Web Appendix for "The Effect of Time-To-Build on Strategic Investment Under Uncertainty" by Goncalo Pacheco de Almeida and Peter Zemsky

Proof of Lemma 2 Suppose $\Delta = 0 \Leftrightarrow x_k \geq Z(x_k, x_{-k})$ for $k = 1, 2$. Then $x_k \geq Z(x_{-k})$ which means that $\partial\pi/\partial x_k = 0$ is a necessary condition for equilibrium. Note that $\partial\pi/\partial z_k \leq 0$ is also a necessary condition for equilibrium. Without loss of generality, consider $x_1 \leq x_2$, which implies that $\partial Z(x_2, x_1)/\partial x_1 = 0$. Then, $\partial\pi/\partial x_1 = a(D - 2x_1 - x_2) - c$ and $\partial\pi/\partial z_1 = a\delta^T(D - 2x_1 - x_2 - c) = \delta^T(\partial\pi/\partial x_1 + c(1-a)) > 0$, which is a contradiction. ■

Proof of Lemma 3 We consider in turn each of the three possible alternative pure strategy equilibrium to Delay, Incremental Cournot, Commit-delay, and Commit-incremental. 1) Both firms invest $x_1 = x_2 = \bar{z}$. Then $x_k = Z(x_k)$ and there is no ex post investment (Lemma 1), which is not possible in a pure strategy SPE (Lemma 2). 2) Both firms commit, x_1, $x_2 > \bar{z}$, and hence $x_1 > Z(x_2)$. This cannot be a SPE as an investment $x_1 = Z(x_2)$ yields strictly higher profits whether or not there is demand. 3) Recall that in an Incremental Cournot equilibrium the firms make the same ex ante investment. Thus, the third alternative is that x_1, $x_2 < \bar{z}$ but $x_1 \neq x_2$. However, with linear demand and symmetric costs, there is only a symmetric solution to the first-order conditions $\pi_1(x_k, x_{-k}) = 0$ for $k = 1, 2$. ■

Proof of Lemma A3 A Commit-delay equilibrium exists iff $\pi_1(0, x_{Ld}) \leq 0$ and $\pi(x_{Ld}, 0) \geq \max_{x \leq \bar{z}} \pi(x, 0)$. A Commit-incremental equilibrium exists iff $x_F > 0$ and $\pi(x_L, x_F) \geq \max_{x \leq \bar{z}} \pi(x, x_F)$. We have $\pi_1(0, x_{Ld}) \leq 0 \Leftrightarrow x_F \leq 0 \Leftrightarrow a \leq a_{di}$.

i) Suppose $T \in [0, \bar{T}]$. Let $N(T) = a_{di} - a_{DC}$. Since $N(\bar{T}) > 0$ and $N'(T) < 0$, $a_{di} < a_{DC}$. We have $a \geq a_D \Leftrightarrow \pi_1(0, 0) \leq 0 \Rightarrow \max_{x \leq \bar{z}} \pi(x, 0) = \pi(0, 0)$. Further, $a \geq a_{DC} \Leftrightarrow \pi(x_{Ld}, 0) \geq \pi(0, 0)$. Since $a_{DC} \geq a_D$ for $T \leq \bar{T}$ (see proof of Lemma A1), a Commit-delay equilibrium exists iff $a_{DC} \leq a \leq a_{di}$. Let $\hat{x}_i = \arg\max_{x \leq \bar{z}} \pi(x, x_F)$ and let $M_i(a) = \pi(x_{Li}, x_F) - \pi(\hat{x}_i, x_F)$. Note that $a > a_{di} \Rightarrow \hat{x}_i > 0$. A Commit-incremental equilibrium exists iff $a > a_{di}$ and $M_i(a) \geq 0$. Since $M_i(a)$ is a quadratic function of a with $M_i(0) < 0$ and $M_i(1) > 1$ and $M_i(a_{di}) = \pi(x_{Ld}, 0) - \pi(0, 0) \geq 0$, a Commit-incremental equilibrium exists iff $a > a_{di}$.

ii) and iii) Suppose $T > \bar{T}$. Let $\hat{x}_d = \arg\max_{x \leq \bar{z}} \pi(x, 0)$ and $M_d(a) = \pi(x_{Ld}, 0) - \pi(\hat{x}_d, 0)$. As with $M_i(a)$, we have $M_d(a)$ a quadratic function of a with $M_d(0) < 0$ and $M_d(1) > 1$. Since $M_i(a_{di}) = M_d(a_{di})$ there exists a unique $a_C \in (0, 1)$ such that a Commitment equilibrium exists iff $a \geq a_C$. Moreover, $M_d(a_{di}) = 0$ at $T = \hat{T} > \bar{T}$ and $a_{di} < a_C$ for $T > \hat{T}$. Hence the Commit-delay equilibrium does not exist for $T > \hat{T}$. Since $x_I > x_F$ and ex ante investments are strategic substitutes, $M_i(a_I) > 0$ and $M_d(a_I) > 0$ and hence $a_I > a_C$. Finally, note that $a_C \geq a_{DC}$ and that for $T > \bar{T}$ we have $a_{DC} > a_D$ and hence $a_C > a_D$. ■

Proof of Proposition 5 i) Consider a Commit-delay equilibrium. The result on market shares follows from $\partial x_{Ld}/\partial T = -(\ln \delta) \frac{(1-a)\delta^T c}{a\left(2-\delta^T\right)^2} > 0$ and the fact that the followers ex post investment is falling in x_{Ld}. Given this, $\partial \pi(0, x_{Ld})/\partial T < 0$. We now show that $\partial \pi(x_{Ld}, 0)/\partial T > 0$, which is sufficient for the difference in expected profits to be increasing. Let $\lambda = \delta^T$ so that $\pi(x_{Ld}, 0) = \frac{\left(\bar{D}(2-\lambda) - c(2-a\lambda)\right)^2}{8a(2-\lambda)}$. Then $\partial \pi(x_{Ld}, 0)/\partial \lambda = -\frac{1}{8}\left(2\bar{D} - \bar{D}\lambda - 2c + c\lambda a\right)(2\bar{D} - \bar{D}\lambda - 4ac + c\lambda a + 2c)/(a(2-\lambda)^2) < 0$ since $x_{Ld} > 0 \Rightarrow 2Da - Da\lambda - 2c + c\lambda a > 0$. Since $\partial \lambda / T < 0$, this establishes that $\partial \pi(x_{Ld}, 0)/\partial T > 0$.

ii) Consider a Commit-incremental equilibrium. We have $\partial x_{Li}/\partial T < 0$ and $\partial x_F/\partial T > 0$ (to see the latter note that x_F is a linear function of c and $\partial x_F/\partial T > 0$ for $c = 0$ and $c = \bar{D}$ while $c \in (0, \bar{D})$ by assumption). This established that the market shares in both periods II and III are getting more similar as T increases. In addition, it follows that $\partial \pi(x_{Li}, x_F)/\partial T < 0$. We now show that $\partial \pi(x_F, x_{Li})/\partial T > 0$ for a sufficiently close to 1, which is sufficient for the difference in expected profits to be decreasing in T. Let $\lambda = \delta^T$ and let $f(a) = \partial \pi(x_F, x_{Li})/\partial \lambda$. Then $f(1) = -\frac{1}{2}(D-c)^2(2-\lambda)/(3-\lambda)^3 < 0$. Further $f'(1) = -\frac{1}{2}(D-c)(D+c)(2-\lambda)/(3-\lambda)^3 < 0$ and $f''(a) = c^2 \frac{23+\lambda^3-17\lambda+\lambda^2}{2a^3(3-\lambda)^3(1-\lambda)^2} > 0$ and hence there exists some $\bar{a} < 1$ such that $\partial \pi(x_F, x_L)/\partial \lambda < 0 \Leftrightarrow \partial \pi(x_F, x_L)/\partial T > 0$ iff $a > \bar{a}$.

iii) We have $\partial a_{di}/\partial T = 2(\ln \delta)\left(3/\delta^T - 2\right)(\bar{D}/c - 1)/\left(3 - \delta^T\right)^2 < 0$. ■

Proof of Proposition 6 i) In a Delay equilibrium $a \leq a_D$ and hence $x_1 + x_2 = x^* = 0$. In a Incremental Cournot equilibrium $a > a_D$ and hence $x_1 + x_2 = 2x_I = \frac{2}{3}x^*$. In

both equililbria $z_1 + z_2 = 2\bar{z} = \frac{2}{3}z^*$. ii) Let $O_d(a) = x_{Ld} - x^*$ and $O_i(a) = x_{Li} - x^*$. Note that both $O_d(a) = 0$ and $O_i(a) = 0$ have the unique solution $a = a_{di}$. As $O_d(1) = -\frac{1}{2}(\bar{D} - c) < 0$ and $O_i(1) = -\frac{1}{2}\left(\bar{D} - c\right)(2 - \delta^T)/(3 - \delta^T) < 0$, we have $x_L + x_F = x_{Ld} \geq x^*$ for $a \leq a_{di}$, which is when a Commit-delay equilibrium exists, and conversely we have $x_L + x_F = x_{Li} + x_F < x^*$ for $a > a_{di}$, which is when a Commit-incremental equilibrium exists. ■

Proof of Proposition 7 i) Denote the social welfare in a Delay equilibrium by $W_D = W(0, 2\bar{z}) = \frac{2}{9a(1-\delta)}\delta^T\left(\bar{D} - ac\right)^2$. By inspection, $\partial W_D/\partial T < 0$, $\partial W_D/\partial a < 0$, $\partial W_D/\partial c < 0$, $\partial W_D/\partial \delta > 0$ and $\partial W_D/\partial \bar{D} > 0$. Suppose $a > a_D$, and denote the social welfare in an Incremental Cournot equilibrium by $W_I = W(2x_I, 2\bar{z}) = \frac{4}{9}W(x^*, z^*)$. By inspection, $\partial W_I/\partial T < 0$, $\partial W_I/\partial a < 0$, and $\partial W_I/\partial \bar{D} > 0$. We have $\partial W_I/\partial c = -\frac{\bar{D}}{a} + \frac{c}{a}\frac{1-a\delta^T(2-a)}{\left(1-\delta^T\right)}$, which is decreasing in a. As $\partial W_I/\partial c = -\bar{D} + c < 0$ at $a = a_D$ the limit value of a for which the equilibrium exists, $\partial W_I/\partial c < 0$.

Denote the social welfare in an Commit-incremental equilibrium by $W_{Ci} = W(x_{Li} + x_F, x_{Li} + Z(x_{Li})) = \frac{1}{8a(1-\delta)}\left(\left(\bar{D} - c\right)^2\frac{\left(8-3\delta^T\right)(4-\delta^T)}{\left(3-\delta^T\right)^2} + 3c^2\frac{\delta^T}{\left(1-\delta^T\right)}(1-a)^2\right)$. By inspection, $\partial W_{Ci}/\partial a < 0$ and $\partial W_{Ci}/\partial \bar{D} > 0$. We have $\partial W_{Ci}/\partial T < 0$ since for $f(\lambda) = \frac{(8-3\lambda)(4-\lambda)}{(3-\lambda)^2}$ we have $f'(\lambda) = 2(2-\lambda)/(3-\lambda)^3 > 0$. Finally, $\partial W_{Ci}/\partial c < 0$ because both ex ante investment and ex post investment are falling in c and both are less than the first best levels (Proposition 6, part (ii)).

Let $W = W(x_{Ld}, x_{Ld} + Z(x_{Ld}))$ denote welfare in a Commit-delay equilibrium. We will use the fact that W is a quadratic function of c. A Commit-delay equilibrium exists for $a \in [a_{DC}, a_{di}]$. Note that $a \leq a_{di} \Leftrightarrow c \geq c_1 = \bar{D}\frac{2-3\delta^T+\delta^{2T}}{2-3a\delta^T+a\delta^{2T}}$. We have $\partial W/\partial c < 0$ since $\partial W/\partial c < 0$ for $c \in \{0, \bar{D}\}$ and $\partial W^2/\partial^2 c > 0$. We have $\partial W/\partial \bar{D} > 0$ since $\partial W/\partial \bar{D} > 0$ for $c \in \{0, \bar{D}\}$ and $\frac{\partial W^2}{\partial c \partial \bar{D}} > 0$ for $c = c_1$. We have $\partial W/\partial a < 0$ since $\partial W/\partial \bar{D} > 0$ for $c \in \{c_1, \bar{D}\}$ and $\frac{\partial W^2}{\partial c \partial a} < 0$ for $c = c_1$. Let $\lambda = \delta^T$. We have $\partial W/\partial \lambda > 0$ since $\partial W/\partial \lambda > 0$ for $c \in \{0, \bar{D}, c_2\}$ where c_2 is defined by $\frac{\partial W^2}{\partial c \partial a} = 0$. Hence $\partial W/\partial T < 0$.

ii) We first compare welfare under Incremental Cournot to Commit-incremental. We have that $W_{Ci} > W_I$ because $2\bar{z} < x_{Li} + Z(x_{Li}) < 2z^*$ and $2x_I < x_{Li} + x_F < x^*$

with the last inequality following from Proposition 6 part (ii). We now compare welfare under Incremental Cournot to Commit-delay. Let $S(a) = (W_{Cd} - W_I)288a$. Note that $S(a)$ is quadratic in a and that a necessary for Commit-delay and Incremental Cournot to both exist is $a \in [a_D, a_{di}]$. $S(a_D) = \left(176 + 35\delta^{2T} - 140\delta^T\right) \left(\bar{D} - c\right)^2 / (\left(2 - \delta^T\right)^2 \delta^T) > 0$, $S(a_{di}) = 4\left(56\delta^T + 44 - 29\delta^{2T}\right)\left(\bar{D} - c\right)^2 / (\delta^T \left(3 - \delta^T\right)^2) > 0$ and $S'(a_{di}) = -4c\left(44 - 31\delta^T\right)(\bar{D} - c)/(3 - \delta^T) < 0$. Hence, $S(a) > 0$ for $a \in [a_D, a_{di}]$. Finally, we compare Delay and Commit-delay, which occurs when $a = a_{DC}$ and $T \in [0, \bar{T}]$. There is no global ranking as $W_{Cd} - W_D > 0$ for $T = 0$ and $W_{Cd} - W_D < 0$ for $T = \bar{T}$. ■

Proof of Proposition 8 i) In a Delay equilibrium we have $R = 1$ for all parameters. One can establish the comparative statics results from the following expressions. In Incremental Cournot,

$$R = \frac{a(1-a)}{(1-\delta^T)\bar{D}/c - \left(1 - a\delta^T - a + a^2\right)}$$

in Commit-incremental,

$$R = \left(1 + \frac{\bar{D}}{c}\frac{(4-g)(1-g)}{a(1-a)(3-g)} - \frac{4 + ag^2 - 3ag - 2g}{a(1-a)(3-g)}\right)^{-1}$$

and in Commit-delay

$$R = \frac{\left((2-\delta^T)\bar{D}/c + (2 + a\delta^T - 4a)\right)a}{\left(4 + 2a - \delta^T(2+a)\right)\bar{D}/c + a\left(2 + \delta^T(a+2)\right) - 4(1+a^2)}$$

ii) In an Incremental Cournot equilibrium $\lim_{T\to\infty} \frac{\partial R}{\partial a} = \frac{(1-2a)c\left(\bar{D}-c\right)}{\left(\bar{D}-c+ac(1-a)\right)^2}$ which is increasing in a for $a < 1/2$.

iii) For Delay $R = 1$ and for Commit-delay $R < 1$. R is increasing in $y_1 + y_2$ and decreasing in $x_1 + x_2$. Hence, to show that R is higher in an Incremental Cournot than in a Commitment equilibrium it is sufficient to show that $x_1 + x_2$ is lower and

$y_1 + y_2$ is higher. The difference in period I investment is

$$2x_I - x_{Li} - x_F = -\delta^T \frac{\left(1-\delta^T\right)\left(\bar{D} - ac\right) + 2c(1-a)}{6a\left(3-\delta^T\right)\left(1-\delta^T\right)} < 0$$

and the difference in period II investment is

$$2(\bar{z} - x_I) - (Z(x_{Li}) - x_F) = \frac{c\,(1-a)}{6\left(1-\delta^T\right)a} > 0.$$

Now suppose that $a_C < a < \min\{a_I, a_{di}\}$ so that there is both an Incremental Cournot and a Commit-delay equilibria exist. Note that the difference in period I investment $2x_I - x_{Ld}$ is decreasing in a while the difference in period II investment $2(\bar{z} - x_I) - Z(x_{Ld})$ is increasing in a. Hence, it is sufficient to show the result for $a = a_{di}$:

$$\begin{aligned}
(2x_I - x_{PL})|_{a=a_{di}} &= -\frac{(\bar{D} - ac)\delta^T}{6a} < 0 \\
2(\bar{z} - x_I) - Z(x_{PL})|_{a=a_{di}} &= \frac{\left(\bar{D} - ac\right)\left(2-\delta^T\right)}{12a} > 0.
\end{aligned}$$

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Proof of Proposition 9 i) If ex post investment is not possible, firm profits are given by $\dot{\pi}(x_k, x_{-k}) = ax_k(D - x_k - x_{-k}) - x_k c$ and social welfare is given by $\dot{W}(x) = au(x) - cx$. Let $\dot{x}^* = \arg\max_x \dot{W}(x)$. The unique Nash equilibrium investments are $x_1 = x_2 = \dot{x}^*/3$. If $a > a_D$ we have $\dot{x}^* = (\bar{D} - c)/a$. The value of the option to wait to society is then $V^S = W(0, 2\bar{z}) - \dot{W}(2\dot{x}^*/3)$ and the value to firms is $V^F = \pi(x_I, x_I) - \dot{\pi}(\dot{x}^*/3, \dot{x}^*/3)$. If $a \leq a_D$ we have $\dot{x}^* = 0$ and hence $\dot{W}(\dot{x}^*) = \dot{\pi}(\dot{x}^*/3, \dot{x}^*/3) = 0$. The option values are then $V^S = W(0, 2\bar{z})$ and $V^F = \pi(0, 0)$. The results in the proposition follow. ■

Proof of Proposition 10 i) We have that in Incremental Cournot:

$$\Delta = \frac{2(1-a)}{(\bar{D}/c + 2a)(1-\delta^T)}$$

in Commit-incremental:

$$\Delta = \frac{\left(3-\delta^T\right)(1-a)}{\left(\left(2-\delta^T\right)\bar{D}/c+1+a(3-\delta^T)\right)\left(1-\delta^T\right)},$$

in Commit-delay:

$$\Delta = 1 - \frac{4a(2-\delta^T)}{(2-\delta^T)\bar{D}/c+2+4a-3a\delta^T}.$$

The comparative statics follow. ii) Like R, Δ is increasing in y_1+y_2 and decreasing in x_1+x_2. Hence, the proof is the same as in Proposition 8 part iii). ■