Which Resources to Accumulate?
Firm Dynamics in a Changing Industry

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Abstract

An important question in strategy is how firms adapt to a fundamental change in industry conditions. I develop a formal model to consider how firms within an industry respond to a shock that leads to a long-run transition in the industry. The model incorporates key aspects of a firm’s strategy in response to a shock: which set of resources to invest in and how much to invest. I consider shocks to resource substitutability, directly affecting the value of sets of resources at a point in time, and ease of resource accumulation over time. The shocks are exogenous but initial firm heterogeneity leads to a more complex, involved transition, from initial response through to long-run changes, due to the potentially different endogenous response of firms to the shocks. I find that a common shock to the industry may lead to non-uniform response across incumbent firms that were previously pursuing similar strategies, and a pattern of entry that includes spikes in entry and periods with very limited entry. Thus, firm dynamics and industry evolution in response to the shocks are not smooth, gradual changes, and the response to a reverse shock is not just the reverse transition.

Keywords: Firm and industry dynamics; Resources; Entry; Exit

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1 Introduction

An important question in strategy is how firms adapt to a fundamental change in industry conditions. An industry may be affected by many types of shocks, for instance, a change in regulations, improvement in technology, or changes in factor or end product markets. Even for a common shock to firms, differences across firms at the time of the shock mean that firms may not respond in a similar manner or at the same time to a shock to the industry. In addition, competitive interactions link a firm’s reaction to a shock to how other firms choose to react, including potential entrants. Hence the evolution of the industry in response to a shock may go through several stages. Thus how a firm responds to a fundamental change in industry conditions depends on the firm’s initial competitive context and how the diverse set of firms in the industry choose to respond.

In response to a shock, two key aspects of a firm’s strategy are which set of resources to invest in and how much to invest. At a point in time the set of accumulated resources determines the firm’s competitive advantage and thus profits in the product market (Barney 2001). Over time, the firm may accumulate resources to strengthen its competitive advantage (Dierickx and Cool 1989). Often these issues are treated separately: resource combinations as a static or long-run issue, and resource accumulation dynamics for a single resource. In this paper, I highlight that in the face of changing industry conditions there is value in considering jointly firm choices of which set of resources to accumulate and how much of each resource to accumulate.

I develop a formal model to consider how firms within an industry respond to a shock that leads to a long-run transition in the industry. I consider shocks that are exogenous and common to all firms. However, there is initial firm heterogeneity in the extent of resource accumulation and firm strategy choices are affected through competitive interactions by the choices of other firms. Consequently, the endogenous response to the shock potentially differs across incumbent firms, and the timing and extent of entry and exit are affected, resulting in a relatively complex, involved industry transition.

The model setup reflects several key aspects of firm dynamics and industry evolution. Considering the evolution of industries, such as over the industry life cycle, a key mechanism highlighted is that successful expansion by a firm supports further expansion, and, in turn, less successful firms exit, when selection effects dominate the ability for firms to adapt to circumstances (Jovanovic 1982, Ericson and Pakes 1995, Klepper 1996, Knott 2003). This is consistent with a process of gradually scaling up internal resources, in line with a broad stream of strategy research that em-
phasizes that firm’s accumulation of resources takes time due to time-compression diseconomies, uncertain outcomes of investment, and effects of competitive considerations (Barney 1986, Dierickx and Cool 1989, Pacheco-de-Almeida, Henderson, and Cool 2008, Pacheco-de-Almeida and Zemsky 2006). Thus there is no instantaneous, easy adjustment by firms to changing industry conditions. Also, entry and exit are important features of industry evolution. A well-documented empirical pattern is that from inception through to long-run maturity the industry life cycle of a nascent industry typically includes a period of rapid growth and high entry, followed by a severe shakeout (Klepper and Graddy 1990, Jovanovic and MacDonald 1994, Klepper 1997, Klepper 2002). The process of ongoing entry and exit, and gradual accumulation of resources leads to substantial heterogeneity across firms, consistent with typical empirical patterns (Dunne, Roberts and Samuelson 1988, Bartelsman, Scarpetta and Schivardi 2003). Nonetheless, the set of heterogeneous firms often comprises strategic groups, sets of firms that pursue similar resource accumulation strategies and thus have similar resource configurations, though the size and pattern of strategic groups and composition of industry leaders may potentially evolve over time (Lee, Lee and Rho 2002, Lee 2003, Desarbo, Grewal and Wang 2009, Dobrev and Carroll 2003, Sutton 2007).

In the model, firm strategy is a choice of which resources to accumulate and how much to accumulate, or, alternatively, the firm may choose to exit the industry. Potential entrants decide whether to enter the industry. Once a firm enters the industry, the firm invests to gradually build up their resources. The firm may accumulate two resources: I refer to firms that invest to accumulate both resources as generalists, and firms that invest to only accumulate one resource as specialists.\textsuperscript{1} Unsuccessful resource accumulation leads firms to exit. At any point in time firms in the industry have heterogeneous resource levels, as firms enter the industry on an ongoing basis and gradually accumulate resources with varying degrees of success. Within each time period, the firms currently in the industry compete in the product market with their profits determined by their current resource levels.

I consider how firms in such an industry respond to an exogenous shock. The basic structure of the model could be used to consider a variety of types of shocks, for instance macroeconomic, technological or regulatory, that affect the resources available to the industry, resource accumulation, the value of resource combinations, and/or demand conditions in the product markets. The main focus of the model is on how firms respond to these changes in underlying resource dynamics,

\textsuperscript{1}Note that the terms generalist and specialist refer to firm choice of resources, not market segments. For ease of presentation I use the terms generalist and specialist instead of resource-generalist and resource-specialist.
within a competitive context. To illustrate, consider a liberalization that removes the requirement for foreign firms to joint venture with local firms in an industry in a developing economy, such as in motorcycles in India in the 1990s. Before the liberalization the main firms are joint ventures that combine technical resources, based on foreign firm’s expertise, and marketing resources, based on local firm’s skills and existing assets, with success dependent on the resulting competitive advantage relative to other firms aiming to leverage similar types of resources. Removal of the joint venture requirement changes the value of resource combinations, which reduces the likelihood of new joint ventures. However, whether an existing joint venture dissolves in part depends on the success of the joint venture in accumulating resources prior to the liberalization. Indeed, in the case of motorcycles in India liberalization led to a divergent response across firms, with some larger joint ventures continuing while other smaller joint ventures dissolved into foreign and local firms.

Based on such situations, but necessarily abstracting from many of the details, I develop the model to focus on choice of which resources to accumulate, in a competitive context, and consider two types of shock that directly affect resource combinations and accumulation: one shock changes the substitutability of the resources available to firms, and the other shock the ease of resource accumulation. I first focus on the shock to resource substitutability. An increase in substitutability leads to a shift in firm strategy away from being generalists, accumulating both resources, towards being a specialist, accumulating just one resource, and vice versa for a decrease in substitutability. This type of shock is of interest as a stylized example of a type of shock highlighted in discussions of different types of industry transitions (McGahan 2004, McGahan, Argyres and Baum 2004). McGahan (2004) argues that changes to the core activities of firms lead to industry transitions distinct from those in an industry life cycle, with firm resource stocks not necessarily directly affected. This is consistent with a shock that does not affect resource accumulation or depletion directly, but does affect the combinations of resources used by firms: in turn, this may lead firms to adjust which resources to accumulate. Also, Jacobides (2005) highlights how substantial vertical disintegration occurred in the mortgage banking industry, even though the main aspects of the services offered by the industry did not change substantially. This is consistent with a shock that does not affect demand directly but does affect the incentives of firms to specialize.

Although an increase in resource substitutability may result in a clear long run shift from, say, 2This is meant to be a stylized example to provide intuition for the type of industry transition considered, as a range of factors impinge on joint venture formation and dissolution that are not the focus of this paper. The evolution of the motorcycle joint ventures and HeroHonda, the largest one, is the subject of a case (Ramaswamy and Sankhe 2003). A broad discussion of the evolution of joint ventures in India is by Kale and Anand (2006), and across a broader set of countries by Moskalev and Swensen (2007).
an industry dominated by generalist firms to an industry with primarily specialist firms, it is less obvious how the industry transitions over time to the new long run pattern, given the potentially varying responses of incumbent firms and possible entry of new firms. A change in resource substitutability changes the mix and levels of resources that firms aim to develop. Consequently, firms change their resource accumulation strategies, even though the shock does not directly affect the costs of resource accumulation. Also, firm profits in the product market change as the distribution of resource stocks across firms changes, even though the shock does not directly affect demand in the product market. Thus an exogenous shock to resource substitutability leads to endogenous effects on resource accumulation and competition in the product market. The focus of the model is on characterizing the transition including the endogenous changes resulting from the exogenous shock.

I first consider the case when the resources initially have low substitutability, leading firms to accumulate both resources. Hence, initially incumbent firms range from small firms with limited stocks of both resources to large firms with substantial stocks of both resources. The shock increases the resource substitutability. In the long-run the effect of the shock is for firms to focus on accumulating just one resource. Consequently, the long-run firm distribution has one set of firms specializing on one resource, including a range of firms with low to high resource stocks of this resource, and another set of firms specializing in the other resource, similarly with a range of resource stock levels. I focus on the transition from the initial to long-run equilibrium.

I find that the incumbents response to the shock is not uniform across firms, even though there is a common shock. Moderately sized incumbents, with moderate levels of both resources, switch to focus on accumulating just one resource. In contrast, large incumbents continue to accumulate both resources whereas the smallest incumbents exit. The reason for this divergence in strategy response to a common shock is that the shock does not directly render obsolete the firm resources. Rather, the shock increases the value of specialization in one resource, relative to accumulation of both resources. This pull towards specialization is strong enough for moderately sized firms to switch strategy, to accumulate one resource and let the other resource deplete gradually, but not for larger firms with high levels of both resources. Thus the response to a common shock initially increases firm heterogeneity in the industry.

In addition, I find entry patterns are not smooth over time. Once firms know of the forthcoming shock the number of firms entering drops to zero, as entrants prefer to enter and accumulate resources only once the change in resource substitutability takes effect. When the shock takes effect
there is a spike in entry, in part as the preceding period of zero entry has reduced the number of firms in the industry. Thereafter entry gradually reduces towards the long run level.

I next consider a shock that decreases resource substitutability. In this case, incumbent response is more uniform. Nonetheless, entry patterns are varied over time. I also consider changes to the ease of resource accumulation, again highlighting variation in entry patterns reflecting the patterns of incumbent response to the shocks.

The paper makes two contributions. First, to characterize the evolution of an industry through shocks to resource substitutability and ease of resource accumulation. A common exogenous shock leads to more complex endogenous response across firms within an industry. The transition from the initial to the new long-run industry equilibrium is not a smooth, gradual change. Also, the transition for a reverse shock is quite different: it is not the same process played backwards as there are incumbency effects due to resource accumulation. This highlights that the expedient of considering the long-run change in an industry is not sufficient to successfully characterize the short and medium run part of the industry evolution, which are nonetheless meaningful time frames from a firm strategy perspective.

The second contribution is to develop a formal model that comprises three levels of analysis: resources, firms and industry. The levels of analysis are linked, as firm choice of which resources to accumulate is determined by the costs of resource accumulation, how firms may combine resources in production, and the product market competition resulting from the choices of all firms in the industry, including whether to enter or exit the industry. In this regard the paper aims to contribute to the growing literature developing formal models to address strategy issues, in line with the call by Adner, Polos, Ryall and Sorenson (2009) and Ghemawat and Cassiman (2007). A key feature of the model set up is that the response of firms to a shock is tracked over successive time periods, as firms adapt to the shock and its knock-on effects. The model enables focus on the firm dynamics during the transition, not just the initial and final long-run equilibrium. The model structure could be leveraged to address other types of industry transitions due to a variety of types of shocks. Due to the dynamic aspects the model is solved numerically.

Following I describe the model set up, equilibrium conditions in the initial stationary state and in the transition in response to a shock, and then the numerical solutions, with detailed description of the algorithm in the appendix.
2 Model Setup

As outlined above, I develop a model to consider how firms adjust over time to a shock that affects the value of resource combinations or the ease of resource accumulation. Below I first describe how a firm accumulates resources and how a firm uses resources in production. I then illustrate with a simple example key aspects of the model set-up. I then add the remaining parts of the model to develop the full dynamic model.

Resource accumulation

The firms in the industry are distinguished by the stock of the firm’s resources at a point in time, denoted by \((r_1t, r_2t)\): hereon I omit the time subscript where this does not cause confusion. In each time period, each resource depletes at rate \(\gamma\) and the firm may invest to increase resource \(r_1\) or \(r_2\) or neither. The benefit of investing in resource \(i\) is \(\eta\) percent growth in stock of resource \(i\). A firm may not increase investment to grow faster than \(\eta\) within a time period: this is a simple, strong form of time-compression diseconomies of resource accumulation.

The cost of investing in resource \(i\), \(I^i(r_i)\), is comprised of \(p_r r_i (\eta - \gamma)\), where \(p_r\) is the per unit cost of investment, plus an additional cost that increases with current level of the resource stock due to diminishing returns. Hence, \(I^i(r_i) = p_r r_i (\eta - \gamma) + J(r_i)\), with \(J'(r_i) > 0\) and \(J''(r_i) > 0\). Due to diminishing returns firms with high enough resource levels would choose not to invest.

The resource accumulation process is kept relatively straightforward for simplicity: a highly stylized process for the firm to attain a better resource configuration. The choice of investment could be generalized, for instance to allow simultaneous investment in both resources, or with choice of investment level. Despite being relatively simple, this set up allows the firm to choose, over successive time periods, alternate investment strategies: for instance, to invest in just one resource, or alternate investment in both resources so as to accumulate both resources.

Within this set up, the substitutability of the resources captures the interdependency across resources at a point in time. The higher the substitutability of resources, the greater the benefit to the firm of having a resource configuration skewed towards one resource. However, an increase in either resource stock, up to the point of diminishing returns, is also beneficial. Thus optimal resource accumulation reflects both resource substitutability at a point in time, a static consideration, and resource accumulation over time, a dynamic consideration.
Production

Each firm is considered to produce a distinct variety of product with the firm’s production determined by its resource stocks $r_1$ and $r_2$. To produce using these resource stocks the firm requires labor which, for simplicity, the firms hires in a labor market with no frictions at cost of $w$ per unit of labor, with $w$ normalized to unity. Hence, in each time period a firm may produce the optimal level based on its resource stocks.

The firm uses its resources to produce based on production technology with constant elasticity of substitution (C.E.S.) $\alpha_t$ between resources $r_1$ and $r_2$. In period $t$, production, $y_t$, is given by:

$$y_t = [(r_{1t}l_{1t})^{\alpha_t-1}\alpha_t + (r_{2t}l_{2t})^{\alpha_t-1}\alpha_t]^{\alpha_t/(\alpha_t-1)}$$

where labor hired is $l_{1t}$ and $l_{2t}$ in period $t$. Based on standard derivations, this production function results in the firm’s marginal cost, $c_t$:

$$c_t = \left[\left(\frac{r_{1t}}{w}\right)^{\alpha_t-1} + \left(\frac{r_{2t}}{w}\right)^{\alpha_t-1}\right]^{-1/(\alpha_t-1)}$$

as wages are set to unity. Hence, an increase in resource levels lowers marginal cost. The effect of a change in mix of resources depends on $\alpha_t$: the higher $\alpha_t$ the greater the benefit to specialization.

In addition, the firm incurs an overhead per-period fixed cost $F$. Thus in the model there is a direct link between resources and production, as summarized by the firm’s production function.\(^3\)

The shock to resource substitutability leads to a change in $\alpha_t$.

Example

In the model, a key strategy choice of firms is whether to accumulate one or both resources. I next illustrate through an example how this strategy choice depends on the interaction of existing

\(^3\)The firm’s production function could be considered to reflect the specific activities conducted by the firm on an ongoing basis so as to generate products/services. Consideration of the activities performed by firms has led to an emphasis on the value of a close fit across activities (Porter 1996). An aspect of fit that has received recent focus is the process by which firms search for these complex combinations of activities (Rivkin 2000), including how competitive interactions affect the search outcomes (Lenox, Rockart, and Lewin 2006 and 2007). In addition, the nature and value of the interdependency across activities, in particular the extent to which these are complements or substitutes, is in part context dependent and in part an industry characteristic (Porter and Siggelkow 2008). Thus in my model I do not have firms needing to determine how to organize operations, as the production function is stable and known. Within the production function the resource stocks enter directly, and the key link between these resource stocks is the extent of substitution.
resource stocks and resource substitutability. The example is kept relatively simple as the aim is to provide intuition for key aspects of the more general dynamic model.

First consider the choice for an incumbent firm of whether to invest to become a generalist or specialist for a generalist firm. Assume firms in the industry have resource configurations \((r_1, r_2)\), with the levels \(r_i\) potentially ranging from \([0, 1]\). Hence, from (1), generalist firms with symmetrical resources \((r, r)\) have \(c_t = \frac{2^{1/(1-\alpha)}}{r}\), and specialist firms with asymmetrical resources \((r, 0)\) have \(c_t = \frac{1}{r}\).

Let firm profits be \(\pi(r_1, r_2) = \frac{A}{c_t^{\sigma-1}} - F\), where \(A\) is a demand factor and I assume \(A > F > 0\), and \(\sigma > 1\) is the elasticity of demand across firms. Thus profits increase the lower marginal cost \(c_t\). Below, in the more general model set up, this specific formulation will be motivated more fully. Let \(I(r)\) be the investment required to grow a resource from current level \(r\) to 1, with \(I(0) > I(r) > I(1) = 0\) for \(r \in [0, 1]\). Hence, for a generalist firm with current resources \((r, r)\) the investment to become: a generalist with resources \((1, 1)\) is \(2I(r)\); or to become a specialist with resources \((1, 0)\) (or \((0, 1)\) is \(I(r)\), assuming that the second resource depletes from level \(r\) to 0 without cost.

The first lemma highlights that the investment choice of an firm to become a generalist or specialist depends on resource substitutability.

**Lemma 1** A firm with current resources \((r, r)\) invests to become a generalist with resources \((1, 1)\), versus a specialist with resources \((1, 0)\) (or \((0, 1)\)), if resource substitutability is sufficiently low: \(\alpha_t < \alpha_r = 1 + (\sigma - 1) \ln(2)/\ln(1 + I(r)/A)\).

**Proof.** For firm with resources \((r, r)\) if invest to be: generalist with resources \((1, 1)\) value is \(A(\sigma-1)/(\alpha_t-1) - F - 2I(r)\); specialist with resources \((1, 0)\) value is \(A - F - I(r)\). Hence, value of generalist is greater than of specialist if \(\alpha_t < 1 + (\sigma - 1) \ln(2)/\ln(1 + I(r)/A) = \alpha_r\), as \((I(r)/A) > 0\).

The critical degree of resource substitutability \(\alpha_r\) depends on the firm’s current resource level, as this affects \(I(r)\). Hence across firms that differ in resource levels, at a given level of resource substitutability firm investment choices may result in a heterogeneous response across firms, with some firms becoming generalists and other firms specialists. This is illustrated in Figure 1 and is the focus of the second lemma:

**Lemma 2** Consider a set of firms with symmetric resource levels \((r, r)\) ranging from \((0, 0)\) to \((1, 1)\). For resource substitutability sufficiently low, \(\alpha_t \leq \alpha_0 = 1 + (\sigma - 1) \ln(2)/\ln(1 + I(0)/A)\), all firms
invest to become generalists. For resource substitutability sufficiently high, \( \alpha_t > \alpha_0 \), there is a resource level \( r^* \) such that: firms with resource levels below \( r^* \), from \((0,0)\) to \((r^*,r^*)\), invest to become specialists with resources \((1,0)\) (or \((0,1)\)); whereas firms with resource levels above \( r^* \), from \((r^*,r^*)\) to \((1,1)\), invest to become generalists with resources \((1,1)\). The cut-off resource level \( r^* \) is given by \( I(r^*) = A(2(\sigma-1)/(\alpha_t-1) - 1) \).

**Proof.** Based on Lemma 1, a firm with resources \((0,0)\) invests to become a generalist if \( \alpha_t \leq 1 + (\sigma - 1) \ln(2)/\ln(1 + I(0)/A) = \alpha_0 \). Hence, if \( \alpha_t \leq \alpha_0 \) all firms invest to become generalists. If \( \alpha_t > \alpha_0 \) then there is a resource level \((r^*,r^*)\) at which the value of investing to become a generalist with resources \((1,0)\) is the same as to become a specialist with resources \((1,0)\) (or \((0,1)\)): \( A2(\sigma-1)/(\alpha_t-1) - F - 2I(r^*) = A - F - I(r^*) \). Hence, \( I(r^*) = A(2(\sigma-1)/(\alpha_t-1) - 1) \). Consequently, firms with resources below \((r^*,r^*)\) invest to be specialists and firms with resources above \((r^*,r^*)\) invest to be generalists.

Hence, a shock that increases resource substitutability may have non-uniform effects across firms. For instance, if initially resource substitutability is sufficiently low (below \( \alpha_0 \)) and incumbent firms have a range of symmetric resource configurations ranging from \((0,0)\) to \((1,1)\), then before the shock the optimal investment for all these firms is to become generalists. However, a sufficiently large increase in resource substitutability (to above \( \alpha_0 \)) will generate a firm response that differs across incumbent firms: smaller firms, with less accumulated resources, will specialize, whereas larger firms, with more accumulated resources, will invest to be generalists. In summary, the industry will switch from having one strategic group of generalist firms to two groups, one pursuing a specialist strategy and the other a generalist strategy.

Next consider a shock to the ease of resource accumulation. For instance, an easing of resource accumulation may be considered as a decrease in the cost of investment \( I(r) \). A lower \( I(r) \) results in a higher \( \alpha_r \) and hence lower \((r^*,r^*)\). Consequently, a greater range of firms pursues a generalist strategy.

In summary, the choice of incumbent firms to pursue a generalist versus specialist strategy depends on both resource substitutability and ease of resource accumulation. Consequently, shocks to either resource substitutability or accumulation may affect the set of firms pursuing each type of strategy.

In the full dynamic model, I extend this basic model in three main ways. First, I specify a demand system that determines revenues per firm. This makes the \( A \) factor endogenous in that it is determined by the resource stocks of all firms in the industry. Consequently, profits per firm
in each period are endogenously determined. Second, I specify how incumbent firms set strategy over successive time periods, including the choice of whether to exit the market. Third I specify the entry process and the value function for a potential entrant to determine whether to enter, so as to make entry endogenous.

The benefit of this more complex set up is to enable characterization of the response to a shock over time. In particular, I first consider the initial pre-shock stationary equilibrium. Next I specify the firm dynamics and industry evolution from first response to the shock through to the long-run transition to the new long-run equilibrium.

**Demand**

I embed the firms in a monopolistic competitive set up. This simplifies considerably what firms need to know so as to set strategy. Each firm is assumed small relative to the mass of firms in the industry\(^4\). The firms each set prices, leading to a distribution of prices in the industry. A key benefit of this demand system is that aggregation of the firm’s prices into a particular price index is such that each firm need just consider the future aggregate price index to set its own firm strategy. In particular, the firm does not need to factor in competitive interactions with other firms on a firm-by-firm basis, as in oligopoly models of industry dynamics that require each firm to respond to each other firm, as in Ericson and Pakes (1995). In essence, for the firm the price index is a summary of the actions of competitors. In this regard, the competitive interactions in my model are more similar to those in Jovanovic (1982) and Hopenhayn (1992). However, a key difference with these models, in addition to the consideration of the transition equilibrium, is that I allow the firms considerable scope for adjusting firm resources over time.

I assume each firm produces a product (or service) differentiated to other firms, with consumer preferences for the differentiated varieties in the industry based on a constant elasticity of substitution (C.E.S.) demand system with elasticity \(\sigma > 1\) (Dixit and Stiglitz 1977). I assume there is a continuum of varieties produced by the firms, denoted by \(\omega \in \Omega\). Total industry revenues are given by \(R_t = Q_t P_t\), where \(Q_t\) and \(P_t\) are, respectively, the aggregate quantity and price indices. Specifically, \(P_t = \left[\int_{\omega \in \Omega} p_t(\omega)^{1-\sigma}\right]^{1/(1-\sigma)}\) is the C.E.S. price index for the aggregated differentiated good and \(Q_t \equiv \left[\int_{\omega \in \Omega} q_t(\omega)^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}\) the C.E.S. quantity index at time \(t\), where \(p_t(\omega)\) and \(q_t(\omega)\) are the price and quantity consumed of the individual varieties \(\omega\). With this demand system

\(^4\)The firms in the model could be interpreted as products for which firms are choosing strategies independent of the strategies of other products of the firm.
the firm’s optimal price is a constant markup of $\sigma/(\sigma - 1)$ over marginal costs, $p_t = (\sigma/(\sigma - 1))c_t$, the firm’s market share is $(P_t/p_t)^{\sigma - 1}$ and hence revenue is $R_t p_t^{\sigma - 1} p_t^{1 - \sigma}$.

**Profits**

In each period, the firm produces taking its current resource levels as given when optimizing hiring of production labor. Hence, the per-period profit from production, $\pi_t$, is:

$$\pi_t = R_t P_t^{\sigma - 1} p_t^{1 - \sigma} - q_t c_t - F$$

The fixed costs result in firms with sufficiently low resource stocks to have negative profits, as marginal costs are high, prices high and thus market shares low leading to insufficient revenues to cover production and fixed costs. As highlighted above, a benefit of the demand system is that for the firm to maximize profits the aggregate industry revenues, $R_t$, and the aggregate price index $P_t$, are sufficient statistics to summarize the necessary information about other firms.

**Firm Strategy Decisions: Dynamic set up**

The specification of production and demand determine firm profits for incumbent firms based on current resource stocks of each firm. I next turn to consider the strategic decisions of the firm that affect development of the firm over successive time periods. For incumbent firms, this is whether to stay in the industry or exit, and if continuing in the market whether to invest in one of the resources or not. For potential entrants, this is whether to enter the industry at that point in time. These policies require firms to set strategy each period based on considering the effect of alternative options on the value of the firm.

A first necessary step is to specify the timing of events within the model to link successive time periods, which are as follows, as illustrated in Figure 2. At the start of each period new entrants pay a sunk cost of entry and thereon are indistinguishable from incumbent firms with the same resource levels. Firms then decide whether to continue or exit. The value of continuation depends on current period profits plus future value of the firm. Current period profits are as specified above. Future value is determined by the firm’s choice of whether to invest, as this determines...
future resource levels. The outcome of the firm’s investment in resources is subject to uncertainty. At the end of the period this uncertainty is resolved, and firms start the next period, as incumbent firms, with their updated resource levels.

The firms in the model are forward-looking, with strategy choice not just determined by current period performance but by maximizing the value of the firm, including current profits and the value of the firm going forward. Clearly, a range of other assumptions could be made based on more limited firm rationality, decision-making complications, or other issues: I return to these assumptions in the discussion section.

Each time period, firms decide whether to continue in the industry or exit based on the maximization of firm value $V_t(r_{1t}, r_{2t})$ comparing the value of continuing, $V^C_t(r_{1t}, r_{2t})$, to the value of exit $V^X_t(r_{1t}, r_{2t})$ which is set to zero for simplicity:

$$V_t(r_{1t}, r_{2t}) = \max \left[ V^C_t(r_{1t}, r_{2t}), V^X_t(r_{1t}, r_{2t}) \right] = \max \left[ V^C_t(r_{1t}, r_{2t}), 0 \right]$$

(3)

Continuing firms produce to generate current profits, and choose whether or not to invest, and if so in which resource. Investment is subject to two forms of uncertainty. Each time period there is an exogenous probability $\delta$ of exit, which is independent of resource levels). Also, the outcome of investment in a resource is stochastic. Thus, there is both exogenous exit due to the death shock and endogenous exit due to bad outcome in investment in resources. Firms discount next period profits at the discount rate $\beta$. The firm strategy choices must satisfy the Bellman equation:

$$V^C_t(r_{1t}, r_{2t}) = \max_{\text{Invest}_t} \left\{ \pi_t(r_{1t}, r_{2t}) - I^1 - I^2 + \beta (1 - \delta) \int_{r_{1t}'} \int_{r_{2t}'} V_{t+1}(r_{1t}', r_{2t}') dG [r_{1t}' | r_{1t}, I_t] dG [r_{2t}' | r_{2t}, I_t] \right\}$$

where $\text{Invest}_t = \{\text{Invest in } r_1; \text{Invest in } r_2; \text{Not invest}\}$

$$I^i(r_i) = \{p_i r_i (\eta - \gamma) + J(r_i) \text{ if invest in } r_i; \ 0 \text{ otherwise}\}$$

Consequently, across potential resource levels there will be four regions differing by firm strategy: exit; invest in resource $r_1$; invest in resource $r_2$; not invest. Firms with sufficiently low resource stocks choose to exit, as the per-period fixed would lead to successive periods of losses before sufficient resource accumulation would start generating profits. At sufficiently high levels of both resources the effect of diminishing returns results in no investment in either resource: a not invest

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5In the above section on resource accumulation the outcome of investment was given as $\eta$ growth in resource stock for simplicity: for the dynamic model this is generalized to allow the outcome to be stochastic with mean $\eta$. 

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region. For other resource levels the shape of regions depends on parameters, as discussed below.

**Entrants**

At the start of each period, new entrants can potentially enter the industry. Entrants arrive into the industry with a range of initial resource levels. An entrant pays a sunk cost of entry, $S$, then discovers its initial resource levels, drawn from a known invariant distribution $G_E(r_1, r_2)$. Thereon, entrants are indistinguishable from incumbent firms with the same resource levels. A prospective entrant therefore faces a net value of entry

$$V_t^E = \int_{r_1', r_2'} [V_t(r_1', r_2')dG_E(r_1, r_2) - S$$

The entry decision is forward looking, as based on $V_t$, and not just reflective of current conditions in the industry (e.g., if entry were only based on current profits). Hence, entry may occur in anticipation of a shock to the industry, not just in response to concurrent industry conditions.

When the value of entry is negative there is no entry. If the value of entry is positive entry occurs. Within a time period, an increase in the number of entrants reduces profits, as the industry becomes more competitive. Hence, the value of entry drops with increased entry. Consequently, the number of entrants is such that the value of entry is zero. I assume that there is a sufficient pool of potential entrants within each time period to achieve this. In summary, the value of entry is either negative and there is no entry, or zero with just enough entry to ensure that the value of entry is non-positive.

3 Initial and final long-run equilibrium: Stationary Equilibrium

In the initial and final long-run equilibrium the parameters do not change over time. I consider a stationary equilibrium in which firm strategy choices, conditional on resource stocks, are invariant over time. For instance, the set of resource combinations that lead to exit does not change. With firm strategies set in a consistent manner and with stable ongoing entry and exit, the overall number of firms and distribution of firms is stable over time. In turn this generates a stable price index, consistent with firm strategy choices. Despite the aggregate stability, the stationary equilibrium comprises substantial firm dynamics, as there is entry, exit, and investment in resources.

A stationary equilibrium must satisfy the following three conditions:
**Firm Value Maximization** All firms' choices for exit/continuation, and, if continuing, for investment, conditional on $r_1$ and $r_2$, must satisfy (3) and (4). In particular, the level of the price index $P_t$ and entry per period determine the current period distribution of firms. The current period distribution of firms is comprised of the distribution of incumbent firms surviving, net of exit, from the prior period, including the effect of their investment choices, plus the distribution of new entrants. The resulting distribution of firms must generate the same distribution of firms as in the prior period. In short, the net effect of entry, investment and exit is to leave the firm size distribution unchanged over time.

**Free Entry** In the stationary equilibrium the net value of entry $V_t^{E}$ must be zero. If the value of entry were positive there would be additional entry until the value of entry falls to zero, since there is an unbounded pool of prospective entrants and entry is not limited beyond the sunk entry cost. If the value of entry were negative, no firms would enter yet some firms would exit due to the exogenous death shock: this would lead to a net decline in the total number of firms which is not consistent with a stationary equilibrium.

**Aggregate Industry Accounting** The distribution of firms over resource stocks implies a distribution of prices (applying the profit maximizing markup rule to firm marginal cost). Aggregating these prices into the C.E.S. demand system price index must yield the $P_t$ used by firms to set strategy in every period.

Below I discuss how these equilibrium conditions are modified when considering the transition equilibrium subsequent to a shock from an initial stationary equilibrium. Next I discuss the stationary equilibrium, with focus on comparing the initial stationary equilibrium and the final, long-run equilibrium consistent with the initial and post shock degree of substitutability of resources.

### 4 Simulated Results

I solve for the equilibrium based on numerical solutions. In the appendix, I describe the algorithm used to generate the solutions and include a more formal statement of the above equilibrium conditions.\(^6\)

\(^6\)In summary, to calculate the stationary state I start with an initial guess for the overall price index $P$ (with time subscript omitted as invariant over time). I then compute firm strategies based on this price index, and consequently the value of entry. I then adjust the price index so as to reach a value of entry closer to zero (i.e., guess for $P$ is reduced if value of entry is positive, and increased if value of entry is negative), and iterate until the value of entry is sufficiently close to zero. These methods have also been used to study the effects of credit constraints on industrial evolution.
Calibration

I next describe how I set the parameters of the model to run the model simulations. The aim is for the model calibration to reflect the typical patterns of firm dynamics within industries, for instance, as in Bartelsman, Scarpetta and Schivardi (2003), Cooley and Quadrini (2001), and Olley and Pakes (1996). The key parameters are described in Table 1, with the main choices highlighted below.

I set each time period to correspond to one quarter, relatively short so as to smoothen out the dynamic processes. The number of grid points is chosen sufficiently high to reduce effects from the discreteness of the grid. The grid size is exogenous to any firm decisions, set wide enough that the exit cutoffs are sufficiently above the lower bounds, and very few firms grow to reach the edges of the grid corresponding to positive resource stocks for both resource stocks. In the scenarios I contrast lower and higher elasticity of substitution of resources, with $\alpha = \{2, 5\}$ and lower and higher benefit to investing in resource $r_i$, with $\eta = \{10\%, 19\%\}$ per year ($\{2.5\%, 4.5\%\}$ per time period).

Stationary state

I first contrast two stationary states with lower and higher resource substitutability, respectively $\alpha = 2$ and $\alpha = 5$, and with lower resource growth rate ($\eta = \{10\%\}$ per year), comparing firm strategies, entry patterns, and the distribution of firms across resource levels. I then compare these two stationary equilibria with other stationary equilibria corresponding to a faster rate of resource accumulation. I structure the discussion around a set of remarks to highlight the key aspects of the stationary states.

- Firm strategies at low and high resource substitutability differ markedly: at low resource substitutability firms accumulate both resources concurrently over time; whereas at high resource substitutability many firms accumulate just one resource and other firms accumulate both resources sequentially.

and the effect of trade opening on industrial evolution (Costantini 2006 and Costantini and Melitz 2008). Similar methods applied to a continuous innovation decision in a general equilibrium setting have also recently been developed by Atkeson and Burstein (2006). The computational methods I use in the current paper apply to a monopolistically competitive sector with a large number of competing firms (where the mass of firms evolves endogenously). Hence, these methods are radically different from the seminal contribution to the computation of such equilibria with a small number of firms under oligopoly in Pakes and McGuire (1994), following the development of the theoretical version of the model in Erikson and Pakes (1995).
Figures 3 and 4 panels (a)(i) and (b)(i) show how firm strategy choices vary with resource levels. Note that the figures are symmetrical along the 45° line from the origin, as the two resources are treated symmetrically throughout.

At low resource substitutability (Figure 3, panel (a)(i)), the exit region is the combination of low resource levels for resource 1 and 2. Firm strategy for firms just outside the exit region is to accumulate the resource for which the firm currently has lower stock levels. Hence, firms invest so as to equalize their resource levels across resources: firms tend to grow towards the 45° line from the origin. Once firms have similar stocks of both resources (and thus the firms are on the 45° line from the origin), firms then alternate investment in the resources so as to gradually build up both resources. At high levels of both resources there is a no investment region due to diminishing returns to resource accumulation.

At high resource substitutability (Figure 4, panel (a)(i)), firms strategy for firms just outside the exit region is to accumulate more of the resource the firm already has more of, and to let the other resource stock deplete. Hence these firms over time become increasingly specialists in just one resource: these firms tends to grow away from the 45° line from the origin. With growth these firms will reach the point of diminishing returns to further investment in the accumulated resource. At this point, firm strategy depends on how much of the other resource remains after ongoing depletion. If the other resource stock is low enough, then the firm strategy is not to invest. If the other resource stock remains sufficiently high, then the firm now starts to invest in this resource, again up to the point of diminishing returns.

The increase in the resource growth rate leads to only minor changes in the strategy regions: comparing in Figures 3 and 4 panels (a)(i) with (b)(i). In summary, there are three typical growth paths of entrants: gradual accumulating of both resources (a generalist), accumulation of one resource (specialist), and accumulation of first one resource and then the other (initially a specialist and eventually a generalist).

- As the shape of the exit region reflects the degree of resource substitutability, the distribution of entrants that continues in the industry, not immediately exiting, differs at low and high resource substitutability.

Figure 5 show the distribution of entrants across firm resource stock levels. Entrants initial initial resource level does not depend on level of resource substitutability. However, entrants that receive an initial draw of resources levels that places them in the exit region immediately exit.
Hence, the entrants that continue have higher average resource levels than entrants that immediately exit, and the continuing entrants then grow through resource accumulation. Thus, the simulations replicate the robust empirical findings that recent entrants are on average smaller, and exhibit higher exit rates than incumbent firms. Also, the distribution of entrants that continue in the industry is endogenous, as dependent on the shape of the exit region. At higher resource substitutability the exit region has a more square shape: for instance, the value of symmetric resource configurations, on the $45^0$ line from the origin, is lower the higher the resource substitutability and hence the exit region cut-off is at higher symmetric resource levels (comparing panels (i) and (ii) in Figure 5).

The distribution of firms across resource levels is the summation of firms surviving from successive entry cohorts. As there is ongoing exit (in part due to the exogenous death shock, in part due to endogenous exit), there is also ongoing entry, as in a stationary equilibrium the number of firms is stable over time. In a stationary equilibrium, each successive cohort of entrants has the same initial distribution and follows the same strategies as other cohorts.

- The distribution of firms across resource levels at low resource substitutability has single-peak with generalist firms having a similar level of both resources. At higher resource substitutability there are two main peaks with specialist firms with high level of just one resource and, depending on the rate of resource growth, an additional peak of generalist firms.

With low resource substitutability (Figure 3 panel (a)(ii)) the mass of firms is on the $45^0$ line from the origin. As firms gradually accumulate resources, the peak of the distribution is at moderate resource levels. With faster resource accumulation the peak shifts towards larger firms (Figure 3 panel (b)(ii)). With high resource substitutability (Figure 4 panel (a)(i)), there are two main peaks where firms have accumulated a high level of one resource and let the other resource deplete to zero. In addition, with faster resource accumulation there is a build up of generalist firms, a third peak (Figure 4 panel (b)(ii)). These firms have accumulated first one resource and then the other resource and consequently have high levels of both resources. In contrast, with low resource substitutability firms with similarly high resource levels would typically have grown with a different pattern, alternating accumulation of each of the resource stocks to maintain similar resource levels.

In summary, variation in the degree of resource substitutability and ease of resource accumulation leads to distinct patterns of firm strategy and consequent firm size distributions. Thus the formation of strategic groups, specialists and generalists, is endogenous.
I next turn to consider the transition between such initial stationary states due to a shift to resource substitutability or to resource accumulation. For instance, I first consider a shift from $\alpha = 2$ to $\alpha = 5$ at lower resource growth rate ($\eta = \{10\% \text{ per year}\}$), as this is sufficient to generate a qualitatively large change in firm strategies, from an industry with generalist firms to an industry with specialist firms and vice versa (from Figure 3 panel (a) to Figure 4 panels (a)). The difference across these stationary states indicate the evolution of the industry in the long run. I next characterize the transition over time from the initial stationary state to the post-shock long-run stationary state.

5 Equilibrium during change in industry conditions

Before discussing the transition equilibrium in response to a shock, I outline the equilibrium conditions for the transition equilibrium as these are more general than for the stationary equilibrium, with the key differences:

**Firm Value Maximization** The price index $P_t$ and entry per period may vary over time, and hence firm strategy, conditional on resource levels, may vary over time. Also, the distribution of firms may also change over time.

**Free Entry** In the transition equilibrium the net value of entry $V_t^E$ must be non-positive. In contrast to the stationary equilibrium, the value of entry may be negative leading to zero entry, as there is no requirement for the number of firms to be stable over time.

**Aggregate Industry Accounting** As in the stationary equilibrium, this must hold in each time period.

In the transition equilibrium the industry starts in an initial stationary state. The shock is unexpected by firms, as their current strategies presume a continuation of the stationary equilibrium. Once the shock is known, firms may then choose to adjust strategy. In the long-run, the industry converges to the stationary equilibrium that is consistent with the post-shock resource substitutability and accumulation. In the equilibrium conditions (and algorithm) this convergence is not imposed: rather, this convergence occurs through a gradual process of adjustment.\footnote{Considering the transition equilibrium, a long enough set of time periods is considered for this gradual process to take effect so that the final time period generates an industry sufficiently close to the new long-run stationary state. In the model successive time periods are distinct from successive iterations performed by the algorithm to converge to an equilibrium. In equilibrium the industry responds to the shock and then converges over time towards the new long-run industry equilibrium. Each iteration of the algorithm corresponds to a guess as to this transition path. Successive iterations seek a transition path that better fits the equilibrium conditions.} The equi-
librium conditions hold throughout the transition from the initial stationary equilibrium through to eventual convergence towards the new long-run stationary equilibrium. Though equilibrium conditions hold, this does not necessarily mean the transition is smooth or gradual, as, for instance, there could be periods of zero entry and entry peaks in equilibrium.

In terms of calibration, two additional aspects need to be specified. First, the total number of time periods, which I set to 200 (i.e., 50 years) as this is long enough to ensure that by the final period the industry has converged close to the stationary equilibrium corresponding to the final set of parameters. Second, the time path of the shock. I consider four cases: an increase and decrease in resource substitutability, and an increase and decrease in resource growth. The change is announced to firms at a point in time that I refer to as the end of period \( t = 1 \). The shock gradually takes effect from \( t = 9 \) to \( t = 17 \), and thereafter is at the new long-run level. The scenarios are illustrated in Figure 6. Thus, firms have a two-year period from announcement to when the change starts, two years of gradual change, and thereafter stable conditions. This timing is meant to reflect, for instance, when a change in regulations, or technology is expected but with some delay in taking effect resulting in an intervening announcement period during which firms may adjust to the anticipated changes. I keep the scenarios relatively simple, although the model (and algorithm) allow for any arbitrary path of changes in parameters over time.

**Firm dynamics during change in resource substitutability**

I first discuss the transition for an increase in resource substitutability, a shift in the industry from mostly generalist firms to mostly specialist firms, highlighting key features of the transition.

- Incumbent response to the shock is not uniform across firms, even though this is a common shock. Incumbents with moderate levels of both resources switch to focus on accumulating just one resource. In contrast, incumbents with already high levels of both resources continue to accumulate both resources and the smallest incumbents exit.

Figure 7 has the evolution of firm strategy choices (panel (a)) and distribution of firms (panel (b)) for selected time periods: \( t = 2 \), the initial response to the shock; \( t = 10 \), as the shock starts

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8In addition, particularly for the transition, an important aspects of model development is to have a consistent clock run throughout the model (e.g., the discount rate, death rate, and fixed costs are all with respect to the chosen unit of time) and to distinguish, in computing the numerical solutions, between the sequence of time periods in the model (in which the clock matters) and the sequence of iterations performed by the algorithm to converge to the equilibrium.
to take effect; \( t = 14 \), midway through the shock taking effect; and \( t = 28 \), three years after the resource substitutability has reached the post-shock level.

The initial response to the shock at \( t = 2 \) leads to a partial change to the strategy choices, in particular for smaller incumbents. Considering firms on \( 45^\circ \) line from the origin (in panel (a) for \( t = 2 \)), from smallest to largest: the smallest incumbent firms exit as there is a slight expansion in the exit region; the mid-sized incumbents switch strategy to specialize in accumulation of one resource; and the largest incumbents continue to accumulate both resources. Not surprisingly, though firm strategies change, the change in the firm size distribution (relative to \( t = 1 \) pre announcement of shock) is more moderate. From when the shock starts to take effect, \( t = 10 \) onwards, the strategy choices are similar to those prevailing in the long-run . However, the distribution of firms remains substantially different to the long-run distribution even at \( t = 28 \) (comparing to Figure 4, panel (a)). By this time the peaks in specialized firms are starting to emerge clearly. However, there remains a substantial peak of large, generalist firms.

These patterns are in line with the introductory example given earlier. Incumbent firm’s response to the shock is divergent as current resources are not made obsolete. Rather, the shock increases the value of specialization in one resource, relative to accumulation of both resources. This pull towards specialization is strong enough for moderately sized firms to switch strategy, to accumulate one resource and let the other resource deplete gradually. However, for larger firms both resources are accumulated to a level that warrants continued investment. In summary, the heterogeneity in firms due to the process of resource accumulation results in a heterogeneous response to a common shock. As a consequence, the industry has for a substantial period of time a distribution of firms that is neither like the initial or long-run distribution of firms.

- Entry is characterized by three phases, based on a shock that becomes known to firms with some anticipation ahead of taking effect. At the announcement of the shock, the number of firms entering drops to zero. When the shock takes effect there is a spike in entry. Thereafter entry drops and the number of firms gradually reduces towards the long run level.

Figure 8 has the time path of selected aggregate variables over the first twenty years (80 time periods). Once the shock is know the value of entry is negative (panel (f)) and thus there is no entry until the shock starts to take effect (panel (e)). Entry is a forward looking decisions. The value of entry is negative as the price index (panel (a)) and thus firm profits (conditional on resource levels) are below the long-run level, whereas the costs of resource accumulation do not change over time.
Thus it is more profitable to enter later on. As entry later on has expected profitability of zero, entry earlier on has negative expected profits. Once entry has a non-negative expected value there is a large spike in entry. Once resource substitutability starts to increase, firm’s marginal costs rise (equation 1) and hence prices rise (conditional on resource levels). The increase in the overall prices index increases the value of entry; however, this increases entry which depresses the price index to the point where entry is break-even. The spike in entry is followed by a gradual reduction in the total number of firms towards the long-run level (panel (d)).

The pattern of entry is not driven by just concurrent industry conditions or long-run changes in number of firms but rather by the specific dynamics of the transition. During the transition the value of entry does not follow either the average value per firm or profits per firm (panel (b)): for instance, the spike in entry is when the average value and profitability of firms is at a minimum. For instance, the average value per firm may not be a relevant measure for a potential entrant to consider as this is the average for a distribution of firms that in part reflects the pre-shock distribution of firms. In particular, this comprises a peak of large incumbent generalists, which are firms with low costs (as with high resource stocks) yet not representative of the firms that entrants are likely to develop into. Also, the moderate long-run change in the total number of firms is not sufficient to determine the pattern of entry over time, as this does not suggest the occurrence of entry spikes.

Consequently, the transition from the initial to the new long-run industry equilibrium is not a smooth, gradual change. Incumbent firms that previously followed similar strategies start to pursue divergent strategies, and entry is very varied over time. I next contrast this transition with a transition for a decrease in resource substitutability.

- The transition for a decrease in substitutability is very different: it is not the same process played backwards as there are incumbency effects due to resource accumulation. Incumbent response is more uniform with a shift to accumulating both resources, and entry has two spikes, one at announcement and one once the shock occurs and no periods of zero entry.

Figures 9 and 10 highlight key aspects of the transition due to a decrease in resource substitutability. In this case the bulk of incumbents are initially specialized: for these firms the strategy shift is immediate towards accumulating the other resource (Figures 9, panel (a) for \( t = 2 \)). That is, the specialist firms all switch towards a generalist strategy. This switch makes sense from a forward looking perspective, though not considering near term profits. The switch leads these for-
Firm dynamics during change in ease of resource accumulation

I consider relatively moderate shifts in rate of resource accumulation as the basic patterns of firm strategies do not change much, comparing panels (a) and (b) in Figures 3 and 4. Hence incumbent firms continue with their existing strategies. In particular, with higher resource substitutability an increase in resource growth rate leads to the emergence of a peak of generalist firms. This is mostly due to entrants, as incumbent specialists continue as specialists.

The entry patterns are in Figure 11. A shock that decreases resource growth induces an immediate spike in entry, reflecting the attractiveness of fast growth ahead of the shock occurring. This is followed by a prolonged period with zero entry, lasting until the shock has taken effect. In contrast, a shock that increases resource growth results initially in a period of zero entry, as potential entrants delay entry to when resource accumulation in easier. Once the change takes effect there is a spike in entry. This may seem odd, as in the long run the number of firms decreases. However, the initial phase with no entry reduces the total number of firms. In summary, despite a fairly limited adjustment by incumbents, the entry patterns are all but smooth.

6 Discussion

The paper develops a formal model for the response of firms to a shock to the industry. The model comprises three levels of analysis: resources, firms and industry. Thus within the model firms choose their internal configuration (how much of each of the two resources to accumulate) and whether to enter or exit, while competing with other firms facing similar choices. The particular aspect of the model set up is that the response of firms to a shock is tracked over successive time
periods, as firms adapt to the shock and its knock-on effects. In particular, the emphasis is on the firm dynamics during the transition, not just the initial and final long-run equilibrium. In this regard the paper aims to contribute to the call for developing formal models to address strategy dynamic issues (Adner, Polos, Ryall and Sorenson (2009) and Ghemawat and Cassiman (2007)) and is complementary to other modeling approaches, such as two-period oligopoly models which are well-suited to consider specific firm-to-firm competitive interactions but with more limited inter-temporal dynamics, long-run models of industry evolution that encompass several stages of an industry evolution, or models of search for optimal organizational configurations that allow for rich variation in firm strategies but often have more limited competitive effects.

Three key features of the model interact to generate more varied and turbulent dynamics: slow-moving resource stocks; linkages across resources; and forward-looking firm policy decisions. Clearly the assumption of slow-moving resource stocks is important to the results: instantaneously flexible firms would lead to very different dynamics. In this regard the results should be taken to suggest the value of considering situations with both a static and dynamic optimization element. I motivated the slow-moving stocks primarily through time-compressing diseconomies in resource accumulation (Dierickx and Cool 1989): other motivations could drive choice of functional form and menu of investment options available to the firm. The importance of gradual adjustment of resource stocks is also highlighted by Sterman, Henderson, Beinhocker, Newman (2007), in their case considered as capacity adjustment, and is consistent with models of industry evolution that have gradually evolving firms.

I consider just two resources but this is already sufficient to lead to interesting issues of which resource to accumulate, trading off static efficiency of specialization versus dynamic benefits of accumulation. This reflects the general concern that with more than one resource present there is the question of how the resources interact in production and thus in the product market (Barney (2001)). In the current set up the substitutability of resources is exogenous to the firm and common across firms. More generally, the choice of firm internal configuration is kept relatively simple in the current context, as the focus is more on the transition dynamics. For instance, the degree of resource substitutability could be considered endogenous, at least to some degree (Porter and Siggelkow 2008), though this would further complicate the model as adding an additional firm choice variable. Also, the internal configuration could be considered to arise through a more complex search process, for instance, represented by a search on an NK landscape (as in Rivkin
Within my model firms base their decisions on discounting expected future profits. In particular, firm response to the shock reflects profit expectations during the transition through to the long-run, not just eventual long-run profits nor just current profits. I consider a major shock to the industry and hence once firms know the change is forthcoming (i.e., post announcement) some degree of anticipation seems reasonable. For instance, Makadok and Barney (2001) highlight the value of firm investment in information, in particular about strategically important resources, and this would motivate search for information in the event of, say, a major shock to the industry. Nonetheless, the results indicate that timing of the shock, such as extent of anticipation, affects substantively firm dynamics during the transition, even if variation in timing does not affect the initial and final stationary states. The importance of timing and anticipation is not surprising in a dynamic context, but this suggests a fruitful direction for future work is to consider the effects of limited firm foresight, and/or firm myopia in setting strategy. This would be in line with assumptions of bounded firm rationality, such as limited ability to forecast demand as in Sterman, Henderson, Beinhocker, Newman (2007), or in setting strategy primarily based on feedback from concurrent performance, as in Jacobides, Winter and Kassberger (2007). Hence, the current results should be considered in the context of firms with relatively full rationality: notwithstanding this, the transition equilibrium is all but smooth and uneventful.

In considering the results several limitations should be highlighted. The model set up is more general than the specific numerical solutions illustrated: however the non-linear dynamics require numerical solutions (e.g., due to potential periods of zero entry) and these are computation intensive. So as to develop the numerical solutions, specific functional forms are used and particular parameter values analyzed. I have varied the key parameters to check that the basic patterns discussed hold beyond the specific solutions highlighted in the paper.

Also, I focused on shocks that affect the value of sets of resources used in production and ease of resource accumulation. These types of shocks are of interest as directly affecting key strategy choices of firms of which resources to accumulate. I have focused on specific types of shocks given the model set up and computational burden. However, the shocks analyzed are specific examples of a broader shock. For instance, resource substitutability is one part of what determines the value of

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9In this regard, the initial stationary state scenarios could correspond to an initial, completed, search by firms for positions on an NK landscape. The transition from the initial stationary state to the eventual new stationary state could correspond to applying a shock to the NK landscape, with firms then adjusting to the new landscape, which is a different process to that of the initial search for competitive positions on the landscape.
sets of resources. An alternative could be to have fixed costs per resource with the shock affecting economies of scope across resources. Also, resource growth is one aspect of resource accumulation with, for instance, an alternative being a shock to the rate of depletion. Hence, a potential direction for future work is to consider a range of other types of shocks, for instance, to the demand system or to the set of resources available, such as due to disruptive technology shocks (Adner 2002, Tushman and Anderson 1986, Anderson and Tushman 1990, Romanelli and Tushman 1994).

With these limitations in mind, I find that a common shock to the industry may lead to non-uniform response across incumbent firms that were previously pursuing similar strategies. This effect is due to the interaction of resource accumulation, a dynamic consideration as affecting change in resources over time, and resource substitutability, a static consideration as affecting value of resources in production. This is in line with the findings that the complementarity of firm’s resources is an important element in understanding incumbent performance in response to industry shocks (Tripsas 1997). An implication is that, at least for the response to these types of industry shocks, caution should be exercised if analyzing static and dynamic issues separately, a typical expedient in considering such situations (Ghemawat and Cassiman 2007, Ghemawat and Levinthal 2008).

I also find that the firm dynamics and industry evolution in response to the shocks are not smooth, gradual changes, and that the response to a reverse shock is not just the reverse transition. The main reason for this is incumbency effects and forward looking entry decisions. For instance, entry patterns often comprise spikes and periods of zero entry. Hence, firm reaction is in part a direct response to the shock, in part a response to the strategies of other firms. This highlights the importance in these types of situations of the relative timing of key processes affecting firm dynamics and industry evolution, including the rate of accumulation and depletion of resources, exit decisions, shifts in market shares across providers. This is consistent with empirical evidence that firm response to uncertainty is not uniform and takes material time to unfold, relative to the underlying competitive dynamics, for instance with firm investment in new technology diffusing slowly across firms (Greve 2008). More generally, this points to the limitations of analyzing industry evolution with a focus on just comparing current conditions to long-run conditions: this may miss much of interest occurring in between within a meaningful time frame for strategy issues.

Also, the findings highlight managerial challenges during such periods of change. For instance, firms that previously were within one strategic group pursuing similar strategies may respond to a shock by pursuing divergent strategies leading to the splitting of the strategic group. Or, recent
entry may not provide indication that entry is currently profitable. Hence, during such periods of industry change firms may not always make optimal decisions. Hence, an interesting extension would be to include the effect of firms making mistakes in setting strategy, for instance in what strategy to pursue and when to do so, to understand the cost of these mistakes in the context of an equilibrium set-up in which other competing firms may also be making such mistakes.

7 Conclusion

An important question in strategy is how firms adapt to a fundamental change in industry conditions. An industry may be affected by many types of shocks, for instance, a change in regulations, improvement in technology, or changes in factor or end product markets. How a firm responds to a fundamental change in industry conditions depends on the firm’s initial competitive context and how the diverse set of firms in the industry choose to respond. Two key aspects of a firm’s strategy in response to a shock are which set of resources to invest in and how much to invest.

I develop a formal model to consider how firms within an industry respond to a shock that leads to a long-run transition in the industry. The shock is exogenous but as there is initial firm heterogeneity this leads to a more complex, involved transition due to the potentially different endogenous response of firms to the shock. I find that a common shock to the industry may lead to non-uniform response across incumbent firms that were previously pursuing similar strategies, and a pattern of entry including spikes in entry and periods with very limited entry. Hence, firm dynamics and industry evolution in response to the shocks are not smooth, gradual changes, and that the response to a reverse shock is not just the reverse transition. The findings highlight the value of jointly considering which set of resources are valuable for a firm to have and how much of a resource to accumulate, and the limitations of analyzing industry evolution with a focus on just comparing current conditions to long-run conditions as this may miss much of interest occurring in between within a meaningful time frame for strategy issues.

References


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Table 1: Calibration: Model Timing and Productivity Grid, Demand and Production Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Explanation of model calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Timing</strong></td>
<td></td>
</tr>
<tr>
<td>Duration of time periods t</td>
<td>Set each time period to correspond to one quarter, so time period short relative to typical empirical unit of analysis. Set total number of time periods such that the by the final period the industry has converged close to the final stationary state.</td>
</tr>
<tr>
<td>Discount rate β</td>
<td>5% per year, thus 1.3% per quarter (i.e., per time period)</td>
</tr>
<tr>
<td><strong>State space grid</strong></td>
<td></td>
</tr>
<tr>
<td>Resource 1 and 2 r_1 and r_2</td>
<td>Set range to [0.5, 6] to allow a sufficiently broad range of firm sizes</td>
</tr>
<tr>
<td><strong>Normalization</strong></td>
<td></td>
</tr>
<tr>
<td>Wage per period w</td>
<td>Normalize monthly wage to one.</td>
</tr>
<tr>
<td>Total industry revenues R</td>
<td>Size of market: Set R=100,000.</td>
</tr>
<tr>
<td><strong>Resource accumulation</strong></td>
<td></td>
</tr>
<tr>
<td>Mean depletion if do not invest in resources r_1 or r_2 γ</td>
<td>Mean -3% decrease per year (equivalent to -1.0% decrease per quarter, with one period corresponding to a quarter)</td>
</tr>
<tr>
<td>Mean increase if invest in resource η</td>
<td>Vary based on scenario: Mean +10% or 19% increase per year (equivalent to +2.5% or 4.5% increase per quarter, with one period corresponding to a quarter)</td>
</tr>
<tr>
<td>Cost of investment I_r</td>
<td>The cost of investment = p_r(η-γ)+I(r). Set p_r=15, and J(r)=exp(η), for each resource. The J(r) is chosen to ensure that the largest firms chose not to invest, thus limiting firm size due to diminishing returns.</td>
</tr>
<tr>
<td><strong>Production</strong></td>
<td></td>
</tr>
<tr>
<td>Elasticity of substitution of resources α</td>
<td>Vary based on scenario: between 2 and 5</td>
</tr>
<tr>
<td>Fixed costs F</td>
<td>Set to 12</td>
</tr>
<tr>
<td><strong>Demand</strong></td>
<td></td>
</tr>
<tr>
<td>Elasticity of substitution σ</td>
<td>Set to 4</td>
</tr>
<tr>
<td><strong>Exit</strong></td>
<td></td>
</tr>
<tr>
<td>Death shock δ</td>
<td>Set to 10% per year, thus 2.5% per quarter (i.e., per time period)</td>
</tr>
<tr>
<td><strong>Entry</strong></td>
<td></td>
</tr>
<tr>
<td>Entrant size</td>
<td>Set entrants as distributed with independent draws on resource r_1 and r_2, with draw lognormal, with mean log(1) and std dev 0.5. This results in entrants with, relative to incumbents, lower average resource levels, and higher exit rates.</td>
</tr>
<tr>
<td>Entry sunk cost S</td>
<td>Set S=60, which corresponds to a quarterly interest charge of 0.6 (i.e., around 6% of per period fixed costs).</td>
</tr>
</tbody>
</table>

Sources: Bartelsman, Scarpetta and Schivardi (2003), Cooley and Quadrini (2001), and Olley and Pakes (1996)
(a) Dependence of choice of investment to become generalist versus specialist on resource substitutability for firms with symmetric resource configurations \((r,r)\)

Note: Line \(\alpha_r = 1 + (\sigma - 1) \ln(2)/\ln(1 + I(r)/A)\) separating investment regions illustrative, as specific shape depends on functional form of \(I(r)\) and value of \(A\) and \(\sigma\). Assumptions of \(I(1) = 0\) and \(I'(r) < 0\) result in \(\alpha_r\) increasing as \((r,r)\) increases from \((0,0)\) to \((1,1)\).

(b) Investment choice for firms initially with symmetric resource configurations in response to resource substitutability \(\alpha > \alpha_0\)

Figure 1: Illustration of investment choice for firms with symmetric resource configurations (a) at different levels of resource substitutability, and (b) across firms at a given level of resource substitutability
Entry Entrants pay sunk cost of entry and then discover their initial resource levels. Entrants thereon are indistinguishable from incumbent firms with the same resource levels.

Continuation / Exit decision Firms decide if to continue in industry or exit

Production decision Firms hire labor for production and generate current period profits

Investment decision Firms decide whether or not to invest to increase resource 1 or resource 2

Transition to next period Uncertainty resolved about transition to next period: death shock for all firms; and resource transitions

Figure 2: Timing and description of events within time periods
Potential growth paths:

For ease of presentation, in (a)(i) the policy regions are labelled, whereas (b)(i) has an example of a firm growth path. In (b)(i) a small firm close to exit boundary grows by accumulating first the resource it has least of (resource 1 in the figure), and then growing both resource stocks (by alternating growth of resource 1 and 2 over successive time periods).

Figure 3: Policy choices and distribution of firms across firm resource stock levels in stationary equilibrium, at lower resource substitutability and at higher and lower resource growth rate.
(a) Higher resource substitutability ($\alpha=5$), and lower resource growth rate ($\eta=10\%$ per year)

(i) Firm policies

<table>
<thead>
<tr>
<th></th>
<th>Invest in resource 1</th>
<th>Not invest</th>
<th>Invest in resource 2</th>
<th>Not invest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exit</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

(ii) Distribution of firms

For ease of presentation, in (a)(i) the policy regions are labelled, whereas (b)(i) has an example of a firm growth path. In (b)(i) small firms close to exit boundary grow along two potential paths. In both cases, the firm first accumulates the resource it has most of, increasing specialization (resource 2 in the figure). The second phase depends on the other resource stock levels (resource 1 in the figure): if low, then the firm does not invest and remains a specialist; if high then the firm invests to grow this resource so as to become a generalist.

(b) Higher resource substitutability ($\alpha=5$), and lower resource growth rate ($\eta=19\%$ per year)

(i) Firm policies

(ii) Distribution of firms

Figure 4: Policy choices and distribution of firms across firm resource stock levels in stationary equilibrium, at higher resource substitutability and at higher and lower resource growth rate.
Figure 5: Distribution of entrants in stationary equilibrium at lower and higher resource substitutability (and at lower resource growth rate).
(a) Anticipated gradual change in resource substitutability:
Announced at $t=1$ and gradually taking effect from $t=10$ to $t=18$

(b) Anticipated gradual change in growth of resource from investment:
Announced at $t=1$ and gradually taking effect from $t=10$ to $t=18$

Figure 6: Timing of shock to resource substitutability and resource accumulation
Figure 7: Transition due to increase in resource substitutability. Shock announced at t=1 and gradually takes effect from t=10 to t=18. Policy choices and distribution of firms across firm resource stock levels shown for selected time periods: just after announcement (t=2), just before shock takes effect (t=10), while shock takes effect (t=14), and after shock (t=28).
Figure 8: Transition due to increase in resource substitutability (announced at t=1 and gradually taking effect from t=10 to t=18). Evolution of selected variables during transition.
Figure 9: Transition due to decrease in resource substitutability. Shock announced at $t=1$ and gradually takes effect from $t=10$ to $t=18$. Policy choices and distribution of firms across firm resource stock levels shown for selected time periods: just after announcement ($t=2$), just before shock takes effect ($t=10$), while shock takes effect ($t=14$), and after shock ($t=28$).
Figure 10: Transition due to decrease in resource substitutability (announced at t=1 and gradually taking effect from t=10 to t=18). Evolution of selected variables during transition.
Shock decreases resource growth rate (from $\eta=19\%$ to $\eta=10\%$ per year)

Shock increases resource growth rate (from $\eta=10\%$ to $\eta=19\%$ per year)

Figure 11: Entry and exit rates during transition in response to shock to resource accumulation
Appendix

A Equilibrium Conditions

Let $\mu_{r_1,r_2,t}$ represent the measure function for producing firms over states $(r_1,r_2)$ in period $t$. This function summarizes all information on the distribution of producing firms across resource levels, as well as the total mass of producing firms in state $(r_1,r_2)$, $M_{v,z,c,t} = \mu_{r_1,r_2,t}(\Upsilon)$. A dynamic equilibrium is characterized by a time path for the price index $\{P_t\}$, the measure of firms in each state, $\{\mu_{r_1,r_2,t}\}$, and the mass of entrants $\{M_{E,t}\}$. Note that a choice of $\{P_t\}$ uniquely determines the time path for $\{V^C_t(r_1,r_2)\}$ and thus determines all the optimal choices for any firm, given its resource levels $(r_1,r_2)$. An equilibrium $\{P_t\}, \{\mu_{r_1,r_2,t}\},$ and $\{M_{E,t}\}$ must then satisfy the following three conditions:

Firm Value Maximization All firms’ choices for exit/continuation, and, if continuing, for investment, conditional on $r_1$ and $r_2$, must satisfy (3) and (4). In the aggregate, this means that $\mu_{r_1,r_2,t}$ is entirely determined by $\mu_{r_1,r_2,t-1}$ and the choices for $\{P_t\}$ and $\{M_{E,t}\}$. Starting with a mass and distribution of firms at time $t-1$, a share $\delta$ of firms receive the exogenous death shock. The remaining $(1-\delta)$ share of firms update resources based on choice of investment. To these firms are added the $M_{E,t}$ new entrants, with a distribution determined by $G_E(r_1,r_2)$. All firms then make their endogenous exit decisions. The remaining firms result in a distribution and mass of firms for every state. In equilibrium this must match the chosen $\mu_{r_1,r_2,t}$.

Free Entry In equilibrium, the net value of entry $V^E_t$ must be non-positive, since there is an unbounded pool of prospective entrants and entry is not limited beyond the sunk entry cost. Furthermore, entry must be zero whenever $V^E_t$ is negative.

Aggregate Industry Accounting The mass and distribution of firms over productivity levels (aggregating over states) implies a mass and distribution of prices (applying the profit maximizing markup rule to firm marginal cost). Aggregating these prices into the C.E.S. demand system price index must yield the chosen $P_t$ in every period.
B Model Algorithm

Following I describe the algorithm for numerically solving the model, focusing on the equilibrium conditions required and the sequence of calculations performed. The demand structure leads to monopolistic competition. In particular, this means that each firm in each time period $t$ need only know industry aggregate outcomes for industry price $P$ from time $t$ onwards, $\{P_t, ..., P_T\}$, to determine its specific policies conditional on its current resources $(r_1, r_2)$. Firm policy choices are whether to $\{Continue, Exit\}$, and, if continuing, whether or not to invest in one of the resources.

The algorithm comprise three steps. Step 1 is to set parameters. Step 2 is to compute the firm policies and firm-size distribution $\{\mu_{r_1, r_2, t}\}$ corresponding to the initial parameter values, the initial stationary state equilibrium at $t = 1$. Within Step 2, there is an iteration over the aggregate price for the stationary state $P_1$. Step 3 computes the firm policies and firm-size distribution for the evolution from the initial stationary state through to period $T$. Within the Step 3, there is an iteration over the price path $\{P_2, ..., P_T\}$.

1) Set initial parameters, including for industry characteristics and grid structure.
2) $P_1$ iteration:

- Choose candidate value for $P_1$.

- Firm Value and Policy Iteration:

  - Compute profit $\pi(r_1, r_2)$ at each resource combination $\{r_1, r_2\}$, based on the specific demand system and production function chosen.
  
  - Pick a candidate value function $V_1(r_1, r_2)$.

  - Determine $\{Continuation/Exit\}$ and choice of investment at each $\{r_1, r_2\}$.

  - Iterate the value function: The set of firm policies over continuation and choice of policy state imply a next iteration value for the value function, $V_1'(r_1, r_2)$, based on computing the value of continuing and comparing to the value of exit.

  - Check whether new $V_1'(r_1, r_2)$ is sufficiently close to $V_1(r_1, r_2)$. If not, continue iteration with $V_1'(r_1, r_2)$. If close enough, return to $P_1$ iteration.

- Check the value of entry. As seek equilibria with positive entry the condition should be close to zero. Compute firm-size distribution $\mu_{r_1, r_2, 1}$.
– If close enough to zero, $P_1$ iteration is complete.
– If not, then adjust candidate $P_1$ accordingly: if condition is positive lower $P_1$, if negative raise $P_1$.

3) $\{P_2, ..., P_T\}$ iteration:

• Choose candidate value for $\{P_2, ..., P_T\}$.

– Compute price corresponding to stationary state at final parameter values.
– Set initial guess for $\{P_2, ..., P_T\}$ based on prices corresponding to initial and final parameter values.

• Firm value and policy iteration

– Firm Value and Policy Iteration for $t = T$:

  * Compute profit $\pi(r_1, r_2)$ at each resource combination $\{r_1, r_2\}$, based on the specific demand system and production function chosen.
  * Pick a candidate value function $V_T(r_1, r_2)$.
  * Determine $\{Continuation/Exit\}$ and choice of investment at each $\{r_1, r_2\}$.
  * Iterate the value function: The set of firm policies over continuation and choice of policy state imply a next iteration value for the value function, $V_T'(r_1, r_2)$, based on computing the value of continuing and comparing to the value of exit.
  * Check whether new $V_T'(r_1, r_2)$ is sufficiently close to $V_T(r_1, r_2)$. If close enough, return to $\{P_2, ..., P_T\}$ iteration. If not, continue iteration with $V_T'(r_1, r_2)$.

– Firm Value and Policy Iteration for $t = \{2, ..., T - 1\}$:

  * Compute profit $\pi(r_1, r_2)$ at each resource combination $\{r_1, r_2\}$, based on the specific demand system and production function chosen.
  * Iterate back to compute $V_{T-1}(r_1, r_2)$ based on $\pi_t(r_1, r_2)$ and $V_T'(r_1, r_2)$, and period T policies, based on computing the value of continuing and comparing to the value of exit. Hence, determine period $T - 1$ policies $\{Continuation/Exit\}$ and choice of investment at each $\{r_1, r_2\}$.
  * Iterate back to period $t = 2$.  

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- Compute value of entry in each time period $t = \{2, ..., T - 1\}$.
- Compute the size-distribution of firms $\mu_{r_1, r_2} = \{\mu_{r_1, r_2, 2}, ..., \mu_{r_1, r_2, T}\}$ consistent with the computed firm policies.
  - Compute $\mu_{r_1, r_2, 2}$ based on $\mu_{r_1, r_2, 1}$ and firm policies computed for $t = 2$.
  - Determine number of entrants:
    * If value of entry negative for $t = 2$, set entry to zero.
    * If value of entry is non-negative, set entry such that:
      - Case 1: If the distribution of incumbents implies a price below $P_2$ then entry is zero, as adding entrants would further distance the firm distribution from the current value of price path
      - Case 2: If the distribution of incumbents implies a price above $P_2$, then add entrants until the firm distribution (including entrants) implies a price equal to $P_2$
  - Iterate forward to compute $\mu_{r_1, r_2} = \{\mu_{r_1, r_2, 3}, ..., \mu_{r_1, r_2, T}\}$.
- Check whether price path $\{P_2, ..., P_T\}$ is close enough to an equilibrium:
  - Objective function to assess equilibrium comprised of two parts:
    * The first part measures the distance between the price path and firm distribution: $(P_{\text{max}} - P)$
    * The second part measures an equivalent gap based on the value of entry: $(P^{fe} - P)$.
      - This is zero if value of entry is negative (to capture instances when this value is close to zero but negative, I consider this to be zero if value of entry/sunk cost of entry is larger than $-10^{-4}$).
      - This is negative if the value of entry is positive. I calculate $P^{fe}$ as what the price in the time period in question would need to change to in order to close part of the gap in value of free entry. Hence, if value of entry is positive the price change is negative so as to lower profitability and thus lower the value of entry. The adjustment is moderated by the extent to which price adjustments for future periods (which have been determined as the algorithm work backs through time periods) are for increases or decreases in prices.

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The objective function is then the Euclidian distance of these two measures: 
\[ ((P_{\text{max}} - P)^2 + (P_{fe} - P)^2)^{\frac{1}{2}} \]

- If objective function not sufficiently small, construct new candidate price path. The suggested price adjustment is the average of \((P_{\text{max}} - P)\) and \((P_{fe} - P)\). The actual price adjustment is only part of the suggested price adjustment, to reduce the risk of cycling over successive iterations of the price path.