1. The mean dollar amount of sales per customer at a toy store located at an airlines terminal from a sample of 36 customers is equal to $20 with a standard deviation of $3.20.

   a) What is the range within which the mean dollar amount of sales of the store will fall with a probability of 99%?

   \[
   n = 36 \quad \bar{x} = 20 \quad S = 3.20
   \]

   for 99% confidence level, \( Z_{\alpha/2} = 2.58 \)

   \[
   \bar{x} - E \leq \mu \leq \bar{x} + E
   \]

   \[
   E = Z_{\alpha/2} \frac{S}{\sqrt{n}} = 2.58 \times \frac{3.2}{\sqrt{36}} = 1.38
   \]

   \[
   20 - 1.38 \leq \mu \leq 20 + 1.38
   \]

   \[
   18.62 \leq \mu \leq 21.38
   \]

   b) What size a random sample should be collected as a minimum if you want to estimate the mean sales amount within $1 with the same level of confidence?

      Maximum error \( E = 1 \)

      Assume \( S \) will remain the same

      \[
      E = Z_{\alpha/2} \frac{S}{\sqrt{n}}
      \]

      \[
      n = \left[ \frac{Z_{\alpha/2}S^2}{E} \right]^2 = \left[ \frac{2.58 \times 3.2}{1} \right]^2 = 68.2
      \]

      \[
      n = 69
      \]
2. A poll for the weekly *Journal du Dimanche* (February 19, 1995) reported that the percentage of the eligible voters who were favorable of Edouard Balladur was 46% with a margin of error (half the width of a 95% confidence interval) at 2.2%. What must have been the sample size in the poll?

\[
E = Z_{\alpha/2} \sqrt{\frac{x(1-x)}{n}} = 0.022
\]

\[
= 1.96 \sqrt{\frac{.46 \times .54}{n}} = 0.022
\]

\[
1.96^2 \times \frac{.46 \times .54}{n} = .022^2
\]

\[
n = \frac{1.96^2 \times .46 \times .54}{.022^2}
\]

\[
= 1972
\]
3. Mark Semmes, owner of the Aurora Restaurant, is considering purchasing new furniture. To help him decide on the amount he can afford to invest in tables and chairs, he wishes to determine the average revenue per customer. The checks for 9 randomly sampled customers had an average of £18.30 and a standard deviation of £6.30.

a) Construct a 95 percent confidence interval for the average check per customer.

\[ n = 9 \quad \bar{X} = 18.3 \quad S = 6.3 \]

As \( n < 30 \) assume population follows a normal distribution. And estimate \( \sigma \) by \( S \) so use a \( t \) distribution

\[ \bar{X} - E \leq \mu \leq \bar{X} + E \]

\[ E = t_{0.025,9} \times \frac{S}{\sqrt{n}} \]

\[ t_{0.025,9} = 2.306 \]

\[ E = 2.306 \times \frac{6.3}{\sqrt{9}} = 4.84 \]

\[ 18.3 - 4.84 \leq \mu \leq 18.3 + 4.84 \]

\[ 13.46 \leq \mu \leq 23.14 \]

b) What additional sample size will be required if Mr Semmes will like the maximum error in his estimation to not exceed £1, at the same level of confidence?

Assume \( n \) will be large enough to use a \( Z \) distribution and \( S \) is still a good estimate of \( \sigma \). At a 95% confidence level

\[ n = \left[ \frac{Z_{0.025} \times S}{E} \right]^2 = \left[ \frac{1.96 \times 6.3}{1} \right]^2 \]

\[ n = 152.47 \cong 153 \]

So the use of \( Z \) is justified

Take an additional sample of \( 153 - 9 = 144 \)
4. Calling Statman? Gillette recently announced plans to launch a new 3-edged razor, called the MACH3. The company spent 6 years and more than $750 million developing this razor. The new razor will sell for approximately $5 with replacement cartridges selling for $1.35 each, a 35% premium over Gillette’s flagship two-blade Sensor razor. The Wall Street Journal (April 14, 1998) calls the product a “calculated gamble” and notes that “Gillette’s master plan has little room for error”.

a) Because of the great secrecy surrounding the development of the razor, Gillette has not done extensive market research. In fact, they have tested the new razor on a sample of only 100 members of the target population (“men who shave”), each of whom signed a non-disclosure agreement. In this sample, 65 indicated that they prefer the MACH3 to the Sensor razor. Use this sample data to construct a 90% confidence interval for the proportion of “men who shave” who prefer the MACH3 to the Sensor.

In sample : \( n = 100 \)
\( x = 65 \)

Sample proportion : \( \frac{x}{n} = \frac{65}{100} = 0.65 \)

A 90% confidence interval for the population proportion is : \( \left[ \frac{x}{n} - E, \frac{x}{n} + E \right] \)

For \( \alpha = 10\% \) \[ E = Z_{\alpha/2} \sqrt{\frac{x (1-x)}{n}} \]

\[ = 1.645 \sqrt{\frac{0.65 \times 0.35}{100}} = 0.078 \]

A 90% confidence interval for the proportion of “men who shave” who prefer the MACH3 is \( [0.65-0.078, 0.65+0.078] = [0.572, 0.728] \)

That is : \( [0.572, 0.728] \)

b) Now that the razor has been announced, Gillette wants to gather more information to improve the accuracy of its market share estimates. Suppose Gillette wants to survey enough “men who shave” to reduce the 90 percent confidence interval of part (a) to a width of \( \pm 2\% \). How many “men who shave” must they sample?

Assuming \( p = 0.5 \) (conservative)

\[ n = \frac{Z_{\alpha/2}^2}{4E^2} = \frac{1.645^2}{4 \times 0.02^2} = 1691.27 = 1692 \]
5. In a recent Gallop Poll of 1300 likely voters in the USA, 672 claimed support for the Democratic candidate.

a) Construct a 99% confidence interval for the proportion of all likely voters who support the Democratic candidate.

\[
\frac{x}{n} = \frac{672}{1300} = 0.517
\]

99% CI \( \Rightarrow Z_{\alpha/2} = 2.58 \)

\[
\frac{x}{n} - E \leq P \leq \frac{x}{n} + E
\]

\[
E = Z_{\alpha/2} \sqrt{\frac{x(1-x)}{n}} = 2.58 \sqrt{\frac{0.517 \times 0.483}{1300}} = 0.036
\]

\[
0.517 - 0.036 \leq P \leq 0.517 + 0.036
\]

\[
0.481 \leq P \leq 0.553
\]

b) What is the probability that the true proportion differs from the sample proportion by more than 0.01?

\[
|E| = 0.01
\]

\[
Z_{\alpha/2} \sqrt{\frac{x(1-x)}{n}} = 0.01
\]

\[
Z_{\alpha/2} \sqrt{\frac{0.517 \times 0.483}{1300}} = 0.01
\]

\[
Z_{\alpha/2} = \frac{0.01}{0.0139} = 0.722
\]

\[
\alpha / 2 \quad \text{(from the table)} = 0.2358
\]

\[
\alpha \quad \text{(for both sides)} = 2 \times 0.2358 = 0.4716
\]

The probability that the true proportion differs from the sample proportion by more than 0.01 is equal to 0.4716.
6. AKBAR and JEFF Co. hires new consultants only from the top business schools in the world. New recruits typically receive a signing bonus in addition to their salary.

   a) In a survey of 25 new offers from AKBAR and JEFF, the sample mean was 24,500 Euros, and the sample standard deviation was 5,200 euros, for the signing bonus. Construct an 80% confidence interval for the mean signing bonus at AKBAR and JEFF. What assumptions do you need to make?

   \[ \bar{X} = 24500 \]

   \[ S = 5200 \]

   \[ n < 30 \text{ so assume population of signing bonus is normally distributed} \]

   \[ n = 25, \sigma \text{ is not known, so } t \text{ distribution} \]

   at \( \alpha = 0.2 \) \( t_{df, \alpha/2} = t_{24,0.1} = 1.318 \)

   80% confidence interval is

   \[ E = t_{df, \alpha/2} \frac{S}{\sqrt{n}} = 1.318 \times \frac{5200}{\sqrt{25}} \]

   \[ = 1371 \]

   \[ \bar{X} - E \leq \mu \leq \bar{X} + E \]

   \[ 24500 - 1371 \leq \mu \leq 24500 + 1371 \]

   \[ 23129 \leq \mu \leq 25871 \]
b) For the next year, one would like to obtain the same width of the confidence interval as in Part (a) but with a confidence level of 99%. What is then the desired sample size for the survey? (What assumptions are you making now?)

Since the required $n$ will surely be bigger than 30, we can use the $Z$. We will assume that $S_x = 5200$ is an accurate estimate of $\sigma$. Then

$$E = Z_{\alpha/2} \frac{S_x}{\sqrt{n}}$$

$$1371 = 2.58 \frac{5200}{\sqrt{n}}$$

$$n = \frac{2.58^2 (5200^2)}{1371^2}$$

$$= 96$$

c) In a separate survey of 100 randomly selected graduates of top b-schools, 44 received a signing bonus and 56 did not. Construct a 90% confidence interval for the proportion receiving a signing bonus.

$$\frac{X}{n} \pm Z_{\alpha/2} \sqrt{\frac{X}{n} \left(1 - \frac{X}{n}\right)} \frac{1}{n}$$

$$0.44 \pm 1.645 \sqrt{\frac{0.44(0.56)}{100}}$$

$$0.44 \pm 1.645(0.0496)$$

$$\pm 0.08$$

$$0.36, 0.52$$
7. Using a sophisticated econometric model, and ISP, a financial analyst has concluded that the monthly returns of the Leima stock can be reasonably well approximated by a normal distribution with $\mu = 0.01$ and $\sigma = 0.03$. Furthermore, the financial analyst is very confident that the monthly returns will follow the same pattern in the foreseeable future, and they are independent from month to month.

a) Given that you invest in the stock for a period of 9 months, find the range within which the mean monthly return for these 9 months will fall with probability 0.95.

$$\mu - E \leq \bar{x} \leq \mu + E$$

$$E = Z_{a/2} \frac{\sigma}{\sqrt{n}} = 1.96 \times \frac{0.03}{\sqrt{9}} = 0.0196$$

$$0.01 - 0.0196 \leq \bar{x} \leq 0.01 + 0.0196$$

$$-0.0096 \leq \bar{x} \leq 0.0296$$

b) Suppose you have just become the top financial manager for Leima and you feel that you must maintain an investment in the Leima stock. What is the probability that your mean monthly return from Leima stock will be more than zero after 24 months?

$$\text{Prob}\left(\bar{x} > 0\right)$$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{0 - 0.01}{0.03/\sqrt{24}} = -1.63$$

$$\text{Prob}\left(Z > -1.63\right) = 0.9484$$

$$= 94.8\%$$
8. In performing an audit, accountants check invoices by drawing random samples for inspection. In one case, the accountants did not have any fraudulent invoices in their sample of 120 invoices. When the company failed, after the accountants had certified its financial statements, a creditor who had relied on the statements sued, claiming that the accountants had been negligent.

a) Suppose that the proportion of fraudulent invoices in the total population of invoices is 0.05. Further assume that the total number of invoices is very large, so that the probability of picking a fraudulent invoice in any one pick remains constant at 0.05. What is the probability that in a random sample of 120 invoices, there will be no fraudulent invoices?

\[ x = \text{number of fraudulent invoices in a sample of } n \text{ invoices} \]

\[ x = \text{Binomial with } n=120 \text{ and } p=0.05 \]

\[ np = 6 \text{ and } n(1-p) = 114 \]

Since \( np \geq 5 \) and \( n(1-p) \geq 5 \), we can approximate this binomial distribution by a Normal distribution with

\[ \mu = np = 6 \quad \text{and} \quad \sigma = \sqrt{npq} = \sqrt{120(0.05)(0.95)} = 2.38 \]

\[ P_{\text{Bin}}(x = 0) = P_{\text{normal}}(x \leq 0.5) = P\left( Z \leq \frac{0.5 - 6}{2.38} \right) \]

\[ = P(Z \leq -2.3) = 0.01 = 1\% \]

b) Suppose that the true proportion of fraudulent invoices, \( p \), is unknown. In a random sample of 500 invoices, there are 13 fraudulent invoices. Construct a 99% confidence interval for \( p \).

\[ n = 500 \]

\[ \frac{x}{n} = \text{sample proportion} = \frac{13}{500} = 0.026 \]

A 99% CI for \( p \):

\[ \frac{x}{n} \pm Z_{\alpha/2} \sqrt{\frac{x(1-x)}{n}} \]

\[ 0.026 \pm 2.58 \sqrt{\frac{0.026(0.974)}{500}} \]

\[ \Rightarrow 0.026 \pm 0.018 \quad \Rightarrow 0.008 \leq p \leq 0.044 \]
A market researcher for a large consumer electronics company wanted to study television viewing habits of residents of a particular small city. A random sample of 40 respondents was selected, and each respondent was instructed to keep a detailed record of all television viewing in a particular week. The results were as follows:

Amount of viewing per week: $\bar{X} = 15.3$ hours, $S = 3.8$ hours;
27 respondents watched the Evening News on at least three weeknights.

Set up 95% confidence-interval estimates for each of the following:

1. The average amount of television watched per week in this city.
2. The proportion of respondents who watch the Evening News at least three nights per week.

If the market researcher wanted to take another survey in a different city:

3. What sample size is required if you wish to be 95% confident of being correct to within ±1 hour and assume the population standard deviation is equal to 5 hours?
4. What sample size is needed if you wish to be 95% confident of being within ±.035 of the true proportion who watch the Evening News on at least three weeknights and no estimate is available based on past experience?

a) 95% confidence interval estimate of the average amount of television watched per week in the city

\[
\bar{X} = 15.3 \quad S = 3.8 \quad n = 40
\]

95% CI \hspace{2cm} \frac{Z_{\alpha/2}}{2} = 1.96

\[
E = \frac{Z_{\alpha/2} \times S}{\sqrt{n}} = 1.96 \times \frac{3.8}{\sqrt{40}} = 1.96 \times \frac{3.8}{6.32} = 1.18
\]

\[
15.3 - 1.2 \leq \mu \leq 15.3 + 1.2
\]

\[
14.1 \leq \mu \leq 16.5
\]

b) 95% confidence interval of the proportion of respondents who watch the evening news at least three nights per week

\[
X = 27 \quad n = 40 \quad \frac{X}{n} = \frac{27}{40} = 0.675
\]

\[
E = \frac{Z_{\alpha/2}}{\sqrt{n}} \sqrt{\frac{X(1 - X)}{n}} = 1.96 \sqrt{\frac{27 \times 13}{40 \times 40}} = 0.145
\]

\[
\frac{27}{40} - 0.145 \leq p \leq \frac{27}{40} + 0.145
\]

\[
0.675 - 0.145 \leq P \leq 0.675 + 0.145
\]

\[
0.53 \leq p \leq 0.82
\]
c) Sample size
\[ n = \left[ \frac{Z_{a/2} \sigma}{E} \right]^2 = \left[ \frac{1.96 \times 5}{4} \right]^2 = 96.04 \]
\[ \approx 97 \]

\[ n = \left[ \frac{Z_{a/2} \sigma}{E} \right]^2 = \left[ \frac{1.96}{0.035} \right]^2 = 784 \]