Aggregate Demand Externalities, Intermediate Inputs and Multiple Equilibria

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Abstract: We show that the presence of intermediate inputs magnifies the multiplier effects of pecuniary aggregate demand externalities in an imperfectly competitive model. In our setting, the presence of intermediate inputs is a necessary condition for the existence of multiple equilibria. Moreover, the range of parameter values for which multiple equilibria are possible is increasing in the share of intermediate inputs.

I. INTRODUCTION

Models with complementarities which lead to multiplier effects and multiple equilibria are used in Macroeconomics to generate large output fluctuations in the absence of large exogenous shocks.[1] Murphy, Shleifer and Vishny (1989) (MSV hereafter) develop a model where complementarities arise from the existence of a pecuniary aggregate demand externality. This externality generates strategic complementarities between firms' decisions; when a firm invests it increases aggregate demand which makes other firm's investment more profitable. The presence of this externality, however, is not enough to generate more than one equilibrium. MSV show that in a model where firms have to decide on a single investment project in the presence of imperfect competition, the equilibrium is always unique. Additional features (such as a wage premium for workers involved in the investment project) need to be introduced to make the strategic complementarity strong enough to generate two equilibria. The need for additional mechanisms raises some questions about the generality and plausibility of the assumptions used to generate the multiplicity of equilibria.

Our paper presents a modification to the basic model of MSV that leads to multiple equilibria. The innovation here is the introduction of intermediate inputs in the production function. We show that if investment is partially done through the purchase of intermediate inputs, then the economy can have two equilibria. More interestingly, the size of the aggregate demand externality becomes larger as the role of intermediate inputs becomes more important. The range of parameter values for which multiple equilibria exist is an increasing function of the share of intermediate inputs. In this sense, this paper confirms an old hypothesis that a

[1] See Matsuyama for a recent survey of this literature.

larger degree of complexity in the economy (in the sense of more interdependence among industries) can lead to larger output fluctuations.[2]

Section II presents the model, Section III solves for the equilibrium and provides the main results of the paper and Section IV concludes.

II. THE MODEL

Our one-period model is based on Murphy, Shleifer and Vishny (1989). There is a representative consumer who owns all claims to profits in the economy, and maximizes

$$U = \int_0^1 ln(x_i) \ di \tag{1}$$

where x_i represents consumption of good i. There is a continuum of sectors normalized on the unit interval. Labor supply is assumed to be inelastic and equal to N.

From the consumer's optimization we obtain identical consumption shares across sectors and unit elastic demand. Define y_i as the expenditure in each sector, then the budget constraint is equal to

$$\int_0^1 p_i x_i = \int_0^1 y_i = Y = \Pi + W \tag{2}$$

Y is aggregate income (expenditure), Π is aggregate profits and W is total wages. Given our normalization, expenditure in each sector, y_i, will be equal to Y.

The market structure is composed of a single monopolist in each sector and a competitive fringe of firms. Competitive firms have access to a CRS technology: $q = n$, where q and n represent production and labor. Thus, the competitive fringe can produce at a marginal cost of 1. The monopolists have access to two possible technologies. They are 'endowed' with a production function

$$q = \lambda_0 n \tag{3}$$

where $\lambda_0 > 1$. Therefore, the monopoly firm can use a technology with lower marginal cost than the competitive fringe. Monopoly firms are also able to im-

[2] See, for example, Means (1935) or Gordon (1990). Rotemberg and Woodford (1995) present calibrations of imperfectly competitive models where the share of intermediate inputs has a significant effect on the magnitude of business cycles. Also, Basu (1995) discusses some implications of intermediate goods for welfare and productivity.

prove upon this production function by paying a fixed cost. This new technology improves the production function to

$$q = \lambda_1 n$$

with $\lambda_1 > \lambda_0$. To upgrade the technology, the monopolist has to combine intermediate inputs and labor in fixed proportions. More precisely, the firm has to purchase $s_m \overline{I}$ units of a composite good and $(1 - s_m)\overline{I}$ units of labor; where $0 < s_m < 1$ is the share of intermediate inputs in the production of the IRS technology and $\overline{I}$ represents the scale of the project.[3] The composite good is made up from all goods according to the aggregator[4]

$$I = \exp\left(\int_0^1 ln(I_i)\ di\right)$$

III. EQUILIBRIUM

We normalize the wage to 1 and assume Bertrand competition. Monopoly firms will never set a price above 1 as they would lose all of the market to the competitive fringe. Also, since both consumers' and intermediate inputs' demand are unit elastic, monopoly firms have no incentive to lower their price below 1. Hence, the price in each sector is 1 and the monopoly firms capture all of the market.

We use the term 'invest' to refer to a firm which upgrades its technology. Firms which decide not to invest have profits equal to

$$\pi_0 = q - \frac{q}{\lambda_0} = a_0 q \tag{4}$$

and firms which decide to invest have profits equal to

$$\pi_1 = q - \frac{q}{\lambda_1} - \overline{I} = a_1 q - \overline{I} \tag{5}$$

where $a_i \equiv (\lambda_i - 1)/\lambda_i$ and the subscripts 0 and 1 refer to the standard and superior technology.

[3] The case $s_m = 0$ is the one analyzed in MSV.

[4] By using a logarithmic function to aggregate intermediate inputs we ensure that the intermediate input demand is also unit elastic.

To solve for the general equilibrium we first solve for aggregate demand. Total demand includes consumption demand and demand for intermediate goods. Total demand has to be equal to gross production, Q. Consumer demand will be given by aggregate income, Y, which is identically equal to wages plus profits. Intermediate inputs' demand will be equal to $\mu s_m \overline{I}$ where μ represents the fraction of monopoly firms investing in the new technology. We then have that

$$Q = Y + \mu s_m \overline{I} = \Pi + N + \mu s_m I$$

Therefore, gross production is equal to

$$Q = \frac{N - \mu(1 - s_m)\overline{I}}{1 - \mu a_1 - (1 - \mu)a_0} \tag{6}$$

We can now solve (4), (5) and (6) for profits in terms of only N and $\overline{I}$. Firms which invest have profits equal to

$$\pi_1 = \frac{a_1\left[N - \mu(1 - s_m)\overline{I}\right]}{1 - \mu a_1 - (1 - \mu)a_0} - \overline{I} \tag{7}$$

Firms which elect not to invest have profits equal to

$$\pi_0 = \frac{a_0\left[N - \mu(1 - s_m)\overline{I}\right]}{1 - \mu a_1 - (1 - \mu)a_0} \tag{8}$$

The nature of the externality can be studied by examining equations (7) and (8). In both equations, the denominator is a function of μ. When a firm decides to invest, it reduces the other firms' denominators, thus increasing profits in both equations. If we subtract (8) from (7) we obtain an expression for the incentive to invest of a single firm as a function of μ, the proportion of firms that have decided to invest. Let $P(\mu)$ be this function.

$$P(\mu) = \pi_1 - \pi_0 = \frac{(a_1 - a_0)N + \mu(a_1 - a_0)\, s_m \overline{I} - (1 - a_0)\overline{I}}{1 - \mu a_1 - (1 - \mu)a_0} \tag{9}$$

From expression (9) we can characterize the symmetric equilibria of the model. $\mu = 0$ is an equilibrium when $P(0) < 0$; $\mu = 1$ is an equilibrium when $P(1) > 0$. We are interested in the conditions under which both equilibria are possible. That is, if no firm invests, any individual firm does not find investing profitable, but if all other firms invest then it is profitable for an individual firm to invest.

RESULT 1: *If intermediate inputs are not used ($s_m = 0$) there exists a unique equilibrium.*

PROOF: In the case $s_m = 0$ (the case of MSV), expression (9) reduces to

$$P(\mu) = \frac{(a_1 - a_0)N - (1 - a_0)\overline{I}}{1 - \mu a_1 - (1 - \mu)a_0} \tag{10}$$

This expression is increasing in μ but its sign is only a function of the other parameters. Either the expression is always positive, in which case the only equilibrium is all firms investing, or it is always negative, in which case the only equilibrium is no firm investing. This occurs because a firm's spillover is positive only if its own profits are positive. Therefore, when the aggregate demand externality solely originates in profits, there cannot be more than one equilibrium.[5]

RESULT 2: *When $s_m > 0$, multiple equilibria are possible. Moreover, the range of values for which both equilbria exist is increasing in the share of intermediate inputs, s_m.*

PROOF: In the case where intermediate inputs enter the production function ($s_m > 0$), expression (9) includes an extra term in the numerator that is an increasing function of μ. It is then possible that $P(\mu)$ is negative for $\mu = 0$, but that it becomes positive for the value $\mu = 1$. If that is the case, the economy will have two equilibria: one in which all firms invest in the superior technology, and a second one in which no firm invests. The intuition is that, in this case, the spillover is directly coming from the purchase of intermediate inputs and not only through profits. Therefore, the previous link between the sign of individual profits and the sign of the effect on aggregate demand is absent. When a firm decides to invest, it could increase demand for other firms even when the profits resulting from the investment project are negative.

[5] This result is not specific to this model where only one investment project is considered. One can easily generalize the model allowing for firms to choose the size of the project and the result still holds.

We can also look at the range of parameter values for which multiple equilibria are possible in terms of the ratio $\overline{I}/N$. This range is equal to

$$\frac{a_1 - a_0}{1 - a_0} < \frac{\overline{I}}{N} < \frac{a_1 - a_0}{1 - a_0 - (a_1 - a_0)s_m}$$

This range is empty for $s_m = 0$ but it exists, and is increasing in s_m, for $s_m > 0$. Therefore the possibility of multiple equilibria increases with the importance of intermediate inputs. Therefore, if multiple equilibria and coordination failures are an important source of output fluctuations, an economy with a larger degree of interdependence across sectors will be more likely to experience large business cycle fluctuations.

IV. CONCLUSION

Complementarities are frequently used in the business cycle literature to generate multiplier effects. The existence of complementarities implies that economic agents' decisions are mutually reinforced which serves as an amplification mechanism to exogenous shocks. When complementarities are large enough, there exists the possibility of multiple equilibria. In this case, fluctuations can occur even in the absence of exogenous shocks. The relevance of these models hinges upon how realistic the mechanisms that generate complementarities are. Among the many mechanisms that have been proposed to create complementarities, pecuniary aggregate demand externalities are probably one of the most simple and intuitive ones; when firms expand production they increase aggregate income and, thus, demand for other firms. However, the work of Murphy, Shleifer and Vishny (1989) shows that this type of externality in a simple imperfectly competitive model cannot lead to multiple equilibria unless other features are added to reinforce the size of the complementarity. Our paper has presented a natural extension to their basic model which leads to the existence of multiple equilibria. The innovation here is the introduction of intermediate inputs. We have shown that the presence of intermediate inputs can create more than one equilibrium. Moreover, the range of parameter values for which multiple equilibria exist is increasing in the share of intermediate inputs. This result stresses the importance of demand linkages across sectors in shaping the magnitude of output fluctuations. Economies that show a greater interdependence across sectors are more likely to exhibit multiple equilibria and therefore are more likely to suffer coordination failures which could lead to large output fluctuations.

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