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Setting price or quantity: Depends on what the seller is more uncertain about

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Abstract We consider a seller with uncertain demand for its product. If the demand curve were certain, then setting price and setting quantity would be equivalent ways to frame the seller's problem of choosing a profit-maximizing point on its demand curve. With uncertain demand, these become distinct sales mechanisms. We distinguish between uncertainty about the market size and uncertainty about the consumers' valuations. Our main results are that (i) for a given marginal cost, an increase in uncertainty about valuations favors setting quantity whereas an increase in uncertainty about market size favors setting price; (ii) keeping demand uncertainty fixed, there is a nonmonotonic relationship between marginal costs and the optimal selling mechanism (setting price or quantity); and (iii) in a bilateral monopoly channel setting, coordination occurs except for a conflict zone in which the retailer's choice of a selling mechanism deviates from the coordinated channel selling mechanism.

Keywords Demand uncertainty · Setting price · Setting quantity · Auctions · Posted prices

JEL Classification M31 · D81

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1 Introduction

For a single-product firm with no uncertainty about demand, the firm can equivalently frame its decision as a choice of price or as a choice of quantity—one variable uniquely determines the other according to the demand curve. However, this is no longer true if the seller does not perfectly control the situation, either because of other strategic players or because of having imperfect information about demand. Setting price or setting quantity captures the relative flexibility the firm retains for adjusting quantity or price:

1. A price-setting firm commits to a price and allows output to adjust, depending on the realization of demand or actions of other players.
2. A quantity-setting firm commits to an output level before the resolution of uncertainty and allows a market-clearing mechanism (such as auctions or promotions) to adjust the price.

Previous research established several conditions where the choice of setting price versus setting quantity is important. Roughly, this choice makes a difference if at least one of the following holds: (a) demand is uncertain, (b) there is strategic value to committing to price or quantity, or (c) there are transaction costs or operational considerations that affect the mechanisms differently.

The purpose of our paper is to understand better how the advantages of setting price versus setting quantity are related to the nature of demand uncertainty. Klemperer and Meyer (1986) is closest to our work, but they studied how this choice depends on the curvature of the cost curve and on the curvature of the demand curve, given a fixed one-dimensional (typically additive) shock to demand.[1] Here we study instead *how this choice depends on the relative mix of uncertainty about market size and uncertainty about valuations*, keeping fixed the curvature of the cost and demand curves.

For this purpose, we model demand uncertainty by introducing, for general demand curves, two concurrent and possibly correlated multiplicative shocks: one that represents uncertainty about market size and one that represents uncertainty about valuations (reservation values).[2] For example, in the case of linear demand, uncertainty about the distribution of valuations is represented by uncertainty about the price intercept; uncertainty about the size of the market is represented by uncertainty about the quantity intercept. We can then meaningfully define "an increase in uncertainty about market size" and "an increase in uncertainty about reservation values" as increasing risk in the distributions of these shocks. To isolate the impact of demand uncertainty, we

[1] An analogous exercise is conducted by Weitzman (1974) in the regulation context. He considers how the choice of regulating quantities or prices (Pigovian taxes), given uncertainty, depends on the curvature of the cost and benefit functions.

[2] This is similar to a distinction made by Marvel and Peck (1995), who derive the implications of the nature of uncertainty on the design of the returns policy for a product.

do not consider issues such as competition, transaction costs, and other factors that were studied previously.

Our main results are that

(i) for a given marginal cost, an increase in uncertainty about valuations favors setting quantity whereas an increase in uncertainty about market size favors setting price (Section 3);
(ii) keeping demand uncertainty fixed, there is a nonmonotonic relationship between marginal costs and the optimal selling mechanism (setting price or quantity; Section 5);
(iii) in a bilateral monopoly channel setting, channel coordination occurs except for a conflict zone in which the retailer's choice of a selling mechanism deviates from the coordinated channel selling mechanism (Section 6).

Each of these results depends on our two-dimensional parametrization of uncertainty, which combines and differentiates between the uncertainty about market size and the uncertainty about valuations that are typically present.

We also make methodological contributions. We define certainty-equivalent (CE) demand and inverse demand curves and show that the price-vs-quantity choice reduces to a choice among these demand curves. We thus decompose the analysis into (a) a conventional choice among demand curves without uncertainty and (b) a characterization of how shifts in uncertainty affect the CE demands. Furthermore, many treatments of demand uncertainty, such Klemperer and Meyer, treat demand and prices as if they could be negative, whereas we take the boundary conditions seriously. This is not a mere technicality since boundary conditions could easily become binding when there is uncertainty about demand.

1.1 Related literature

In addition to Klemperer and Meyer (1986) and Weitzman (1974), described above, there are two main subsets of the related literature on the choice of price versus quantity mechanisms.

Strategic interactions Ever since the works of Cournot and Bertrand, researchers have shown that strategic interaction creates a preference for sellers between setting price and quantity. Klemperer and Meyer (1986) consider competition under uncertain demand. Vives (1999) provides a comprehensive survey of the implications of issues such as product differentiation, asymmetric information and repeated interactions on the relative merits of setting price or quantity. Further on, Fershtman and Judd (1987) and Sklivas (1987) show that the presence of intermediaries leads not only to double marginalization but also to additional strategic effects. Miller and Pazgal (2001) show how this impacts the preferences of managers which in turn has implications for decision delegation.

Transaction costs in auctions Because the leading example of a quantity-setting mechanism is a multi-unit auction, our analysis is also a comparison of posted prices and auctions. The prices-vs-auctions literature has grown with the introduction on eBay of the "buy it now" option. However, that literature has considered quite distinct question from this paper—and hence our results are complementary—because it has largely considered auctions for a fixed quantity and hence is not about the trade-off between preserving flexibility in price and in quantity. Instead, part of that literature has studied the transaction costs associated with auctions when buyers arrive stochastically over time, so that holding an auction requires waiting to assemble buyers rather than (as with a posted price) allowing buyers to purchase as they arrive; other papers have examined how risk aversion can make it optimal to augment an auction with a buy-it-know option.

Among the papers on single-unit auctions are the following: Wang (1993) considers the impact of transaction costs, particularly a storage cost that introduces impatience on the part of the seller. Matthews (2004) considers instead the impact of impatience on the part of the buyers. Other papers have considered how it can be beneficial to augment an auction with a buy-it-now price if either buyers are risk averse (Budish and Takeyama 2001) or sellers are risk averse (Hidvegi et al. 2006). Wang et al. (2008) provide a theoretical and empirical analysis driven by bidders' transactions costs.

There are several papers in which a seller with a fixed multiunit supply and stochastically arriving buyers may simultaneously use auctions and posted prices. These represent a linking of the revenue management literature, which traditionally considered only posted prices in such a setting, and the multiunit auctions literature. Etzion et al. (2006) consider the ability to use such dual channels to price discriminate based on impatience. Caldentey and Vulcano (2007) extend the analysis to account for impatience by both sellers and buyers and to allow the seller to ration the total quantity sold.

2 Model of demand uncertainty

2.1 Setting quantity or setting price

We use a stylized model to study the implications of demand uncertainty for the sales mechanism of a single risk-neutral seller of a good or a service with constant marginal cost c. If the demand curve were known, the seller would pick the point (p, q) on the demand curve that maximizes its profit $(p - c)q$; "choosing price" and "choosing quantity" would merely be two equivalent ways to frame the decision problem. However, with uncertain demand, these sales mechanisms are not equivalent to each other.

Setting quantity The firm can set its output or capacity to q and let the market price adjust to the uncertain value $\tilde{P}(q)$ (a random variable whose distribution depends on q).

Setting price Alternatively, the firm can set a price p and let its output adjust to meet the uncertain demand $\tilde{Q}(p)$.

The realized profits for the quantity and price mechanisms are, respectively,

$$\tilde{\Pi}_q(q) = \tilde{P}(q)q - cq \qquad \text{and} \qquad \tilde{\Pi}_p(p) = (p - c)\tilde{Q}(p),$$

leading to expected profits of

$$\Pi_q(q) = E\big[\tilde{P}(q)\big]\, q - cq \qquad \text{and} \qquad \Pi_p(p) = (p - c)E\big[\tilde{Q}(p)\big].$$

Then the firm's maximum expected profits for the two mechanism are

$$\Pi_q^* = \max_q \Pi_q(q) \qquad \text{and} \qquad \Pi_p^* = \max_p \Pi_p(p).$$

Our goal is to understand how the choice of the optimal mechanism (i.e., the difference between Π_q^* and Π_p^*) depends on the nature of the demand uncertainty.

2.2 Parametrization of demand uncertainty

We distinguish between two types of demand uncertainty.

Uncertainty about market size On one extreme, the firm may know the distribution of the consumer's characteristics in the market but not the number of consumers (market size). Denote the known per capita demand by $g(p)$ and let $\tilde{n}$ be the number of consumers in the market. Then demand as a function of price is $\tilde{Q}(p) = \tilde{n}g(p)$. If we let f be the inverse of g then the inverse demand is $\tilde{P}(q) = f(q/\tilde{n})$.

Uncertainty about valuations Alternatively, the firm may know the exact size of the market but not the valuations (in the case of unit demand) or the marginal valuations (in the case of multi-unit demand) of the consumers. Unlike uncertainty about market size, such uncertainty is not inherently one-dimensional. We restrict attention to a one-dimensional parametrization in which a single random variable $\tilde{a}$ scales each consumer's valuation or marginal valuations linearly. If we let $f(q)$ be the inverse demand curve (hence marginal valuation curve) of the market when $\tilde{a} = 1$, then the inverse demand for other realizations of $\tilde{a}$ is $\tilde{P}(q) = \tilde{a}\, f(q)$. If g is the inverse of f then the demand curve is $\tilde{Q}(p) = g(p/\tilde{a})$.

Combining these two kinds of uncertainty, the inverse demand curve and the demand curve are given, respectively, by

$$\tilde{P}(q) = \tilde{a}\ f(q/\tilde{n}) \qquad \text{and} \qquad \tilde{Q}(p) = \tilde{n}\, g(p/\tilde{a}),$$

where g is the inverse of f. We thereby have a general specification of uncertain demand that explicitly distinguishes between uncertainty about market size ($\tilde{n}$) and uncertainty about valuations ($\tilde{a}$). For example, in the case of unit demand with a finite number of consumers, any demand curve has both a quantity and price intercept. A change in $\tilde{n}$ is then a movement of the quantity intercept alone; a change in $\tilde{a}$ is a movement of the price intercept alone.

2.3 Linear case

As a special case we consider linear demand, where we have a two-dimensional family of demand curves that we parametrize by the quantity intercept $\tilde{n}$ and the price intercept $\tilde{a}$. This matches our general specification when $f(q) = 1 - q$ and $g(p) = 1 - p$. Then the inverse demand and demand curves are

$$\tilde{P}(q) = \tilde{a}(1 - q/\tilde{n}) \qquad \text{and} \qquad \tilde{Q}(p) = \tilde{n}(1 - p/\tilde{a}). \tag{1}$$

This specification corresponds, for example, to the case of unit demand with $\tilde{n}$ consumers whose valuations are uniformly distributed on $[0, \tilde{a}]$. Figure 1 illustrates this by showing an example in panel (a) in which only $\tilde{a}$ is uncertain and an example in panel (b) in which only $\tilde{n}$ is uncertain.

The formulas for linear demand in Eq. 1 do not take into account that price and quantity cannot be negative. The fully specified formulas are

$$\tilde{P}(q) = \max\{\tilde{a}(1 - q/\tilde{n}), 0\} \qquad \text{and} \qquad \tilde{Q}(p) = \max\{\tilde{n}(1 - p/\tilde{a}), 0\}. \tag{2}$$

Such formulas are unnecessarily pedantic when there is no demand uncertainty; we understand that the firm will limit its price and quantity decisions to the region where the simpler formulas in Eq. 1 are correct. However, with uncertainty, a firm's optimal price (or quantity) could be such that, for some realizations of demand, the non-negativity constraint in (2) is binding and the

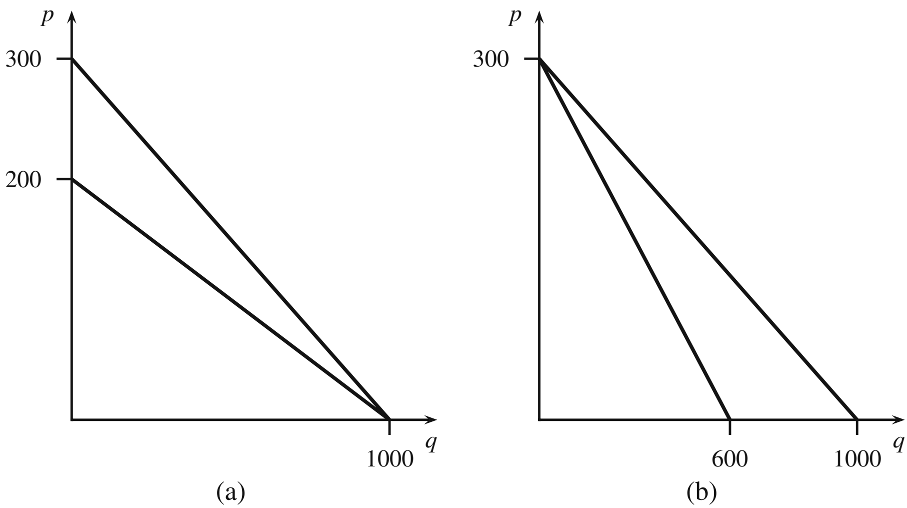


Fig. 1 Possible linear demand curves if $g(p) = 1 - p$. In panel **a**, n is known to be 1000 and $\tilde{a}$ is either 200 or 300. In panel **b**, a is known to be 300 and $\tilde{n}$ is either 600 or 1000

quantity (or price) is zero. This is a consequence of the fact that no demand curve can be everywhere weakly convex, an issue that matters when there is uncertainty. We consider it in details in Section 4.

3 The relative advantage of price versus quantity

We show in Sections 3 and 4 that greater uncertainty about market size would favor setting price and greater uncertainty about valuations would favor setting quantity. To provide some intuition for our main results, first consider special cases in which only one source of uncertainty is present.

Uncertainty is only about market size (a is known) In this scenario, setting price is optimal because the full-information optimal price does not depend on market size. This is seen by writing the firm's price-setting problem as maximizing markup times volume:

$$\max_p \; (p - c)\,(n\, g(p/a))\,.$$

The parameter n merely scales up the objective function. Thus, the firm can achieve the first best by choosing the price p^* that maximizes $(p - c)\, g(p/a)$ and letting the quantity adjust to $ng(p^*/a)$.

Uncertainty is only about valuations (n is known) and c $= 0$ Here we can write the firm's full-information problem as one of choosing q to maximize revenue:

$$\max_q \; a\, f(q/n)\, q.$$

The parameter a merely scales the objective function. The firm can achieve the first best by choosing the quantity q^* that maximizes $f(q/n)\, q$ and letting the price adjust to $a\, f(q^*/n)$.

3.1 Comparative statics with respect to risk

We extend these results to our model in which both types of uncertainty are present. We characterize how the difference $\Pi^*_p - \Pi^*_q$, which measures the value (positive or negative) of switching from flexible prices to flexible quantities, changes when either valuations become more uncertain or market size becomes more uncertain. "More uncertain" means "more risky"—in the sense of Rothschild and Stiglitz (1970)—except that we must take into account that $\tilde{n}$ and $\tilde{a}$ might be dependent.

Definition 1 Given random variables $\tilde{x}_1$, $\tilde{x}_2$, and $\tilde{y}$, we say that $\tilde{x}_2$ *is riskier than* $\tilde{x}_1$ *given* $\tilde{y}$ if, for almost every realization y of $\tilde{y}$, the distribution of $\tilde{x}_2$ conditional on $\tilde{y} = y$ is riskier than the distribution of $\tilde{x}_1$ conditional on $\tilde{y} = y$.

For conciseness, we often say "$\tilde{n}_2$ is riskier than $\tilde{n}_1$" when we mean "$\tilde{n}_2$ is riskier than $\tilde{n}_1$ given $\tilde{a}$". Our main results are the following.

1. An increase in uncertainty about market size favors a price mechanism ($\Pi_p^* - \Pi_q^*$ rises) because it reduces the expected profit of the quantity mechanism without affecting the expected profit of the price mechanism.
2. An increase in uncertainty about valuations favors a quantity mechanism ($\Pi_p^* - \Pi_q^*$ falls) because it reduces the expected profit of the price mechanism without affecting the expected profit of the quantity mechanism.

We explain these results in terms of shifts in the expected demand curves and then show that the changes in expected profit occur not merely around the optimum but for any value of the decision variable.

3.2 Certainty-equivalent demand curves

Recall that expected profit as function of q and as a function of p are given by

$$\Pi_q(q) = E\big[\tilde{P}(q)\big]\,q - cq \qquad \text{and} \qquad \Pi_p(p) = (p-c)E\big[\tilde{Q}(p)\big].$$

We can convert each decision problem (setting quantity or setting price) into a familiar problem without uncertainty by defining

$$P(q) = E\big[\tilde{P}(q)\big] \qquad \text{and} \qquad Q(p) = E\big[\tilde{Q}(p)\big],$$

so that the objective functions become

$$\Pi_q(q) = P(q)\,q - cq \qquad \text{and} \qquad \Pi_p(p) = (p-c)\,Q(q)\,.$$

That is, a quantity-setting (resp., price-setting) firm faces the same objective—thus the same solution and maximum profit—as if it had the deterministic inverse demand curve $P(q)$ (resp., demand curve $Q(p)$). For this reason, we call $P(q)$ and $Q(p)$ the *certainty-equivalent* (CE) inverse demand and demand curves. Whereas without uncertainty a firm's inverse demand and demand curves would be inverses of each other and hence equivalent, $P(q)$ and $Q(p)$ are, in general, different curves.

The choice of sales mechanism can thus be viewed as the choice between two demand curves by a firm with deterministic demand. The distribution of $(\tilde{n}, \tilde{a})$ affects this choice entirely through its impact on the CE demand curves. We exploit this viewpoint whenever possible in our analysis.

When we need to emphasize the dependency of these functions and values on the distribution of $(\tilde{n}, \tilde{a})$, we will write $P(q; \tilde{n}, \tilde{a})$, $Q(p; \tilde{n}, \tilde{a})$, $\Pi_q(q; \tilde{n}, \tilde{a})$, $\Pi_p(p; \tilde{n}, \tilde{a})$, $\Pi_q^*(\tilde{n}, \tilde{a})$, and $\Pi_p^*(\tilde{n}, \tilde{a})$.

Example 1 Figure 2 illustrates the CE demand curves for linear demand, meaning that $f(q) = 1 - q$ and $g(p) = 1 - p$. Over the range of quantities and prices for which the boundary of the linear demand curves are not reached with positive probability, we have the following CE demand curves:

$$P(q) = E[\tilde{a}] - qE[\tilde{a}/\tilde{n}] \qquad \text{and} \qquad Q(p) = E[\tilde{n}] - pE[\tilde{n}/\tilde{a}]. \tag{3}$$

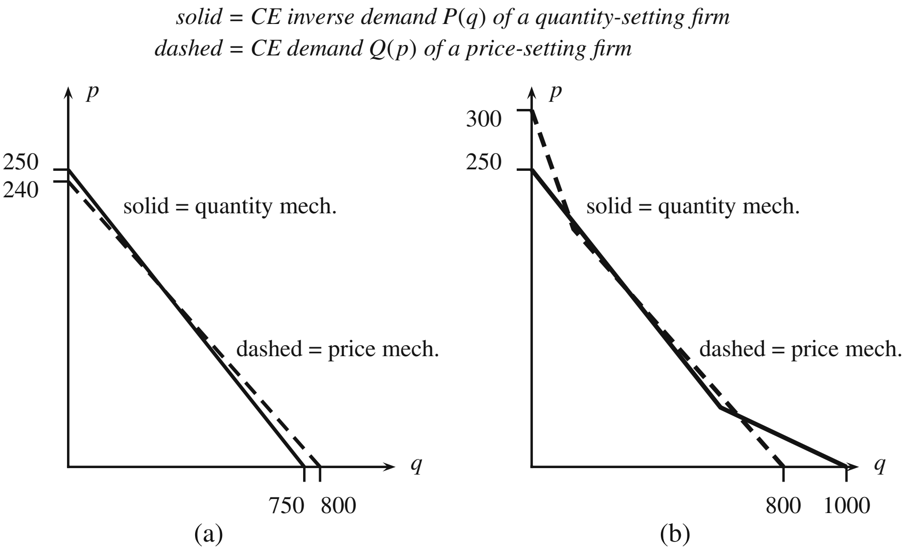


Fig. 2 Certainty-equivalent demand curves for linear demand. Demand is linear, $\tilde{n}$ and $\tilde{a}$ are independent, $\tilde{n}$ equals 600 and 1000 with equal probability, and $\tilde{a}$ equals 200 and 300 with equal probability. Panel **a** ignores the non-negativity condition; panel **b** takes it into account

For example, suppose that $\tilde{n}$ equals 600 and 1000 with equal probability, that $\tilde{a}$ equals 200 and 300 with equal probability, and that $\tilde{n}$ and $\tilde{a}$ are independent. Then the CE demand curves based on the formulas in (3) are shown in Fig. 2a. When we take into account the boundary conditions, applying the formulas in Eq. 2, we obtain the curves in Fig. 2b.

3.3 Revenue curves and main results

Some of the upcoming results assume that the revenue curves (as a function of quantity and as a function of price) are strictly concave (in order to apply Jensen's inequality in the proof, not because we rely on second-order conditions). Below we define these revenue curves and explain how such assumptions will be treated.

Denote revenue as a function of quantity by $\tilde{R}(q) \equiv \tilde{P}(q)q$ and revenue as a function of price by $\tilde{S}(p) \equiv p\tilde{Q}(p)$. Observe that these curves, which equal

$$\tilde{R}(q) = \tilde{a}\, f(q/\tilde{n})\, q \qquad \text{and} \qquad \tilde{S}(p) = p\tilde{n}\, g(p/\tilde{a})\,,$$

are strictly concave for all realizations of $\tilde{a}$ and $\tilde{n}$ if and only if the "canonical" revenue curves

$$\hat{R}(q) = f(q)\, q \qquad \text{and} \qquad \hat{S}(p) = p\, g(p)$$

(respectively) are strictly concave.

However, because $\hat{R}(q)$ and $\hat{S}(p)$ are positive, they could be strictly concave on all of $[0, \infty)$ only if they were strictly increasing. Therefore, we assume instead that $\hat{R}(q)$ is strictly concave on an interval $[0, \hat{q}]$ and that $\hat{S}(p)$ is strictly concave on an interval $[0, \hat{p}]$. Let $n^{\min}$ and $a^{\min}$ be the greatest lower bounds on the supports of $\tilde{n}$ and $\tilde{a}$. Then $\tilde{R}(q)$ is strictly concave on $[0, n^{\min}\hat{q}]$ for all realizations of demand and $\tilde{S}(p)$ is strictly concave on $[0, a^{\min}\hat{p}]$ for all realizations of demand, respectively. We explore the significance of these assumptions in Section 4.

3.3.1 Main results

Theorems 1 and 2 show that the increase in uncertainty about market size causes $\Pi^*_p - \Pi^*_q$ to rise, while the increase in uncertainty about valuations causes $\Pi^*_p - \Pi^*_q$ to fall. More generally, by Theorem 1:

1. *For a price-setting firm,* the increase in uncertainty about market size has no effect on the CE demand and therefore it has no effect on the profit function or on the maximum profit.
2. *For a quantity-setting firm,* the increase in uncertainty about market size dampens the CE inverse demand and hence reduces the profit function and the maximum profit—all under an assumption that the revenue function is strictly concave on a large-enough domain (see Section 4).

Theorem 1 *Suppose $\{\tilde{n}_1, \tilde{n}_2, \tilde{a}\}$ are such that $\tilde{n}_2$ is riskier than $\tilde{n}_1$ given $\tilde{a}$.*

1. *For all p, $Q(p; \tilde{n}_2, \tilde{a}) = Q(p; \tilde{n}_1, \tilde{a})$ and hence $\Pi_p(p; \tilde{n}_2, \tilde{a}) = \Pi_p(p; \tilde{n}_1, \tilde{a})$. Therefore, $\Pi^*_p(\tilde{n}_2, \tilde{a}) = \Pi^*_p(\tilde{n}_1, \tilde{a})$.*
2. *Suppose $\hat{R}(q)$ is strictly concave on $[0, \hat{q}]$. For all $q \in [0, n_2^{\min}\hat{q}]$, $P(q; \tilde{n}_2, \tilde{a}) < P(q; \tilde{n}_1, \tilde{a})$ and hence $\Pi_q(q; \tilde{n}_2, \tilde{a}) < \Pi_q(q; \tilde{n}_1, \tilde{a})$. Therefore, if a solution to $\max_q \Pi_q(q; \tilde{n}_2, \tilde{a})$ lies in $[0, n_2^{\min}\hat{q}]$, then $\Pi^*_q(\tilde{n}_2, \tilde{a}) < \Pi^*_q(\tilde{n}_1, \tilde{a})$.*

By Theorem 2:

1. *For a quantity-setting firm,* the increase in uncertainty about valuations has no effect on the CE inverse demand and therefore it has no effect on the profit function or on the maximum profit.
2. *For a price-setting firm,* the increase in uncertainty about valuations dampens the CE demand and hence reduces the profit function and the maximum profit—all under an assumption that the revenue function is strictly concave on a large enough domain (see Section 4).

Theorem 2 *Suppose $\{\tilde{n}; \tilde{a}_1, \tilde{a}_2\}$ are such that $\tilde{a}_2$ is riskier than $\tilde{a}_1$ given $\tilde{n}$.*

1. *For all q, $P(q; \tilde{n}, \tilde{a}_2) = P(q; \tilde{n}, \tilde{a}_1)$ and hence $\Pi_q(q; \tilde{n}, \tilde{a}_2) = \Pi_q(q; \tilde{n}, \tilde{a}_1)$ and $\Pi^*_q(\tilde{n}, \tilde{a}_2) = \Pi^*_q(\tilde{n}, \tilde{a}_1)$.*

2. *Suppose* $\hat{S}(p)$ *is strictly concave on* $[0, \hat{p}]$*. Then, for all* $p \in [0, a_2^{\min}\hat{p}]$*,* $Q(p; \tilde{n}, \tilde{a}_2) < Q(p; \tilde{n}, \tilde{a}_1)$ *and hence* $\Pi_p(p; \tilde{n}, \tilde{a}_2) < \Pi_p(p; \tilde{n}, \tilde{a}_1)$*. Therefore, if a solution to* $\max_p \Pi_p(p; \tilde{n}, \tilde{a}_2)$ *lies in* $[0, a_2^{\min}\hat{p}]$*, then* $\Pi_p^*(\tilde{n}, \tilde{a}_2) < \Pi_p^*(\tilde{n}, \tilde{a}_1)$.

The proofs of these two theorems are identical except for a substitution of the roles of $\tilde{a}$ and $\tilde{n}$, of f and g, et cetera. Therefore, the Appendix contains only the proof of Theorem 1.

3.3.2 Remark on the standard additive-shock model

A standard representation of uncertain linear demand is $\tilde{Q}(p) = \tilde{\alpha} - \beta p$, where β is known. The single parameter α scales both the horizontal and vertical intercepts; in our model, this means that $\tilde{n} = \tilde{\alpha}$ and $\tilde{a} = \tilde{\alpha}/\beta$. As a consequence, the CE inverse demand and demand curves are equivalent (assuming that p and q are such that formulas (1) do not result in negative values.) Therefore, this representation has the knife-edge property that the seller is indifferent between setting price and setting quantity no matter what is the distribution of $\tilde{\alpha}$.

This observation is interesting for two reasons. First, it means that the widely used additive demand model (see e.g. Petruzzi and Dada 1999; Bhardwaj 2001; Chod and Rudi 2005) is very special. It also highlights that explicit modeling of the nature of demand uncertainty, which is the subject of our paper, is important. Second, it is also a benchmark that is useful for ranking the sales mechanisms. If we deviate from additive demand model by increasing the uncertainty about market size (valuations), then by Theorem 1 (Theorem 2) we know that setting price (quantity) is better than setting quantity (price). We summarize this conclusion in Proposition 1.

Proposition 1 *Suppose that demand is linear.*

1. *If* $\tilde{n} = \beta\tilde{a} + \tilde{x}$*, where* $\beta > 0$ *and* $E[\tilde{x} \mid \tilde{a}] = 0$ *almost surely, then setting price is better than setting quantity.*
2. *If* $\tilde{a} = \tilde{n}/\beta + \tilde{x}$*, where* $\beta > 0$ *and* $E[\tilde{x} \mid \tilde{n}] = 0$ *almost surely, then setting quantity is better than setting price.*

4 Decreasing marginal revenue assumptions

The assumptions in Theorems 1 and 2, under which an increase in uncertainty about valuations (resp., market size) favors setting quantity (resp., setting price), are that the revenue curves be concave on a large-enough domain. This section characterizes conditions under which the assumptions hold. We develop the following ideas.

1. An empirical regularity is that demand becomes weakly more elastic at higher prices.

2. This implies that the "strict concavity of $\hat{R}$" and "strict concavity of $\hat{S}$" conditions hold on a range $q \in [0, \bar{q}]$ and $p \in [0, \bar{p}]$, respectively, that includes the optimal quantity and price as long as there is not too much uncertainty and the marginal cost is low enough.
3. For the effect of market-size uncertainty on the expected profit of a quantity-setting firm, higher marginal cost merely strengthens the result (relaxes the condition on the amount of uncertainty).
4. In contrast, for the effect of valuation uncertainty on the expected profit of a price-setting firm, higher marginal cost weakens the result (strengthens the condition on the amount of uncertainty).

As noted, the property that demand becomes weakly more elastic at higher prices is an empirical regularity; we refer to it as "monotone elasticity". Linear demand has monotone elasticity as does the following variant of constant-elasticity demand: $q = n(p^{-b} - k)$, where $b \in (0, 1)$ and $k > 0$. Given our parametrization of demand uncertainty, a realization of the demand curve has monotone elasticity if and only if the canonical demand curves f and g have monotone elasticity. Assume that this is true.

Consider first the canonical revenue curve $\hat{R}(q)$. Marginal revenue can be written as

$$\hat{R}'(q) = p\left(1 + \frac{1}{e}\right),$$

where p is the price $f(q)$ when quantity is q and e is the elasticity at the point (q, p) on the demand curves. (Note that e is negative and we have assumed that it becomes greater in magnitude for higher p and lower q.) One can use this formula to show the following.

R1. $\hat{R}$ is maximized at the point q^r at which $e = -1$.
R2. $\hat{R}$ is strictly concave on an interval $[0, \hat{q}]$, where $\hat{q} > q^r$.
R3. $\hat{R}$ is single peaked, meaning that it is decreasing for $q > q^r$.

Likewise, consider the canonical revenue curve $\hat{S}(p)$. Marginal revenue can be written

$$\hat{S}'(p) = q\,(1 + e)\,.$$

One can use this formula to show the following.

S1. $\hat{S}$ is maximized at the point p^r at which $e = -1$.
S2. $\hat{S}$ is strictly concave on an interval $[0, \hat{p}]$, where $\hat{p} > p^r$.
S3. $\hat{S}$ is single peaked, meaning that it is decreasing for $p > p^r$.

Example 2 With linear demand, where $f(q) = 1 - q$ and $g(p) = 1 - p$, the revenue curves are shown in Fig. 3. They are quadratic (and hence strictly concave) functions up until the intercepts, and hence we can take $\hat{p} = 1$ and $\hat{q} = 1$. The revenue maximizing values are $q^r = 1/2$ and $p^r = 1/2$.

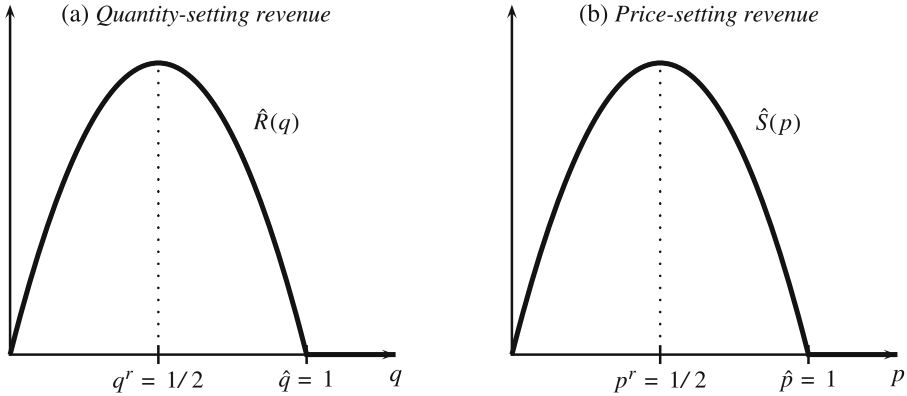


Fig. 3 The canonical revenue curves for linear demand: $f(q) = 1 - q$ and $g(p) = 1 - p$. **a** Quantity-setting revenue. **b** Price-setting revenue

Thus, over the domains $[0, n^{\min}]$ for quantities or $[0, a^{\min}]$ for prices, the CE demand curves and the profit functions will shift as predicted by the second parts of Theorems 1 and 2. Theorem 1 is illustrated in Fig. 4, with independent $\tilde{n}$ and $\tilde{a}$. The dashed line assumes that $\tilde{n}$ equals 700 or 900 with equal probability; the solid line assumes that $\tilde{n}$ equals 600 or 1000 with equal probability. As predicted by Theorem 1, the quantity-setting CE inverse demand curve, and hence the profit function, shift downward for $q \in [0, 600]$. For high enough values of q, the shift is in the opposite direction.

Figure 5 illustrates Theorem 2, with independent $\tilde{n}$ and $\tilde{a}$. The dashed line assumes that $\tilde{a}$ equals 240 or 260 with equal probability; the solid line assumes that $\tilde{a}$ equals 200 or 300 with equal probability. As predicted by Theorem 2, the

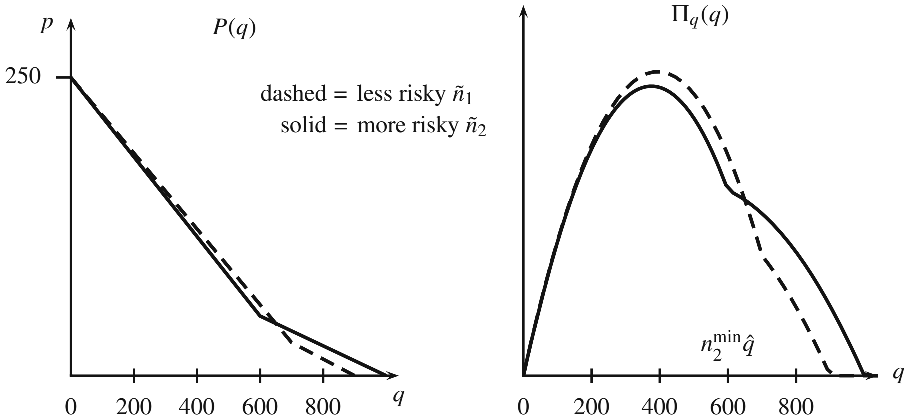


Fig. 4 Illustration of Theorem 1 (effect of higher uncertainty about market size on quantity-setting CE inverse demand and profit function) for an example of linear demand; $\tilde{n}$ and $\tilde{a}$ are independent. *Dashed curves* are for less risky $\tilde{n}_1$, equal to 700 or 900 with same probability. *Solid curve* is for more risky $\tilde{n}_2$, equal to 600 or 1000 with same probability

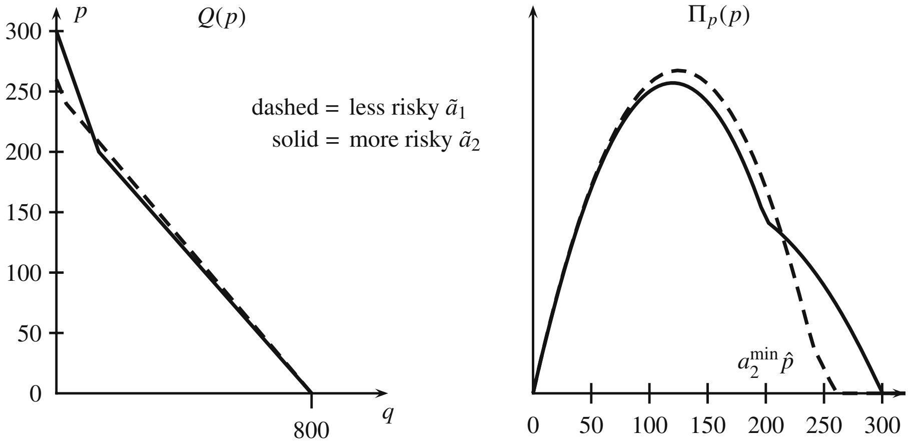


Fig. 5 Illustration of Theorem 2 (effect of higher uncertainty about valuations on price-setting CE inverse demand and profit function) for an example of linear demand. $\tilde{n}$ and $\tilde{a}$ are independent. *Dashed curves* are for less risky $\tilde{a}_1$, equal to 240 or 260 with same probability. *Solid curve* is for more risky $\tilde{a}_2$, equal to 200 or 300 with same probability

price-setting CE demand curve shifts to the left, and hence the profit function shifts downward, for $p \in [0, 200]$. For high enough values of p, the shift is in the opposite direction.

Already R2 and S2 indicate that there are interesting domains on which the comparisons of the profit functions in Theorems 1 and 2 hold. Next consider what it takes for these domains to be large enough for the comparisons of maximum profit to hold. We need to show that a solution to $\max_q \Pi_q(q; \tilde{n}_2, \tilde{a})$ lies in $[0, n_2^{\min}\hat{q}]$ and that a solution to $\max_p \Pi_p(p; \tilde{n}, \tilde{a}_2)$ lies in $[0, a_2^{\min}\hat{p}]$. Observe, for example, that this is true in Figs. 4 and 5.

Consider Theorem 2 and a quantity-setting firm. For any realization of the inverse demand curve f, revenue—and hence profit—is a decreasing function of q on $[nq^r, \infty)$. Therefore, $n^{\max}q^r$ is an upper bound on the quantity that can maximize expected profit. As long as $n_2^{\max}q^r < n_2^{\min}\hat{q}$, that is, $n_2^{\max}/n_2^{\min} < \hat{q}/q^r$, we can conclude that the shift in the CE inverse demand curve causes the maximum expected profit to fall. If marginal cost is positive, we could weaken the restriction because the profit-maximizing quantity would be lower and hence more likely to lie in $[0, n_2^{\min}\hat{q}]$.

Consider Theorem 1 and a price-setting firm. Now we must be more restrictive because the profit-maximizing price necessarily gets pushed up to the region where the revenue curve is not concave as the marginal cost rises. Otherwise, however, the argument is similar. The condition $a_2^{\max}/a_2^{\min} < \hat{p}/p^r$ guarantees that the price that maximizes expected revenue is in the strictly concave region of the revenue curve. As long as the marginal cost is small enough, the profit-maximizing price is also in this region. We summarize these two observations in Proposition 2.

Proposition 2

1. *Suppose that* $\{\tilde{n}_1, \tilde{n}_2, \tilde{a}\}$ *are such that* $\tilde{n}_2$ *is riskier than* $\tilde{n}_1$ *given* $\tilde{a}$. *If* $n_2^{\max}/n_2^{\min} < \hat{q}/q^r$ *then* $\Pi_q^*(\tilde{n}_2, \tilde{a}) < \Pi^*(\tilde{n}_1, \tilde{a})$.
2. *Suppose that* $\{\tilde{n}; \tilde{a}_1, \tilde{a}_2\}$ *are such that* $\tilde{a}_2$ *is riskier than* $\tilde{a}_1$ *given* $\tilde{n}$. *If* $a_2^{\max}/a_2^{\min} < \hat{p}/p^r$ *then there is a* $\bar{c}$ *such that* $\Pi_p^*(\tilde{n}, \tilde{a}_2) < \Pi_p^*(\tilde{n}, \tilde{a}_1)$ *for* $c \in [0, \bar{c}]$.

On the other hand, consider linear demand with fixed $\{\tilde{n}, \tilde{a}_1, \tilde{a}_2\}$, where $\tilde{a}_2$ is riskier than $\tilde{a}_1$. Then for high-enough marginal cost the optimal price ends up where the CE demand curve for $\tilde{a}_2$ is to the right of the one for $\tilde{a}_1$, and the firm is better off with the more uncertain valuations. The intuition is as follows. Suppose the marginal cost is high and therefore the firm sets a high price. It can do no worse than to have zero sales and this downside risk is not affected by an increase in uncertainty about valuations. However, some times it will be lucky and sell at high price; this upside becomes more likely when there is greater uncertainty about valuations.

5 How the price-versus-quantity decision depends on the marginal cost

We now examine, for the case of linear demand, how the choice of sales mechanism depends on the magnitude of the marginal cost. We first ignore the boundary constraint (i.e., we assume that the uncertainty about market size and valuations is not too large) and then show how the answer changes when we take it into account. As mentioned above, for any form of demand, marginal revenue cannot be decreasing everywhere (since revenue has to be positive), and therefore the results of our analysis are rather general, and not just an artifact of linear demand specification.

5.1 Analysis ignoring non-negativity constraints

Consider the example of the CE demand curves in Fig. 2a, which do not take into account the non-negativity constraints on quantity and price. Recall that (i) the choice of sales mechanism for a firm that faces uncertain demand is equivalent to (ii) the choice of CE demand curves for a firm with certain demand but two demand curves from which to choose. A firm with decision problem (ii) would always choose a point on the outer (i.e., upper) envelope of the two demand curves. We can thus frame the choice of sales mechanism as:

1. choose a profit maximizing point on the outer envelope of the CE demand curves $P(q)$ and $Q(p)$; then
2. see whether the solution comes from the $P(q)$ curve or the $Q(p)$ curve.

Figure 6a shows the outer envelope of the CE demand curves from Fig. 2a. The kink, where the outer envelope switches between $P(q)$ and $Q(p)$, is marked with a dot. Because of the kink, the optimal price/quantity on this

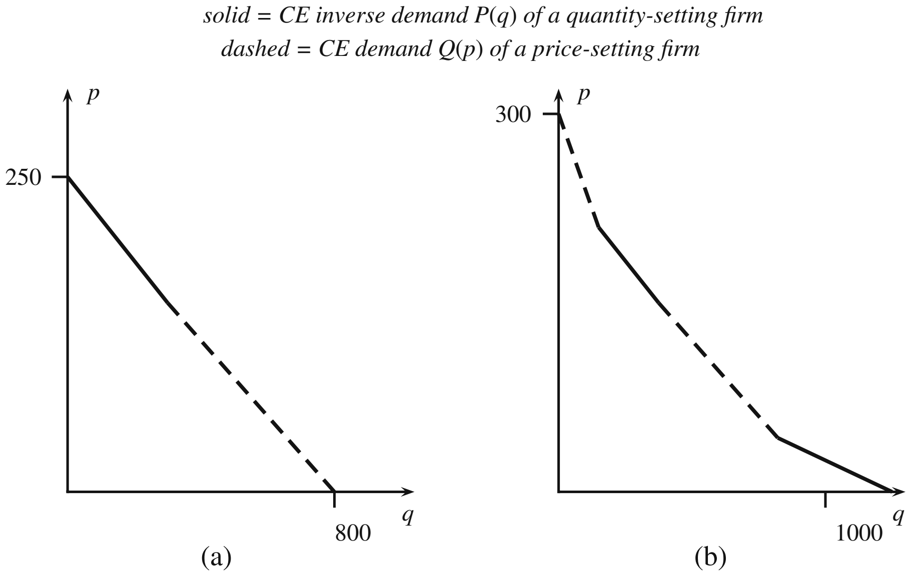


Fig. 6 Outer envelope of the CE demand curves for the example of linear demand in Fig. 2. Panel **a** ignores the non-negativity condition, as in Fig. 2a; panel **b** takes into account the condition, as in Fig. 2b

outer envelope does not vary continuously with marginal cost; in fact, it will jump over the kink as the marginal cost rises. Nevertheless, as a simple application of monotone comparative statics, we know that the optimal quantity is decreasing and the optimal price is increasing as the cost rises because the objective function $\pi(q; c) = pq - cq$ has strictly decreasing differences in q and c. Therefore, as the marginal cost rises, the optimal price/quantity may switch from the dashed section (meaning that setting price is optimal) to the solid section (meaning that setting quantity is optimal) but not vice versa.

To make sure that this conclusion is robust (as long as we ignore the non-negativity conditions) we just have to check that the CE curves can cross only as shown in Fig. 2a.

The formulas for the intercepts of the CE demand curves are shown in Table 1 for easy reference.

Proposition 3 *Consider linear demand and ignore the non-negativity constraints. Then either the CE demand curves do not cross or $P(q)$ crosses $Q(p)$ from above, as illustrated in Fig. 2a.*

Table 1 Intercepts of the CE inverse demand and demand curves for the linear case, ignoring non-negativity constraints

Intercept	$P(q)$	$Q(p)$
Quantity	$E[\tilde{a}]/E[\tilde{a}/\tilde{n}]$	$E[\tilde{n}]$
Price	$E[\tilde{a}]$	$E[\tilde{n}]/E[\tilde{n}/\tilde{a}]$

Proof We are ruling out the case in which the CE curves cross the other way, which would mean that the quantity intercept of $Q(p)$ is lower than that of $P(q)$ whereas the price intercept of $P(q)$ is lower than that of $Q(p)$. From Table 1, this translates into the following inequalities:

$$E[\tilde{n}] < \frac{E[\tilde{a}]}{E[\tilde{a}/\tilde{n}]} \qquad \text{and} \qquad E[\tilde{a}] < \frac{E[\tilde{n}]}{E[\tilde{n}/\tilde{a}]}. \tag{4}$$

By multiplying the two inequalities together (LHS×LHS and RHS×RHS), then canceling $E[\tilde{n}]E[\tilde{a}]$ from the resulting inequality, and then rearranging, we obtain $E[\tilde{a}/\tilde{n}]E[\tilde{n}/\tilde{a}] < 1$. This is impossible: for any nonnegative random variable $\tilde{z}$, $E[\tilde{z}]E[1/\tilde{z}] \geq 1$. □

Proposition 3 leads to our main result on how the price-versus-quantity decision depends on the magnitude of marginal cost.

Corollary 1 *Suppose that demand is linear and consider the range of cost c such that the non-negativity constraints are not reached. If for some marginal cost c^* setting quantity is better for the seller, then it is better for any $c > c^*$. If for some marginal cost c^* setting price is better for the seller, then it is better for any $c < c^*$.*

The result of Corollary 1 is, to some extent, counterintuitive. One would think that if marginal cost is high, setting price is safer, since it will guarantee that the product is never sold at a loss (i.e., below the marginal cost). However, this intuition is based on hitting the non-negativity constraint for quantity, as shown in the next subsection. The intuition for Corollary 1 comes from Proposition 3, showing the way the CE demand curves might cross.

5.2 Full analysis with non-negativity constraints

We now consider how the answer changes when we take into account the non-negativity constraints.

Consider the example of the CE demand curves in Fig. 2b. Figure 6b shows their outer envelope. The kinks, where the outer envelope switches between $P(q)$ and $Q(p)$, are marked with a dot. As can be seen from Fig. 6b, the story is more complicated than in Fig. 6a. If the CE curves cross three times, as in this picture, then the optimal sales mechanism could switch from quantity to price to quantity and then again to price as the marginal cost rises.

We can slightly simplify the possibilities by assuming that the bottom right kink (crossing point) occurs at quantities beyond those that could maximize profit. This is true in the example because the quantity is greater than $n^{\max}/2$, which bounds the optimal quantity. There are then three possible zones:

1. at low enough costs, the firm sets price;
2. at an intermediate range of costs, the firm sets quantity;
3. at an upper range of costs, the firm sets price.

Table 2 Intercepts of the CE inverse demand and demand curves for the linear case, taking into account the non-negativity constraints

Intercept	$P(q)$	$Q(p)$
Quantity	$n^{\max}$	$E[\tilde{n}]$
Price	$E[\tilde{a}]$	$a^{\max}$

To ensure that these conclusions are robust, we must verify that the CE demand curves cross as in Fig. 2b. In fact, this is true. Table 2 shows the intercepts of the CE curves. Along the vertical axis (low quantity, high price), the price-setting curve $Q(p)$ must dominate. As long as the firm sets price below $a^{\max}$, it gets some sales; in contrast, when setting quantity it can never get an expected price that exceeds $E[\tilde{a}]$. By a similar argument, along the horizontal axis (high quantity, low price) the quantity-setting curve $P(q)$ dominates. In fact, there are three possibilities.

1. *The CE curves ignoring boundaries never cross because setting price dominates everywhere.* Then the true CE curves cross only once, at low price and high quantity, in a region that is never optimal even for zero marginal cost. Therefore, setting price is always optimal.
2. *The CE curves ignoring boundaries never cross because setting quantity dominates everywhere. Then the true CE curves cross only once, at high price and low quantity.* If $c = 0$ then setting quantity is optimal; if $c \geq E[\tilde{a}]$ then setting price is optimal; in between, there is a single cost at which the optimal mechanism switches from quantity to price.
3. *The CE curves ignoring boundaries cross as illustrated in Fig. 2a.* Then the true CE curves cross at this point and at two other points, one above and one below. If the optimal price/quantity at zero marginal cost is above the middle crossing point, then this case ends up the same as case 1. Otherwise, at $c = 0$ and for $c \geq E[\tilde{a}]$, setting price is optimal. There may be a single interval of costs between 0 and $E[\tilde{a}]$ for which setting quantity is optimal.

Results of this section (in particular, Corollary 1), where we have shown how the price-versus-quantity decision depends on the marginal cost, are relevant for manufacturer–retailer interactions. The retailer faces the problem we have so far studied in this paper, but with its marginal cost equal to the wholesale price set by the manufacturer. Because of double marginalization, this wholesale price is greater than the marginal cost of the manufacturer. As a consequence, the price-versus-quantity preferences of the retailer and the manufacturer might not be aligned. We study this issue in the following section.

6 Manufacturer–retailer interaction

We now consider a supply chain with an upstream manufacturer and a downstream retailer, both risk neutral. The manufacturer has constant marginal cost c. We restrict the manufacturer to a linear posted-price mechanism and denote its price—which becomes the marginal cost of the retailer—by w. The retailer, on the other hand, can either set quantity or set price. We consider three cases.

1. The manufacturer can stipulate the mechanism used by the retailer (this is meant to be a benchmark).
2. The retailer must commit to one of the two mechanisms before the manufacturer sets w.
3. The retailer selects its sales mechanism after the manufacturer sets w.

We assume that demand is linear and that the uncertainty is low enough relative to the cost that the boundaries of the linear demand curves are not relevant. (The higher is the cost, the less uncertainty there can be about demand so that the boundaries are avoided.) Thus, we use the formulas for linear demand in Eq. 1, which do not take into account the non-negativity conditions. Alternatively, we rely on Section 5.1, and not on Section 5.2. We use subscript m (resp., r) to denote variables for the manufacturer (resp., retailer).

6.1 Mechanism is selected before the wholesale price is set

Our analysis begins with the observation that the "certainty equivalent" approach extends to the supply chain if the mechanism is selected (by the manufacturer or retailer) in advance of the wholesale price w. That is, the quantity, price, and profit (or expectations thereof) for the parties are the same as for a supply chain whose certain demand curve equals the mechanism's certainty equivalent demand curve.

For example, suppose the retailer is committed to setting quantity. Let $P(q)$ be the CE inverse demand curve. Given w, the retailer chooses q to maximize $(P(q) - w)q$, just as it would if its inverse demand curve were known to be $P(q)$. Let $Q_r(w)$ be the solution as a function of w. Then, whether demand is certain or uncertain, the manufacturer chooses w to maximize $(w - c)Q_r(w)$.

Suppose instead the retailer is committed to setting price. Let $Q(p)$ be the CE demand curve. Then the retailer chooses p to maximize $(p - w)Q(p)$, just as it would if its demand curve were known to be $Q(p)$. Let $Q_r(w)$ be the resulting *expected* sales as a function of w, which would equal its actual sales if its demand curve were certain. Either way, the manufacturer chooses w to maximize $(w - c)Q_r(w)$.

Therefore, the choice of sales mechanism by the manufacturer or by the retailer is equivalent to the choice of CE demand curve $P(q)$ or $Q(p)$ in a supply chain with two possible demand curves. Our next step is to review the equilibrium profits for a supply chain with known demand.

The preceding observation does not depend on the assumption that demand is linear, but we now restrict attention to this case. Suppose, then, that the inverse demand and demand curves are known to be

$$p = a(1 - q/n) \qquad \text{and} \qquad q = n(1 - p/a).$$

One can show that the manufacturer sets $w = (1/2)(a + c)$ (the midpoint between the constant marginal cost c and the intercept a) and the retailer then sets $p = (3/4)a + (1/4)$ (the midpoint between its constant marginal cost w and the intercept a). The manufacturer's price w is the same as the retail price of the coordinated channel. However, because the retailer's price p is higher than w, sales are half of what they would be in the coordinate channel. Therefore, the manufacturer's profit is one half of the profit of the coordinated channel. In the supply chain, the manufacturer and retailer sell the same amount, but the manufacturer's markup is twice that of the retailers: $(1/2)(a - c)$ versus $(1/4)(a - c)$. Therefore, the manufacturer's profit is twice that of the retailers.

With uncertainty and ignoring the boundaries, the CE inverse demand and demand curves are linear. Applying the summary of the previous paragraph, we see that, for either sales mechanism, the retailer's profit is half that of the manufacturer, which in turn is half that of the coordinated channel:

$$\Pi_q^* = 2\Pi_{qm}^* = 4\Pi_{qr}^* \qquad \text{and} \qquad \Pi_p^* = 2\Pi_{pm}^* = 4\Pi_{pr}^*.$$

It follows that whichever CE demand curve (sales mechanism) gives the highest profit to the coordinated channel also gives the highest profit to the manufacturer and to the retailer. It does not matter which party chooses the sales mechanism — the decision would be the same. Furthermore, all our results from Sections 3–5 about how the nature of the uncertainty and the marginal cost c affect the choice of sales mechanism for the coordinated channel apply to the choice of sales mechanism for the supply chain.

6.2 Retailer selects the mechanism after the wholesale price is set

Let M' be the mechanism the manufacturer would select if it controlled the choice, let w' be the price the manufacturer would select given that the mechanism is set to M', and let Π'_m be the resulting profit for the manufacturer.

Here we suppose that the retailer selects the mechanism after w is set, and consider whether the outcome for the manufacturer is still $\{M', w', \Pi'_m\}$ or it earns a lower profit than Π'_m because the retailer can choose the sales mechanism after observing the wholesale price. We refer to parameters where the latter holds as a "conflict zone" because the manufacturer would prefer to be able to force the mechanism M' on the retailer and not allow for the retailer's discretion.

Such a conflict occurs if, given w', the retailer would not choose sales mechanism M'. (We saw in Section 5 that the marginal cost influences the choice of sales mechanism.) The manufacturer cannot achieve profit Π'_m because either (a) it modifies w away from w', so that the retailer chooses m', or (b) the retailer uses a mechanism that yields a profit lower than Π'_m for the manufacturer.

Consider the CE curves shown in Fig. 2a and suppose that $c = 0$. For the coordinated channel, the optimal price for either demand curve is one half the vertical intercept; we can see that the price-setting curve $Q(p)$ dominates in this region. Hence this is the mechanism that the manufacturer would like to specify and that the retailer would also choose if it had to commit to a mechanism in advance. Let w_p^* be the manufacturer's optimal price if the retailer had to set price; w_p^* is one half the intercept of the dashed curve. Given w_p^*, the retailer would set a price halfway between w_p^* and the vertical intercept—in the region where the quantity-setting curve $P(q)$ dominates. Therefore, the retailer prefers the quantity-setting mechanism.

Given our assumption that the boundaries of the demand curves are not relevant, we know that an increase in marginal cost can cause a switch from setting price to setting quantity but not vice versa. Therefore, the only type of conflict that can exist is as in the previous example: the manufacturer would prefer to commit to a price-setting mechanism but, given the manufacturer's wholesale price, the retailer would end up setting quantity.

This happens for an intermediate level of uncertainty about the valuations of the consumers. With very little uncertainty about $\tilde{a}$, the CE demand curves intersect near the vertical axis, and setting price will be optimal for both the manufacturer and for the retailer given w. With a lot of uncertainty about $\tilde{a}$, setting quantity dominates for the manufacturer and hence also for the retailer.

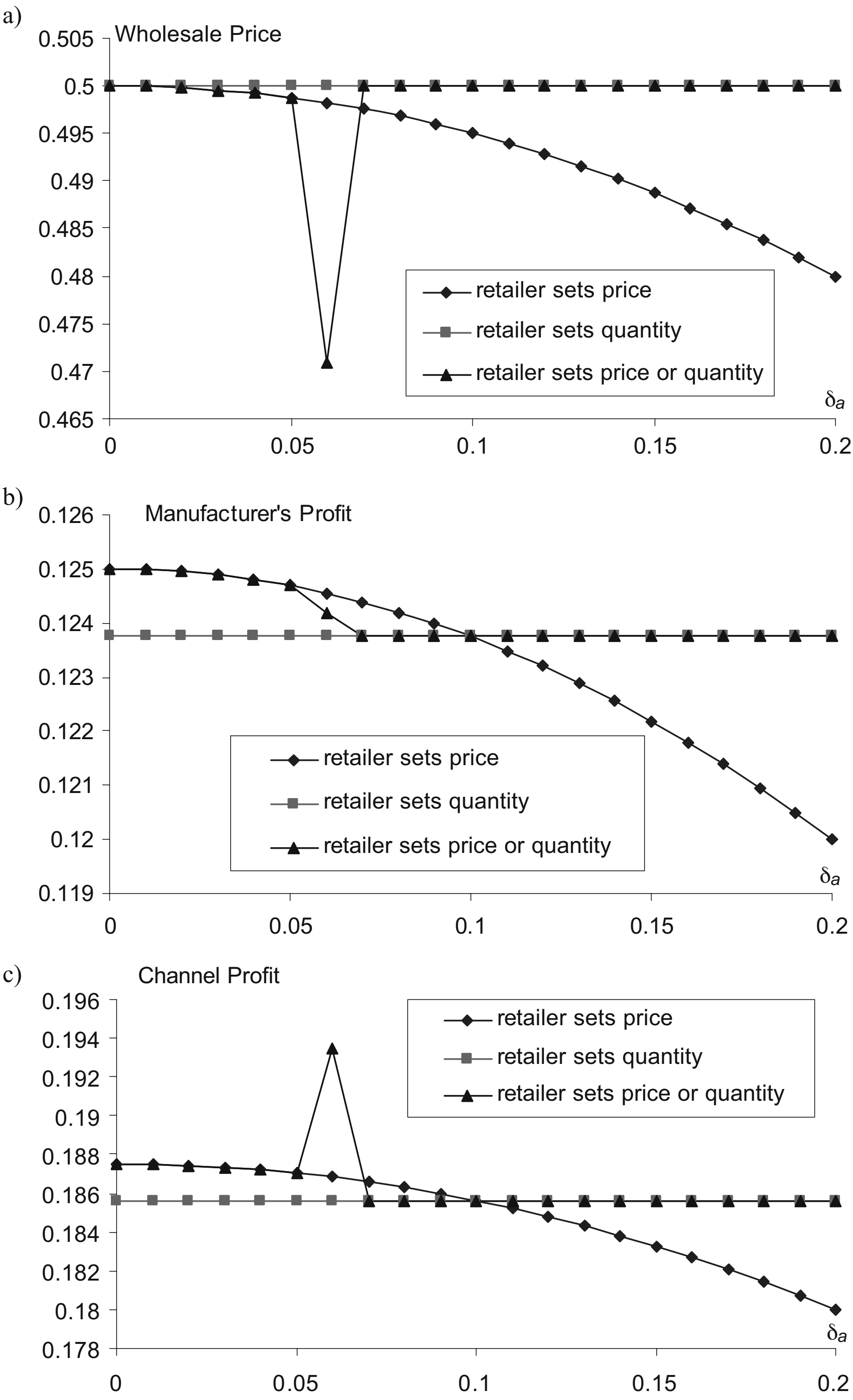
What does the manufacturer do in this conflict zone? It would not naively set w_p^* when it knows that the retailer would choose to set quantity. There are two possibilities: (a) the manufacturer may lower w to below w_p^* in order to induce the retailer to set price; or (b) the manufacturer may accept that the retailer will set quantity and so chooses the price $w_q^* > w_p^*$ that is optimal given that mechanism.

Consider the following example. Let $\tilde{a}$ and $\tilde{n}$ be independent, with $\tilde{n}$ equal to $1 - \delta_n$ and $1 + \delta_n$ with equal probability and $\tilde{a}$ equal to $1 - \delta_a$ and $1 + \delta_a$ with equal probability. Parameters δ_n and δ_a capture uncertainty about market size and valuations, respectively.

Fix $c = 0$ and $\delta_n = 0.1$. Figure 7 plots wholesale price, manufacturer's profit, and total channel profit as functions of δ_a under three possible scenarios:

1. the retailer sets price;
2. the retailer sets quantity;
3. the retailer chooses to set price or quantity.

In the range $\delta_a \leq 0.05$, the uncertainty about valuations is relatively low and thus the manufacturer is better off when the retailer sets price, which is also in the retailer's interests. In the range $\delta_a > 0.1$, the uncertainty about valuations is large (compared to uncertainty about market size, $\delta_n = 0.1$) and thus the manufacturer is better off when the retailer sets quantity; this is also in the retailer's interests. Hence $\delta_a \leq 0.05$ and $\delta_a \geq 0.1$ correspond to "non-conflict" zones. In the range $0.05 < \delta_a < 0.1$, we observe the conflict zone: the manufacturer is better off if the retailer sets price, but the retailer prefers to set quantity when faced with the wholesale price w_p^*. For $\delta_a = 0.06$ the manufacturer is better off by lowering the wholesale price to 0.47, so that the retailer then prefers to set price. For higher values of δ_a the manufacturer is better off letting the retailer to set quantity and thus setting the wholesale price


a)
Wholesale Price
0.505
0.5
0.495
0.49
0.485
0.48
0.475
0.47
0.465
0
0.05
0.1
0.15
0.2
retailer sets price
retailer sets quantity
retailer sets price or quantity
δ_a
b)
Manufacturer's Profit
0.126
0.125
0.124
0.123
0.122
0.121
0.12
0.119
0
0.05
0.1
0.15
0.2
retailer sets price
retailer sets quantity
retailer sets price or quantity
δ_a
c)
Channel Profit
0.196
0.194
0.192
0.19
0.188
0.186
0.184
0.182
0.18
0.178
0
0.05
0.1
0.15
0.2
retailer sets price
retailer sets quantity
retailer sets price or quantity
δ_a

◀ **Fig. 7** Wholesale price **a**, manufacturer's profit **b**, and total channel profit **c** as a function of δ_a for cases where the retailer sets price, the retailer sets quantity, and the retailer chooses between setting price or quantity

at w_q^*. In other words, in the conflict zone the manufacturer must pay careful attention to how its choice of wholesale price determines whether the retailer sets price or quantity as its decision variable. Finally, observe that the total channel profit has a peak at $\delta_a = 0.06$. The intuition is as follows: The lower is the wholesale price, the better is channel coordination; since at that value of δ_a the manufacturer is better off by lowering the wholesale price, total channel profit goes up.

7 Conclusions

We examine how uncertainty about the demand curve influences a seller's choice to set price or quantity. In particular, the seller could be uncertain about the market size (e.g., the number of buyers) for its product or about how much consumers are willing to pay for the product. We show that greater uncertainty about market size favors setting price and greater uncertainty about valuations favors setting quantity. We also find a nonmonotonic relationship between marginal cost and the choice of sales mechanism.

Framing the issues differently, we show that the seller needs to consider which of these two decision variables it should be more flexible on. Clearly, there are different costs to the seller of retaining flexibility of prices versus flexibility of quantity. Our analysis allows the seller to be informed on how the nature of uncertainty and the magnitude of marginal cost affect this trade-off.

We also consider the channel setting with a single manufacturer and a single retailer. The interests of the manufacturer and retailer are aligned if there is prevailing uncertainty about either market size or about valuations. If both uncertainties are comparable, then there exists a "conflict zone" in which the manufacturer prefers the retailer to set price but the retailer prefers to set quantity. In this situation the manufacturer must either (a) lower the wholesale price to the level at which the retailer prefers to set price or (b) let the retailer set quantity and adjust the wholesale price accordingly. The manufacturer would then be better off if it could force the retailer to set the price.

The focus of our paper is on the effect of the nature of demand uncertainty on the basic operation mode (to set price or to set quantity) preferred by a seller. We also show how the results extend to the channel setting. We expect that the effect of the uncertainty about different parameters of demand curve on the seller's choice of price versus quantity, illustrated in the simple monopolistic one-shot sales model in this paper, will prevail under more sophisticated and realistic scenarios, such as auctions versus posted prices.

An interesting direction for future theoretic research could be the impact of the nature of demand uncertainty on other types of decisions, e.g., related to inventory, either for a single seller or for the channel, possibly with competition. On the empirical side, it would be important to distinguish and estimate uncertainty about market size and uncertainty about valuations.

Acknowledgements We thank Oleg Chuprinin and Kaifu Zhang for assistance, and the associate editor and two anonymous referees for helpful and constructive comments.

Appendix: Proof of Theorem 1

Observe that

$$E\big[\tilde{Q}(p)\big] = E[\tilde{n}\, g(p/\tilde{a})] = E\big[E[\tilde{n}\, g(p/\tilde{a}) \,|\, \tilde{a}]\big] = E\big[E[\tilde{n} \,|\, \tilde{a}]\, E[g(p/\tilde{a})]\big], \quad (5)$$

$$E\big[\tilde{P}(q)\big] = E[\tilde{a}\, f(q/\tilde{n})] = E\big[E[\tilde{a}\, f(q/\tilde{n}) \,|\, \tilde{a}]\big] = E\big[\tilde{a}\, E[f(q/\tilde{n}) \,|\, \tilde{a}]\big]. \quad (6)$$

In each equation, the first equality is by substitution of the formulas for $\tilde{Q}(p)$ and $\tilde{P}(q)$. The second equality is by iterative expectations. The third equality follows because we can treat $\tilde{a}$ as a constant term when conditioning on it and hence move it out of the expectation.

Suppose that $\tilde{n}_2$ is riskier than $\tilde{n}_1$ given $\tilde{a}$. By definition, this means that the conditional distribution of $\tilde{n}_2$ is riskier than the conditional distribution of $\tilde{n}_1$, conditioning on $\tilde{a}$.

One implication is that $E[\tilde{n}_1 \,|\, \tilde{a}] = E[\tilde{n}_2 \,|\, \tilde{a}]$ almost surely. Therefore, by Eq. 5, $E[\tilde{Q}(p; \tilde{n}_1, \tilde{a})] = E[\tilde{Q}(p; \tilde{n}_2, \tilde{a})]$. This gives us part 1 of the theorem.

A second implication is that $E[u(\tilde{n}_2) \,|\, \tilde{a}] < E[u(\tilde{n}_1) \,|\, \tilde{a}]$ almost surely for any strictly concave u. In particular, suppose that

(a) $n \mapsto f(q/n)$ is a strictly concave function of n on an interval that contains the supports of $\tilde{n}_2$ and $\tilde{n}_1$.

Then $E[f(q/\tilde{n}_2) \,|\, \tilde{a}] < E[f(q/\tilde{n}_1) \,|\, \tilde{a}]$ almost surely, and hence by Eq. 6 we have $E\big[\tilde{P}(q; \tilde{n}_2, \tilde{a})\big] < E\big[\tilde{P}(q; \tilde{n}_1, \tilde{a})\big]$. To conclude the proof of part 2 of the theorem, we need to show that (a) holds for $q \in (0, n_2^{\min}\hat{q}]$ if

(b) $\hat{R}(q)$ is strictly concave on $[0, \hat{q}]$.

Since $\tilde{n}_2$ is riskier than $\tilde{n}_1$ conditional on $\tilde{a}$, $[n_2^{\min}, \infty)$ is an interval that contains the supports of $\tilde{n}_2$ and $\tilde{n}_1$. (Recall that $n_2^{\min}$ is the greatest lower bound of the support of $\tilde{n}_2$.)

For any function $u(x)$ on $\mathbb{R}$ and any $\lambda > 0$, u is strictly concave on $[z', z'']$ if and only if $z \mapsto u(\lambda z)$ is strictly concave on $[z'/\lambda, z''/\lambda]$. Therefore, $n \mapsto f(q/n)$ is strictly concave on $[n_2^{\min}, \infty)$ if and only if $n \mapsto f(1/n)$ is strictly concave on $[n_2^{\min}/q, \infty)$. The latter holds for all $q \in (0, n_2^{\min}\hat{q}]$ if and only if it holds for the largest such q, i.e., if and only if $n \mapsto f(1/n)$ is strictly concave on $[\hat{q}, \infty)$.

The proof of Theorem 1 is then completed by using Lemma A.1. It says that $n \mapsto f(1/n)$ is strictly concave in n on $[\hat{q}, \infty)$ if and only if $\hat{R}(q) = f(q)\, q$ is strictly concave in q on $[0, \hat{q}]$.

Lemma A.1 *Let $F: \mathbb{R}_+ \to \mathbb{R}_+$ be differentiable. Define $G(x) \equiv xF(x)$ and $H(y) \equiv F(1/y)$. Let $\hat{y} \in \mathbb{R}_{++}$. Then $H(y)$ is strictly concave on $[\hat{y}, \infty)$ if and only if G is strictly concave on* $[0, 1/\hat{y}]$.

Proof We provide a simple proof for differentiable F by showing that $H''(y) < 0$ for all $y \in [\hat{y}, \infty)$ if and only if $G''(x) < 0$ for all $x \in [0, 1/\hat{y}]$. The lemma can also be proved algebraically without differentiability; we omit the details.[3] Observe that

$$G'(x) = xF'(x) + F(x), \qquad H'(y) = -\frac{1}{y^2}F'(1/y),$$

$$G''(x) = xF''(x) + 2F'(x), \qquad H''(y) = \frac{1}{y^4}F''(1/y) + \frac{2}{y^3}F'(1/y).$$

After canceling $1/y^3$ in the expression for $H''(y)$, $H''(y) < 0$ is equivalent to

$$\hat{H}(y) \equiv \frac{1}{y}F''(1/y) + 2F'(1/y) < 0.$$

By substituting $x = 1/y$, we see that $\hat{H}(y) < 0$ for all $y \geq \hat{y}$ if and only if $G''(x) < 0$ for all $x \leq 1/\hat{y}$. □

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