm decisions. The grid is set wide enough such that the exit cutoffs are sufficiently above the lower bound. I set the number of grid points to 9x60x12 = 6480; high enough that there are sufficient grid points to reduce any effects from the discreteness of the grid. For instance, a finer grid allows for the exit region to more smoothly adjust over time.

I next discuss the finance shock, and demand and production parameters. The cost of external finance depends on two components, each of which may vary over time: the baseline level, \( \phi_t^b \), and the spread between highest and lowest capital stock firms, \( \phi_t^s(z) \):

\[
\phi_t(z) = \phi_t^b + \phi_t^s(z)
\]

In the initial stationary state, I set the costs of external finance to \( \phi^b = 1.0 \) and \( \phi^s = 0.2 \), resulting in a range of \( \phi \) from 1.0 for firms with greatest capital stock, \( z = 50 \), to 1.2 for firms with least capital stock, \( z = 0.8 \). The change in cost of external finance is announced at the end of
period \( t = 1 \), and the change occurs in period \( t = 2 \). That is, there is a step-change in the cost of external finance: the baseline level, \( \phi^b_t \), and the spread between highest and lowest capital stock firms, \( \phi^s(z) \), both increase. The cost of external finance during the shock rises to \( \phi^b = 1.2 \) and \( \phi^s = 0.4 \), resulting in a range of \( \phi \) from 1.2 for firms with greatest capital stock, \( z = 50 \), to 1.6 for firms with least capital stock, \( z = 0.8 \). This new level is sustained for eight periods, before gradually returning in a linear path to the pre-shock levels (see Figure 2).

The main demand parameters is the elasticity of substitution between varieties, which I set to \( \sigma = 4 \). The weights of the Cobb-Douglas production function are: productivity \( \alpha_o = 1/3 \), capital stock \( \alpha_z = 1/3 \), and labour \( \alpha_l = 1/3 \), with an overhead cost \( F = 30 \). Throughout the calibration I set the capital stock variables such that capital stock has a high share in production but depreciates fast. This is in part to capture a broad definition of capital, for instance including working capital. The choice of labor to hire is set relative to choice of a firm with no financial constraints: 100% (the same), 80% or 60%.

Next I discuss my choices for the productivity and capital stock transitions, and entry and exit (see Table 2). I set the stochastic productivity transition based on a lognormal distribution. For each firm, the draw is from a distribution with mean corresponding to the current firm’s productivity. The standard deviation is the same across all firms (with truncation of extreme changes in productivity, in part to avoid accumulation of firms at the edge of the productivity grid). Thus each firm has the same probability of experiencing a similar percent increase or decline in productivity.\(^7\)

The firm’s choices of investment is constrained to be from zero to three times depreciation: this is not so restrictive, as allowing growth rates of 24% per period (i.e., 140% per year). I set the stochastic evolution of capital stock based on a lognormal distribution, with mean corresponding to the firm’s depreciated capital stock plus investment (see equation 2), with truncation of extreme outcomes.

In terms of exogenous exit, the death shock is set to 15% per year, which is higher than the firm level exit rates observed empirically (of around 3-7% per year). Finally, I specify the distribution of potential entrants over productivity levels as lognormal with \( \log(1.5) \) mean and 0.5 standard deviation, and over capital stock as lognormal with \( \log(1.5) \) mean and 0.1 standard deviation,

\(^7\)Note that, although the productivity transition has no effect on the firm’s expected productivity, this is not true for the effect on firm’s expected profit. The profit function rises steeply with productivity - hence in expectation, profits rise with a productivity shock. The death shock in part compensates for this effect (otherwise no firm would ever want to exit). Another modeling alternative would be to specify a mean reverting productivity transition.
and initial cash to zero. In the stationary state, the endogenous exit region will always include
the minimum productivity, capital stock and cash point, so that some entrants with low enough
productivity and/or capital stock draws will choose to immediately exit and not produce. Overall,
in the simulations, entrants enter with an average productivity, capital stock and cash lower than
that of incumbent firms. Thus, the simulations replicate the robust empirical findings that recent
entrants are on average smaller, and exhibit higher exit rates than incumbent firms. The entry sunk
cost is set to $S = 50$, which is equivalent to a per-period interest charge of 2% of fixed costs (at the
5% annual discount rate $\beta$).

**Stationary state**

I first describe the numerical properties of the stationary states with $\phi^b = 1.0$ and $\phi^s = 0.2$, and thus
ranging from $\phi = [1.0, 1.2]$ across firms depending on the firm capital stock, but not productivity
or cash. The firm policies depends on the firm’s productivity, capital stock and cash. At a given
level of cash, the exit region comprises the lowest levels of productivity and capital stock. In the
presence of costs to external finance, $\phi \neq 1$ for all firms, at higher levels of cash the exit region is
smaller: with additional cash on hand, the firm has greater ability to invest and thus less incentive
to exit (see Figure 4, panel (a), in particular the exit boundary for low capital stock). With no
costs of external finance the exit boundary does not depend on cash holding, and is less sensitive
to capital stock$^8$.

Within the continuation region, the firm’s labor, investment and dividend polices vary depending
on the firm’s state variables, giving rise to regions of the state space within which firms pursue
similar types of policies. The labor policy is given by the proportion of labor a firm hires relative
to what the firm would hire with no financial constraints. Cash holding affects labor policy. With
low levels of cash, only the firms with high capital stock hire the labor similar to of there were
no financial constraints, whereas with high cash all firms do: Figure 4, panel (b), comparing the
left and right arrays. In terms of investment policy, firms may invest in each period from zero
to three times depreciation. Firms with high productivity and low capital stock invest the most,
so as to grow fast. Firms close to the exit boundary tend to invest less, and firms that have
reached sufficient size tend to invest only enough to replace depreciated capital stock. Considering
the set of productivity, capital stock combinations that leads firms to only replace capital stock

$^8$Capital stock is subject to sunk costs, as capital stock may not be sold off by the firm but just decreased through
gradual depreciation and at exit only part of the value of the capital stock is recovered. Hence, even with no financing
costs exit does not just depend productivity.
(i.e., the frontier of the largest firms that invest), this slopes from low productivity and low capital stock towards high productivity and high capital stock. As firm cash increases, this frontier shifts towards firms with higher capital stock, and varies more with firm productivity. An increase in cash holding alleviates pressure from external financing costs, thus raising investment in general and more so for the most productive firms (Figure 4, panel (c)). In terms of the change in cash holding, with low cash balances there is a range of behavior: Firms closer to the exit region decrease their cash holding, whereas the most productive firms accumulate cash. In contrast, firms with higher cash balances mostly aim to reduce their cash balance (Figure 4, panel (d)).

Consequently there is substantial variation across firms in the policies pursued. In particular, cash holdings affect policy choices reflecting the non-trivial costs of external finance for many firms. Table 3 contains the aggregate variables across firms, with the first column for \( t = 1 \) stationary state. In particular, within policy choices, the average level of labor hiring (relative to the unconstrained level) is 94\%: thus costs of external finance are sufficient to distort labor choice. On average the cost wedge of external finance for firms that do use external finance is 1.19 (Table 3 under external finance). Also dividend policy comprises a mix of some firms paying out and some firms receiving inflows of external finance (Table 3, under policy choices).

Also, the policy patterns highlight the basic, ongoing firm dynamics part of the stationary equilibrium. Firms with low capital stock and better productivity draws grow aggressively, over time reducing external financial dependence through increasing profitability (due to higher capital stock) and by accumulating cash (that reduces cost to finance working capital). Firms with worse productivity draws move towards the exit region, the more so if the firm has low capital stock or low cash, and may indeed exit if productivity drops sufficiently. The optimal firm policies generate firm dynamics that in the stationary state result in the distribution of firms across the set of states \((v, z, c)\) not changing over time. That is, the effects of entry, exit and transitions across the set of states offset to generate a stationary distribution. In the stationary state, the distribution of incumbent firms is comprised of firms with higher capital stock, higher productivity and higher cash relative to entrants. The distribution across productivity draws reflects the stochastic evolution of productivity together with exit of the least productive. The distribution across capital stock is double-humped: the hump at lower capital stock reflects recently entered firms, whereas the hump at higher capital stock comprises firms at high capital stock for which the optimal policy is to reinvest to cover depreciation (and thus remain at a stable capital stock, leading to an accumulation of firms at these levels of capital stock. The distribution across cash balances shows that most
firms have some cash. These patterns are illustrated in Figure 3.

A change in the cost of external finance shifts the firm’s optimal policies. For instance, in the stationary states the number of entrants is always positive as there is forced exit through the death shock. However, a sufficient shift in the cost of external finance may reduce the value of entry to the extent that there are periods of zero entry. The next section characterizes the changes in firm policies during a financial shock and the consequent changes in firm dynamics.

Dynamics during financial shock

The firms start in the stationary state, with stable parameter values. The last such period is referred to as $t = 1$. The financial shock takes effect at $t = 2$, with no pre-announcement, with, as described above, an increase in the cost of external finance especially for firms with lower capital stock. The higher costs of external finance persist for eight periods, until $t = 9$, before gradually returning to pre-shock levels over the subsequent 12 periods. Note that throughout other parameters are kept constant, in particular industry total revenues are stable so there is no concurrent change in aggregate demand.

The evolution of aggregate variables is in Table 3, with the evolution of firm policies and other variables over time: at the initial stationary state ($t = 1$); the onset of the crisis ($t = 2$); the average during the peak of the crisis ($t = 3$ to $t = 9$); and the average over the subsequent 24 periods that include the 12 periods during which the cost of external finance gradually returns to pre-shock levels. In Figure 6 the solid line plots are the period-by-period value of selected variables: Note that in the long run the equilibrium converges towards the stationary state equilibrium.

At the onset of the financial shock, firm policies immediately adjust to the increase in costs of external finance. Figure 5 highlights how firm policies at the onset of the crisis ($t = 2$) differ from firm policies in the initial stationary state ($t = 1$), and how this varies with firm cash holding. The exit boundary adjust somewhat, more so at low cash holdings.$^9$ Firms with low cash reduce labor hiring, whereas firms with high cash do not change policy (in particular as already at 100% these firms could not increase labor hiring). Investment differs sharply across cash holding: decreasing with low cash holding and increasing for high cash holding. The increased investment for firms with high cash holding is because the shock of the cost of external finance does not affect firms equally. Thus for firms least dependent on external finance there is limited change to the costs

$^9$The relatively modest changes in the exit boundary reflect the relatively coarse productivity grid, as with a finer grid there would be more variation. However, in setting up the grid, with an eye to computer run times, a finer grid was emphasized for capital stock and cash than for productivity.
of investment; however the benefits increase due to competitive dynamics within the industry. In effect, tighter external finance acts as a barrier to growth, limiting competitive pressures. In terms of financial policy at the onset of the crisis both low and high cash firms tend to increase cash, especially if closer to the exit region. Thus at the onset of the financial crisis the firm size distribution across capital stock and cash holding shift towards higher values: Figure 3, panels (b) and (c). In part this reflects increased exit, and reduced entry (Table 3, bottom two rows), in part the shift in investment and cash accumulation policies.

In aggregate firms reduce their need for external finance at the onset of the crisis (Table 3). Cash flow from operations turns positive, due to cutting back of investment and labor hiring by firms with lower cash balances. The use of current cash balances to support working capital is mitigated by the benefit of holding cash balances for use in subsequent time periods. Dividend outflows drop substantially, immediately: in contrast, firms increase sharply cash accumulation. Dividend inflows, use of external finance, drops less dramatically at the onset of the crisis, more so in subsequent periods. At the onset of the crisis cutting back in labor and investment reduces need for external finance: however, the major reduction comes from accumulating greater internal financial resources, cash balances, and this takes time.

The immediate rise in exits is more than compensated for by a decrease, to zero, of entry. Thus the total number of firms drops. The overall price index rises, reflecting the effect of fewer firms dominating the effect of exit of weaker firms. That is, in equilibrium part of the adjustment process is to increases prices to consumers. This effect is evident as I consider a pure financial crisis, with no effect on aggregate demand effect. In effect, the price rise reflects the firms in part passing on to consumers the rise in the cost of an input, external finance.

For the remainder of the crisis, through to period $t = 9$, the shift to positive cash flow from operations is sustained but not increased (Figure 6, panel (h)). Firms in aggregate accumulate cash more so than in the stationary state, but much less so than during the sharp adjustment in period $t = 2$. Hence firms raise the quantity of labor hired closer to the unconstrained optimal level, more similar to pre-crisis levels. Investment continues to remain relatively stable, due to offsetting effects across firms. Relative to $t = 2$, exit drops and entry rises but not enough to stem a gradual decline in number of firms. With stable investment and drop in number of firms there is a sustained decline in aggregate capital stock.

As the gradual transition back towards pre-crisis cost of finance looms and then starts, there is a gradual unwinding of the accumulated cash balances and consequent payout of large dividends,
relative to pre-crisis levels. Exit and entry rates steadily climb, with entry out-pacing exit. The
increased competitiveness in the industry depresses prices and induces lower investment. Hence,
capital stock continues to drop substantially: as compared to during the crisis, this is for lack of
investment, whereas during the crisis this was due to drop in number of firms (Figure 6, panel (c)
for investment and (d) for capital stock).

In summary, the response to the crisis involves a very sharp reaction early on to accumulate
cash by most firms, however labor and investment policies exhibit substantial heterogeneity in
response. As aggregate demand is not affected, the rise in financing cost that hits some firms most
hard provides opportunities for other firms to expand. Once firms have adjusted cash stocks so
as reduce external financial dependency, firm operating policies shift towards pre-crisis patterns,
especially in regard to labor hiring. In terms of investment and capital stock, during the rest of
the crisis and in the transition back towards the long-run stationary equilibrium, aggregate capital
stock steadily declines, with the margin for adjustment changing over time. During the crisis
the number of firms drops, and in the gradual transition back towards the long-run investment is
depressed. Overall, the effect of the crisis persists much longer than the duration of high costs to
access external finance.

Comparison of dynamics during financial shock with demand shock

Next I highlight the differences in response to a financial shock as compared to a demand shock.
The demand shock is modeled with a similar time path: an immediate 30% drop in aggregated
demand, sustained for twelve periods, with then a gradual return to pre-crisis levels. The evolution
of the key variables is in Figure 6. The main purpose of the comparison to the demand shock is to
highlight that the response of firms in the model does vary depending on the type of shock applied:
that is the patterns discussed above are not generic responses to a shock, rather the response to
a shock to the cost of external finance. Consequently, I next will focus on a few key differences,
rather than describe in detail the response to the demand shock. In response to a demand shock,
entry rates drop to zero for a number of periods (panel (l)): with depressed current demand and
incumbent firms remaining in the industry for the return of higher demand, current prices drop
making entry less attractive (panel (a)). Entry rates pick up once demand starts to return to
pre-crisis levels. In aggregate firm cash stocks drop substantially (panel (e)), in part as as firms
use cash to weather the crisis and so have lower cash flow from financing (panel (i)) and as the total
number of firms drops (panels (k)). The change in composition of firms, with less entrants, leads
to a rise in labor hiring closer to the optimal (panel (b)), as large incumbents tend to be hire labor
closer to the unconstrained optimum. Overall investment drops immediately as does aggregate
capital stock (panels (c) and (d)), in part due to the decline in number of firms and in part as
there is no substantial increased investment by some firms (as compared to in the financial crisis
scenario). In summary, the responses differ both in terms operation and financial policies, and in
terms of near-term and medium-term response.

Dynamics during financial shock with only operational, no financial, policy choices: the no cash
scenario

I now return to consider pure financial shocks, comparing the base case to a situation in which firms
may only vary their operational policies, labor and investment, as there are no financial policies. I
refer to this as the no cash scenario, as firms do not have cash in this scenario: thus, as discussed
above, dividends (paid out or external finance inflows) are fully determined by choice of labor and
investment. This scenario is meant to represent a firm in which the financial side is not adjusted in
response to the shock: only the operational side. The evolution of the key variables is in Figure 7.
In short, firm policy responses are more extreme than in the base case: there is no way to build up
a cash buffer, either pre-crisis or early on in the crisis, to reduce the exposure to external finance
costs. In particular, even large firms fund working capital through external finance and thus are
exposed to the substantial rise in external finance cost wedge: this is in contrast to the base case
in which these firms have or may rapidly build up cash to cover most or all of their working capital
needs. Thus in the no cash scenario cash flow from operations and financing rise substantially
(panels (h) and (i)). Note that the cash flow from operations in this case is always negative for
all firms as all firms have zero cash. This substantial change in cash flows is achieved by reducing
labor hiring, relative to the optimal (panel (b)) and decreasing in investment (panel (c)). Part
of the change is due to the composition of firms changing, as entry stops for a number of periods
(panel (l) leading to a sharp decline in the number of firms (panel (k)).

Dynamics during financial shock with cost of external finance homogeneous across firms

I next compare the base case with a set-up in which the external cost of finance is uniform across
firms and thus the financial shock is also uniform across firms, with the evolution of key variables
in Figure 8. As external finance has a uniform cost even in the stationary state, there is some
difference between the stationary states with non-uniform and uniform cost of external finance.
Nonetheless, comparing the two scenarios what is evident is that the overall dynamic patterns are very similar. One difference is that labor hiring returns closer to the pre-crisis levels faster with a homogeneous shock (panel (b)). The reason for this is that smaller firms face a less severe financial shock in the uniform scenario than the base case. Consequently, these firms are able to more quickly increase cash balances to enable operational decisions to return close to pre-crisis levels. Overall, the main reason for the similarity across the uniform shock and base case shock is that even a uniform costs of external finance has a heterogenous effect on firm policies. As firms are heterogeneous in their operational and financial characteristics, the firms have varying sensitivity to the cost of external finance. At the extreme, large firms with high cash balances are barely affected by changes in the cost of external finance, net of competitive feedback effects. In summary, the results in the base case scenario are primarily driven by the heterogeneity in the firms, not by the heterogeneity in the financial shock.

5 Conclusion

In this paper, I build a dynamic model of firm-level adjustment to a financial shock that jointly addresses firm operating and financing decisions. I analyze the equilibrium transition from an initial stationary state through a financial crisis and back, eventually, to the pre-crisis stationary equilibrium. I highlight how the firm policy responses vary across firms and over time as the crisis unfolds: this is the core contribution of the model set up. I highlight how the firm’s initial response to the crisis is to rapidly raise cash. Firms with limited initial financial resources also cut labor and investment. However, other firms do not cut labor and increase investment. After the initial response, the transition through the crisis back to pre-crisis conditions is faster for firm cash holding and labor hiring than for investment. Also, entry and exit transition back to pre-crisis levels relatively slowly. I compare the response to the financial crisis to three other scenarios to highlight that the policy responses are driven by: the characteristics of a financial shock, as response to a demand shock is very different; the inclusion of operational and financial policy choices, as the removal of financial policy substantially changes firm responses; and underlying heterogeneity of the firms that results in heterogeneous sensitivity to external finance, less so that the cost of external finance is non-uniform across firms.

Returning to the managerial challenge of addressing a financial crisis, "managing for value" while "managing for cash", the model results suggest that the wide variation across firms in responses, the adjustment of policies as the crisis unfolds, and the strong links between operational and financial
policies make for a formidable challenge. That is, common approaches to the crisis or what other firms do may not be a good guide for action, the frequency of changes in policy could be well above that of normal times, and careful consideration needs to be given to how firm policies fit together. This is in addition to the other concurrent challenges in facing a crisis highlighted at the start: overall macroeconomic developments, variation in how the crisis affects different financial instruments available to the firm, and, last but not least, the internal organizational challenge of adapting organizational processes and decision making to the situation. The relative frequency of crises and the potential for long-term effects arising from crises suggest a rich opportunity for further research, tough in a situation for managers, firms and society at large.

References


Table 1: Calibration: Model Timing and Productivity Grid

<table>
<thead>
<tr>
<th>Variable</th>
<th>Empirical evidence*</th>
<th>Explanation of model calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Timing</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Duration of time periods</td>
<td></td>
<td>Time period corresponds to one quarter, short relative to typical empirical unit of analysis. Total number of time periods so that by final period industry has converged close to final stationary state.</td>
</tr>
<tr>
<td><strong>Discount rate</strong></td>
<td>5% per year, thus 1.3% per quarter (i.e., per time period)</td>
<td></td>
</tr>
<tr>
<td><strong>State space grid</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity v</td>
<td>Relative size of largest to smallest firms often over 100x [B et al]</td>
<td>Set v range to [0.5, 10] to allow a sufficiently broad range of firm sizes.</td>
</tr>
<tr>
<td>Capital stock z</td>
<td>Relative size of largest to smallest firms often over 100x [B et al]</td>
<td>Set z range to [0.8, 50] to allow a sufficiently broad range of firm sizes.</td>
</tr>
<tr>
<td>Cash c</td>
<td></td>
<td>Set c range to [0, 140] to allow a sufficiently broad range.</td>
</tr>
<tr>
<td><strong>Normalization</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factor prices</td>
<td></td>
<td>Normalize monthly wage to one, and price of capital to ten.</td>
</tr>
<tr>
<td><strong>Scaling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of workers L</td>
<td>Choice of L scales size of market: Set L=100,000.</td>
<td></td>
</tr>
<tr>
<td><strong>Finance costs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost wedge for external finance φ</td>
<td>New share premium of 30% used by [C and Q]</td>
<td>Set baseline for all firms to $q_i = 1.0$ pre-crisis (rising to 1.2 during crisis) and additional spread between lowest and highest capital stock firms $q_i = 0.2$ pre-crisis (rising to 0.4 during crisis). Hence, pre-crisis set $q = {1,1.2}$, depending on firm capital stock, $z$; increase to $q = {1.2,1.6}$ during crisis, thus increasing both average across firms and spread between highest and lowest capital stock.</td>
</tr>
<tr>
<td>Timing of shock</td>
<td>Typically fairly sudden onset of crisis, some persistence of crisis, and gradual return to pre-shock conditions</td>
<td>Set policy shock to start with no pre-announcement or lag (i.e., in t+2), last for 8 periods (i.e., two years), and gradually return to pre-shock levels over 12 periods (i.e., 3 years).</td>
</tr>
<tr>
<td><strong>Demand</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity of substitution σ</td>
<td></td>
<td>Set to 4.</td>
</tr>
<tr>
<td><strong>Production</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Weights α</td>
<td>Weights around 1/3 for capital, 2/3 labour for telecom equipment industry [O and P]</td>
<td>Set weights in Cobb-Douglas production function as $α_v=1/3, α_z=1/3, and α_l=1/3$.</td>
</tr>
<tr>
<td>Labor hiring l</td>
<td>Three choices of quantity of labor to hire, relative to optimal with no financial constraints: 100%, 80%, and 60%</td>
<td></td>
</tr>
<tr>
<td>Fixed costs F</td>
<td>Set F=30 so that for firms on average fixed labor cost is around 2/3 of total labor cost</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Calibration: Evolution of Productivity, Capital Stock, and Entry

<table>
<thead>
<tr>
<th>Variable</th>
<th>Empirical evidence*</th>
<th>Explanation of model calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Productivity transitions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transition for firm productivity</td>
<td></td>
<td>Productivity evolves according to truncated lognormal evolution with mean log(v) and 0.6 standard deviation (hence, mean zero change in productivity). Also, truncate increase/decrease to future productivity to within [0.5x,2x] current v</td>
</tr>
<tr>
<td><strong>Capital stock transition</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment outcome</td>
<td></td>
<td>Maximum investment 30 per period (i.e., maximum growth of 24% per quarter, about 140% per year). Realization of increase in capital stock stochastic, with truncated lognormal evolution with mean log(z(1-δ)+f)) and 0.1 standard deviation, with increase/decrease of capital stock to within [0.9x,1.1x] of mean</td>
</tr>
<tr>
<td><strong>Cost of investment</strong></td>
<td></td>
<td>The cost of investment is the expected increase in capital stock scaled by a factor that increases exponentially with investment and with firm current capital stock: this is just to ensure that firms havea bound to potential size, in effect introducing diminishing return to investment</td>
</tr>
<tr>
<td>Depreciation δ of capital stock</td>
<td>7% per year used by [C and Q], and 4% for buildings and 12% for equipment per year used by [O and P]</td>
<td>Set δ, 8% per quarter so around 28% per year</td>
</tr>
<tr>
<td><strong>Exit</strong></td>
<td>Exit rate ~3-7% per year [B et al]</td>
<td>Set to 15% per year (with additional exit from firm productivity dropping below exit productivity cutoff), thus 4% per quarter (i.e., per time period)</td>
</tr>
<tr>
<td>Death shock δ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquidation value of capital stock α</td>
<td>Set α,50%</td>
<td></td>
</tr>
<tr>
<td><strong>Entry</strong></td>
<td>Entrants smaller than incumbents on average. Also, around 50% of entrants survive to 7 years, with 20% hazard in year 1 and around 10% hazard thereafter [B et al]</td>
<td>Set entrants as distributed with independent draws on productivity and capital stock. Productivity draw lognormal, with mean log(1.5) and std dev=0.5. Capital stock draw lognormal, with mean log(1.5) and std dev=0.1. Cash zero. This results in entrants with, relative to incumbents, lower average productivity and capital stock, and higher exit rates.</td>
</tr>
<tr>
<td>Entry sunk cost S</td>
<td>Set S=50, which corresponds to a quarterly interest charge of 0.6 (i.e., around 2% of per period fixed costs).</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Financial shock: Evolution of key variables

<table>
<thead>
<tr>
<th>Average value of variable across time periods</th>
<th>Initial t=1, stationary state</th>
<th>t=2</th>
<th>t=3 to 9</th>
<th>t=10 to 16</th>
<th>t=17 to 21</th>
<th>t=22 to 30</th>
<th>t=31 to 60</th>
<th>Long-run stationary state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total demand, '000</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Baseline φ</td>
<td>1.00</td>
<td>1.20</td>
<td>1.20</td>
<td>1.13</td>
<td>1.03</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Spread in φ: from lowest to highest capital stock</td>
<td>0.20</td>
<td>0.40</td>
<td>0.40</td>
<td>0.33</td>
<td>0.23</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Price index</td>
<td>0.92</td>
<td>0.99</td>
<td>1.02</td>
<td>1.04</td>
<td>0.98</td>
<td>0.92</td>
<td>0.92</td>
<td>0.92</td>
</tr>
<tr>
<td><strong>Policy choices</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor: % relative to no financial constraints</td>
<td>94%</td>
<td>91%</td>
<td>92%</td>
<td>92%</td>
<td>94%</td>
<td>94%</td>
<td>94%</td>
<td>94%</td>
</tr>
<tr>
<td>Investment</td>
<td>13,856</td>
<td>13,695</td>
<td>14,040</td>
<td>12,889</td>
<td>12,316</td>
<td>13,405</td>
<td>13,939</td>
<td>13,856</td>
</tr>
<tr>
<td>Dividends</td>
<td>14,385</td>
<td>11,812</td>
<td>19,959</td>
<td>20,926</td>
<td>15,690</td>
<td>12,670</td>
<td>14,359</td>
<td>14,385</td>
</tr>
<tr>
<td>- Dividend outflows</td>
<td>24,720</td>
<td>20,972</td>
<td>26,914</td>
<td>29,291</td>
<td>27,168</td>
<td>23,778</td>
<td>24,630</td>
<td>24,720</td>
</tr>
<tr>
<td>- Dividend inflows</td>
<td>(10,335)</td>
<td>(9,160)</td>
<td>(6,954)</td>
<td>(8,365)</td>
<td>(11,478)</td>
<td>(11,108)</td>
<td>(10,272)</td>
<td>(10,335)</td>
</tr>
<tr>
<td>Increase in cash</td>
<td>2,101</td>
<td>9,033</td>
<td>3,057</td>
<td>2,378</td>
<td>1,980</td>
<td>2,293</td>
<td>2,097</td>
<td>2,101</td>
</tr>
<tr>
<td><strong>Cash flow</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operations cash flow</td>
<td>(1,582)</td>
<td>2,406</td>
<td>6,760</td>
<td>3,992</td>
<td>(3,898)</td>
<td>(2,293)</td>
<td>(1,510)</td>
<td>(1,582)</td>
</tr>
<tr>
<td>Financing cash flow</td>
<td>17,802</td>
<td>16,908</td>
<td>23,818</td>
<td>25,365</td>
<td>20,469</td>
<td>16,314</td>
<td>17,755</td>
<td>17,802</td>
</tr>
<tr>
<td><strong>External finance</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>External financing cost wedge φ for firms use external finance</td>
<td>3.417</td>
<td>5,097</td>
<td>3,859</td>
<td>4,439</td>
<td>4,779</td>
<td>3,644</td>
<td>3,397</td>
<td>3,417</td>
</tr>
<tr>
<td>1.19</td>
<td>1.58</td>
<td>1.59</td>
<td>1.46</td>
<td>1.26</td>
<td>1.19</td>
<td>1.19</td>
<td>1.19</td>
<td>1.19</td>
</tr>
<tr>
<td><strong>Firm stocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital stock</td>
<td>8,630</td>
<td>8,340</td>
<td>8,469</td>
<td>8,376</td>
<td>8,228</td>
<td>8,203</td>
<td>8,608</td>
<td>8,630</td>
</tr>
<tr>
<td>Cash</td>
<td>78,515</td>
<td>76,620</td>
<td>79,885</td>
<td>76,249</td>
<td>73,654</td>
<td>79,100</td>
<td>78,637</td>
<td>78,515</td>
</tr>
<tr>
<td><strong>Number of firms</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1,400</td>
<td>1,226</td>
<td>1,167</td>
<td>1,177</td>
<td>1,375</td>
<td>1,460</td>
<td>1,399</td>
<td>1,400</td>
</tr>
<tr>
<td>Entrants</td>
<td>139</td>
<td>-</td>
<td>81</td>
<td>124</td>
<td>198</td>
<td>138</td>
<td>138</td>
<td>139</td>
</tr>
<tr>
<td>Exits</td>
<td>139</td>
<td>174</td>
<td>99</td>
<td>105</td>
<td>137</td>
<td>151</td>
<td>139</td>
<td>139</td>
</tr>
</tbody>
</table>

33
<table>
<thead>
<tr>
<th>Event</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry</td>
<td>Entrants pay sunk cost of entry and then discover their initial productivity, capital stock and cash</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuation / Exit decision</td>
<td>Entrants and incumbent firms decide if to continue in industry or exit</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor hiring decision</td>
<td>Firms decide labor to hire for production; profits realized after payment of labor and investment, before payment of dividends</td>
</tr>
<tr>
<td>Investment decision</td>
<td>Firms decide how much to invest to increase capital stock</td>
</tr>
<tr>
<td>Working capital needs</td>
<td>Determined by operating decisions plus current cash</td>
</tr>
<tr>
<td>Dividend decision</td>
<td>Firms decide dividends (positive is payout, negative is access to external finance), and thus retained earnings to hold as cash</td>
</tr>
<tr>
<td>Transition to next period</td>
<td>Uncertainty resolved about transition to next period: death shock for all firms; productivity transition and capital stock transition; cash updated</td>
</tr>
</tbody>
</table>

Figure 1: Timing and description of events within time periods
Figure 2: Evolution of Cost of External Finance.
Figure 3: Distribution of firms at $t=1$, $t=2$ and $t=6$, across: Productivity (top panel); Capital stock (middle panel); and Cash (bottom panel)
Figure 4: Policies at initial stationary state: Entry/Exit, Labor, Investment and Cash retention policies at low and high levels of Cash across all Productivity and Capital Stock values
Figure 5: Firm policies at $t = 2$, relative to firm policies at $t = 1$
Figure 6: Comparison of Financial Shock and Demand Shock
Figure 7: Comparison of Financial Shock with and without cash as state variable
Figure 8: Comparison of Financial Shock that is heterogeneous versus uniform across firms
Appendix

A Model Algorithm

Following I describe the algorithm for numerically solving the model, focusing on the equilibrium conditions required and the sequence of calculations performed. The demand structure leads to monopolistic competition. In particular, this means that each firm in each time period $t$ need only know industry aggregate outcomes for industry price $P$ from time $t$ onwards, $\{P_t, ..., P_T\}$, to determine its specific policies conditional on its current productivity $v$, capital stock $z$, and cash $c$. Firm policy choices are whether to $\{\text{Continue, Exit}\}$, and, if continuing, labor to hire, investment, and dividends.

The algorithm comprise three steps. Step 1 is to set parameters. Step 2 is to compute the firm policies and firm-size distribution $\mu_{v, z, c, 1}$ corresponding to the initial parameter values, the initial stationary state equilibrium at $t = 1$. Within Step 2, there is an iteration over the aggregate price for the stationary state $P_1$. Step 3 computes the firm policies and firm-size distribution for the evolution from the initial stationary state through to period $T$. Within the Step 3, there is an iteration over the price path $\{P_2, ..., P_T\}$.

1) Set initial parameters, including for industry characteristics and grid structure.

2) $P_1$ iteration:

- Choose candidate value for $P_1$.

- Firm Value and Policy Iteration:
  - Compute profit $\pi(v, z; l)$ at each productivity $v$ and capital stock $z$ and potential choice of labor to hire, based on the specific demand system and production function chosen.
  - Pick a candidate value function $V_1(v, z, c)$.
  - Determine $\{\text{Continuation/Exit}\}$ and choice of policy for labor, investment and dividends at each $\{v, z, c\}$.
  - Iterate the value function: The set of firm policies over continuation and choice of policy state imply a next iteration value for the value function, $V'_1(v, z, c)$, based on computing the value of continuing and comparing to the value of exit.
  - Check whether new $V'_1(v, z, c)$ is sufficiently close to $V_1(v, z, c)$. If not, continue iteration with $V'_1(v, z, c)$. If close enough, return to $P_1$ iteration.
- Check the value of entry. As seek equilibria with positive entry the condition should be close to zero. Compute firm-size distribution \( \mu_{v,z,c,1} \).
  - If close enough to zero, \( P_1 \) iteration is complete.
  - If not, then adjust candidate \( P_1 \) accordingly: if condition is positive lower \( P_1 \), if negative raise \( P_1 \).

3) \( \{P_2, ..., P_T\} \) iteration:

- Choose candidate value for \( \{P_2, ..., P_T\} \).
  - Compute price corresponding to stationary state at final parameter values.
  - Set initial guess for \( \{P_2, ..., P_T\} \) based on prices corresponding to initial and final parameter values.

- Firm value and policy iteration
  - Firm Value and Policy Iteration for \( t = T \):
    * Compute profit \( \pi(v, z; l) \) at each productivity \( v \) and capital stock \( z \) and potential choice of labor to hire, based on the specific demand system and production function chosen.
    * Pick a candidate value function \( V_T(v, z, c) \).
    * Determine \( \text{Continuation}/\text{Exit} \) and choice of policy for labor, investment and dividends at each \( \{v, z, c\} \).
    * Iterate the value function: The set of firm policies over continuation and choice of policy state imply a next iteration value for the value function, \( V'_T(v, z, c) \), based on computing the value of continuing and comparing to the value of exit.
    * Check whether new \( V'_T(v, z, c) \) is sufficiently close to \( V_T(v, z, c) \). If close enough, return to \( \{P_2, ..., P_T\} \) iteration. If not, continue iteration with \( V'_T(v, z, c) \).
  - Firm Value and Policy Iteration for \( t = \{2, ..., T - 1\} \):
    * Compute profit \( \pi(v, z; l) \) at each productivity \( v \) and capital stock \( z \) and potential choice of labor to hire, based on the specific demand system and production function chosen.
* Iterate back to compute $V_{T-1}(v, z, c)$ based on $\pi_t(v, z)$ and $V'_T(v, z, c)$, and period
T policies, based on computing the value of continuing and comparing to the value
of exit. Hence, determine period $T - 1$ policies {Continuation/Exit} and choice of
policy for labor, investment and dividends at each $\{v, z, c\}$.

* Iterate back to period $t = 2$.

- Compute value of entry in each time period $t = \{2, ..., T - 1\}$.
- Compute the size-distribution of firms $\mu_{v, z, c} = \{\mu_{v, z, c, 2}, ..., \mu_{v, z, c, T}\}$ consistent with the com-
puted firm policies.
  - Compute $\mu_{v, z, c, 2}$ based on $\mu_{v, z, c, 1}$ and firm policies computed for $t = 2$.
  - Determine number of entrants:
    * If value of entry negative for $t = 2$, set entry to zero.
    * If value of entry is non-negative, set entry such that:
      · Case 1: If the distribution of incumbents implies a price below $P_2$ then entry is
        zero, as adding entrants would further distance the firm distribution from the
        current value of price path
      · Case 2: If the distribution of incumbents implies a price above $P_2$, then add
        entrants until the firm distribution (including entrants) implies a price equal to
        $P_2$
    - Iterate forward to compute $\mu_{v, z, c} = \{\mu_{v, z, c, 3}, ..., \mu_{v, z, c, T}\}$.
- Check whether price path $\{P_2, ..., P_T\}$ is close enough to an equilibrium:
  - Objective function to assess equilibrium comprised of two parts:
    * The first part measures the distance between the price path and firm distribution:
      $(P^{max} - P)$
    * The second part measures an equivalent gap based on the value of entry: $(P^{fe} - P)$.
      · This is zero if value of entry is negative (to capture instances when this value is
        close to zero but negative, I consider this to be zero if value of entry/sunk cost
        of entry is larger than $-10^{-4}$).
· This is negative if the value of entry is positive. I calculate $P^{fe}$ as what the price in the time period in question would need to change to in order to close part of the gap in value of free entry. Hence, if value of entry is positive the price change is negative so as to lower profitability and thus lower the value of entry. The adjustment is moderated by the extent to which price adjustments for future periods (which have been determined as the algorithm work backs through time periods) are for increases or decreases in prices.

* The objective function is then the Euclidian distance of these two measures: $((P^{max} - P)^2 + (P^{fe} - P)^2)^{\frac{1}{2}}$

− If objective function not sufficiently small, construct new candidate price path. The suggested price adjustment is the average of $(P^{max} - P)$ and $(P^{fe} - P)$. The actual price adjustment is only part of the suggested price adjustment, to reduce the risk of cycling over successive iterations of the price path.