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Information overload in a network of targeted communication

Timothy Van Zandt*

As the costs of generating and transmitting information fall, the main bottlenecks in communication are becoming the human receivers, who are overloaded with information. For networks of targeted communication, I discuss the meaning of information overload, provide a theoretical treatment as the outcome of strategic interaction between senders, and examine mechanisms for allocating the attention of receivers. Such mechanisms increase the cost of sending messages and thereby shift the task of screening messages from the receivers to the senders, who know the contents of the messages. If the communication cost is low, then a tax on sending messages benefits all the senders if either the tax is redistributed to them as lump-sum transfers or their information about the receivers is sufficiently accurate.

1. Introduction

■ The physical resources for communication, such as data networks and the postal service, are scarce. Attempts have always been made to allocate these resources efficiently through price mechanisms. However, human communication is not merely the transmission of bits from one computer to another; information must go from brain to brain. The cost of physical communication resources has fallen so much that the relatively scarce resource is now the human attention needed to process and understand information. For example, compare the cost of mailing a working paper to a colleague with the opportunity cost of the time it takes your colleague to read and understand your paper. Yet human attention is not priced in networks.

Should it be? Human attention is not a passive resource; rather, each person controls the allocation of her attention to the information she receives. Since information is freely disposable, the mere fact that people receive more information than they can process does not mean they receive too much.

Nevertheless, complaints about information overload are common. This article provides an interpretation of information overload such that "nearly rational" decision makers can be made worse off by receiving more information. The decision makers are limited in their ability to process information and are forced to search among messages—without knowing their content, of course—if they receive more than they can process. An increase in the amount of information can be detrimental if it is accompanied by a decrease in the average value of the information. I construct a model of networked communication—messages are targeted to individual receivers based on imperfect demographic data—in which precisely this link between quantity and quality

* INSEAD; tvz@econ.insead.edu.

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arises endogenously as the outcome of strategic interaction between senders; this link is due to the fact that human attention is an unpriced resource.

Consider in more detail this last point. If there were only a single sender, then he would not choose to overload any receivers, preferring to decide himself which messages receivers process rather than having receivers do so blindly, just as a single sender would not overload an e-mail network to the point that messages are dropped. But when there are multiple senders, information overload is an externality between the senders, as each sender's messages crowd out messages of other senders. Thus, inadequate screening by senders arises because the physical cost of generating and transmitting information no longer rations access to receivers' attention.

My analysis is carried out via a simple model in which there is a one-time, one-way flow of information from several senders to a large population of receivers, and in which communication is targeted in the sense that senders can send their messages to any subset of receivers. An example is the communication from firms to consumers via targeted advertising, such as direct mail, e-mail, and fax. Targeted communication contrasts with untargeted or broadcast communication; in the latter, messages are sent either to all receivers or to none. The Web, billboards, and television are means of untargeted communication.

What differentiates my model theoretically from a standard commons game is that I consider a large population of unpriced resources—corresponding to the large population of receivers—and derive my main new insights from the aggregate properties of the equilibria and the relation to the imperfect marketing data of the senders. In my treatment of mechanisms for allocating attention, I do not restrict attention to ones that treat each receiver independently, decomposing the problem into many independent resource pricing problems. Such mechanisms, in which senders are charged a different cost for each receiver they target, are excessively complex to implement. Instead, I consider the effects of uniform changes in communication costs, either as exogenous technological parameters or as the result of taxes. Some of my main results are that, if transmission costs are low enough, (a) the senders unanimously benefit from the imposition of a tax on their communication if the revenues are returned to them as lump-sum transfers and (b) if the senders' marketing data are accurate enough, then they unanimously benefit from an increase in communication costs even if the extra costs are not reimbursed.

I began this Introduction by discussing information overload as a complaint of receivers, yet I have shifted the point of view to that of the senders. One of the messages of this article is precisely that information overload is a consequence of the strategic interaction among senders of messages and that they also are directly harmed by their collective overexploitation of the receivers' attention. To focus on this strategic interaction, I model the receivers passively and do not study the complex screening methods they might use to cope with information overload. Most of my formal analysis centers, therefore, on the strategic interaction between senders and on the senders' welfare.

However, I can still draw conclusions from the model about the welfare properties of the equilibria from the point of view of the receivers. The interests of senders and receivers do not coincide perfectly, but they do coincide in part. A sender benefits from having a receiver process a message only if the message influences the actions of the receiver—and this is precisely when the message also benefits the receiver. Thus, when senders reduce their screening and send messages they believe unlikely to interest the receivers, the receivers find that they are searching among messages that are more numerous but of less interest on average. Using this link, I am able to show that if the senders' transmission cost is low enough, then the aggregate welfare of receivers rises with this cost even though some receivers (those about whom the senders have inaccurate information) are better off with a lower communication cost. The message of this article is supported by a recent laboratory experiment by Kraut et al. (2002), who found that charging for e-mail can increase the effectiveness of communication.

Although this article is not specifically about communication within organizations, it has clear applications. If the agents in the organization all have the same preferences, i.e., if the agents act as a team, then information overload does not arise. The problem, however, is that agents gain advantages (e.g., prestige, promotions) by getting the attention of others, especially their superiors. Therefore we expect to see too much communication. The challenges managers face in rationing their attention have long been documented, such as by Mintzberg (1989). We observe that organizations use a variety of measures to restrict communication and ration attention, such as imposing limits on the length of a memo and restricting access to top management so that others have to signal the importance of what they have to say through perseverance.

The rest of the article is organized as follows. Section 2 is a literature review. Section 3 introduces a game with costly targeted communication. Section 4 shows that the game has an equilibrium, and I illustrate the game with a few examples. Section 5 shows that the total payoffs of the senders are often not maximized because senders communicate too much. I then consider mechanisms for alleviating this inefficiency: Section 6 studies pricing and bidding mechanisms that are tailored to each receiver type; Section 7 considers whether efficiency can be achieved by using surcharges that are uniform across receivers. Section 8 examines how the payoffs of the senders vary with the cost of communication in a neighborhood of zero. Section 9 takes up the welfare of receivers, and Section 10 concludes. All proofs are in the Appendix.

2. Literature review

■ Research on information overload is scattered in the fields of computer and information science, marketing, law, psychology, and economics. The research I have found has various objectives and various approaches, all distinct from those of this article. Here is an incomplete review.

Researchers in information science are interested in developing computer-mediated communication systems to increase the efficiency of information screening by receivers in order to reduce the relative scarcity of attention in electronic networks. Early examples are Malone et al. (1987) and Lai, Malone, and Yu (1988).

In a brief essay, Denning (1982) called attention to the problem of information overload. He also suggested several ways of restricting communication in order to reduce information overload, such as setting up different paths for different types of mail, restricting access to mailboxes to specifically authorized users, and allocating attention by bids attached to each message. Only the bidding mechanism is discussed in this article.

As discussed in Section 1, information overload is a problem because senders screen too little and so receivers end up poorly informed. If senders cannot selectively screen (e.g., if the messages are sent by nature), then there is no benefit to reducing the flow of information to receivers unless people cannot ignore incoming messages or systematically err by processing too many messages. Psychologists have studied the ability of humans to ignore incoming messages (for example, Libowski, 1975).

In the marketing literature, Jacoby, Speller, and Kohn (1974), Malhotra (1982), and Keller and Staelin (1987) have experimentally tested the following hypothesis: consumers choose to process too much information and thus may benefit from unselective restrictions on their access to information. This literature has ignored the inverse relationship between the quantity and quality of information that I study in this article.

In my model, the receivers can be viewed as searching among their messages; hence the model is related to the economics literature on consumer search. My model and results are different from those of that literature, however. Consumer welfare may decline in some search models when there are more firms in the market, but this is because equilibrium prices rise. Examples include Diamond (1971), Satterthwaite (1979), Stahl (1989), and Rosenthal (1980). In my model, the terms of the opportunities offered by senders are fixed exogenously, but the receivers' welfare may decline when the senders stop screening and when receivers are thus less likely to learn about the opportunities in which they are interested.

A leading interpretation of my model is as advertising (although price competition is not included in the model). Butters (1977), Stegeman (1991), Robert and Stahl (1993), Stahl (1994), and Grossman and Shapiro (1984) study competition between advertisers, but this competition is via prices rather than for the buyers' attention; each buyer processes all the advertisements she receives. A decrease in advertising costs may reduce firms' profits, but this is because it heightens price competition and not because it reduces the effectiveness of communication (as in my model); a reduction in the advertising cost always increases total surplus.

3. Model

■ The first key aspect of communication that I want to capture is the informational difference between a *sender* of a message, who costlessly knows its contents but is not fully informed about the interests of potential receivers, and a *receiver*, who costlessly knows her own interests but is not fully informed about the contents of messages without first processing them. The second key aspect is the externality between senders of messages as they compete for the scarce attention of receivers. Furthermore, I want to study a network in which communication is targeted rather than broadcast, meaning that messages are sent to individual receivers and that one of the decisions made by senders is whom to target—based on imperfect information about the receiver's interests.

I construct a model that brings these points to the forefront. Communication takes place once and is unidirectional. A finite number $n > 1$ of senders, indexed $j = 1, \ldots, n$, transmit messages to a large population of receivers. Each sender has a single message. It is unstructured, which means that (a) it can be read either in its entirety or not at all and (b) a receiver cannot distinguish between messages when choosing which ones to read. Each message informs a receiver of an opportunity or recommendation, which I call an offer. The terms of the offer are fixed. Hence, a sender's only decision is choosing whom to send messages to. A receiver is not aware of a sender's offer unless she receives and processes his message. A senders' payoff increases as a function of the number of receivers who receive, process, and respond to his message.

Before imposing additional simplifying assumptions, I note the following examples:

(i) Senders are organizers of seminars in a university and receivers are faculty and students. Each organizer can send an announcement about the seminar he is organizing to anyone in the university through the intramural mail or by e-mail. The organizer's status depends on the number of attendees.

(ii) Senders are safety groups in a firm and receivers are employees. Each group can send a recommendation to the employees, using intramural mail or e-mail, about the safety hazards it is responsible for. The groups are evaluated by the safety records.

(iii) Senders are firms and receivers are consumers. Each firm offers a transaction with exogenously given terms. For example, the transaction might be the sale of a product at an exogenously given price. Each firm can send any set of consumers a description of the transaction, a web address, an order form, or a toll-free number through the mail. The firm's profits depend on how many consumers accept the transaction.

I borrow terminology from the last example of firms and consumers.

Senders decide simultaneously whom to target. Each receiver then receives between zero and n messages. She can process up to m messages costlessly but cannot process more. So that information overload is potentially a problem, I assume $1 \leq m < n$. Because messages are indistinguishable when they arrive and cannot be partially processed, each message received is processed with equal probability. In particular, a receiver who receives ℓ messages processes all of them if $\ell \leq m$ and processes each one with probability m/ℓ otherwise.

This particular search technology is simpler than sequential search in which a cost is paid for each message processed, but it is not crucial to the results in this article. More important is my assumption (standard in the screening literature) that messages are unstructured, which precludes screening based on partial processing of messages. Such screening is an important mechanism for dealing with information overload. However, as much as such screening merits study and is important for a measurement of the effects studied here, it does not alter the qualitative points that I make. Each decision by a receiver of whether to process other parts of a message is still made based on partial information about the content and relevance of these parts. When searching through working papers, for example, being able to read titles in order to decide whether to read abstracts, abstracts in order to decide whether to read introductions, and so on, is a structured search process that improves communication but does not eliminate the possibility that we choose not to read papers that, *ex post,* we would have been happy to have read, or that we read papers that, *ex post,* we wish we had not bothered with.[1]

After processing up to m messages, the receiver either responds to a message if interested or does not. Whether a receiver is interested in a message does not depend on what other messages she processes. Thus, senders are competing only for the attention of the receiver and not directly with the other offers.

Particularly in the advertising scenario in which senders are firms, the design of the offers (for example, the design and pricing of products) and the direct competitive interaction when the offers are substitutes or complements are interesting issues. I abstract from these staples of the IO literature because they have been well studied in models without information overload and are not essential to the competition for attention. The interaction between price competition and competition for attention is left for future research. Note, however, that even in advertising much of the competition is for attention. Marketers are worried about how to get their advertising messages heard over the din of other advertisers, most of whom are selling products that are neither direct substitutes nor complements.

The most intricate part of my model is its treatment of receivers and of the senders' information about them. I use a reduced form that is motivated informally here but can be derived from a primitive model described in Van Zandt (2003). Senders cannot perfectly observe which messages a receiver would be interested in. They have common beliefs about this based on, for example, demographic and marketing data. I call these beliefs about a receiver her *type* and represent them by $t \in [0, 1]^n$, where $t = \langle t_1, \ldots, t_n \rangle$ and t_j is the probability that the receiver is interested in sender j's message. Thus, if sender j sends one of the ℓ messages received by a receiver of type $\langle t_1, \ldots, t_n \rangle$, then the probability that the receiver reads j's message is $\min\{1, m/\ell\}$ and the probability that she also responds to it is $t_j(\min\{1, m/\ell\})$.

Let $T \equiv [0, 1]^n$ be the set of types. The distribution of types in the population is given by a measure γ on T, normalized so that $\gamma(T) = 1$. I have in mind a large population, so I typically assume that γ is dispersed in some sense. If the demographic characteristics are good indicators of the interests of receivers, then γ places greater mass near the corners of T, where a type is an n-tuple of zeros and ones and hence the receiver's interests are known. In the other extreme, if the senders have poor information about the receivers, then γ is concentrated around its mean. If each receiver is interested in one and only one message, then the entire mass of γ is on the $(n-1)$-dimensional unit simplex Δ^{n-1}. This special case is used for some graphical examples because the set of types has one less dimension.

I require that each sender treat receivers of the same type the same way, targeting all or none of them. This is not a strong restriction, given the dispersion of types; senders can divide up a pool of receivers by dividing up similar types rather than dividing up receivers of the same type. Given this restriction, a typical pure strategy for a sender j is a Borel subset X_j of T, where X_j is the set of types that the sender targets. I let $\mathcal{B}$ be the set of Borel subsets of T, which thus denotes each sender's set of pure strategies. A strategy profile for the senders is denoted $X = \langle X_1, \ldots, X_n \rangle \in \mathcal{B}^n$.

The following notation keeps track of how many senders communicate to a given receiver type. Given a strategy profile $X = \langle X_1, \ldots, X_n \rangle$, let $X_{-j} \equiv \langle X_1, \ldots, X_{j-1}, X_{j+1}, \ldots, X_n \rangle$. For $t \in T$, let $X_j(t) = 1$ if $t \in X_j$ and $X_j(t) = 0$ otherwise; that is, $X_j(\cdot)$ denotes the indicator function of X_j. Define also $\#X(t) \equiv \sum_{j=1}^{n} X_j(t)$ and $\#X_{-j}(t) \equiv \sum_{i \neq j} X_i(t)$. Hence, $\#X(t)$ is the number of senders who target t, and $\#X_{-j}(t)$ is the number of senders other than j who target t.

If a receiver processes a message in which she is interested, then she accepts the offer and the sender is said to have made a "sale." The expected sales of sender j given a strategy profile X

[1] The article "There's Still No Quick Fix for Dumping the E-mail Junk," by Rob Pegararo (*International Herald Tribune*, July 24, 2000, p. 19), observes that even software filters are imperfect at screening out junk mail: "They cannot catch everything. Worse, yet, sometimes they go too far, trashing the things you do want to see." They are even worse at letting through just the junk mail that would be of interest.

are

$$\sigma_j(X) \equiv \int_{X_j} t_j \left(\min\{1, m/\#X(t)\}\right) \, d\gamma(t). \quad (1)$$

I treat expected sales as realized sales to simplify the discussion. I assume that sender j earns a surplus $s_j > 0$ per transaction, so j's gross payoff given X, before discounting communication costs, is $s_j \sigma_j(X)$.

Sending messages may be costly for a sender because it requires resources such as envelopes, paper, or bandwidth; these costs are nearly proportional to the number of receivers targeted. The sender also has fixed costs of preparing the original copy of the message and obtaining the mailing lists and marketing data. I disregard these fixed costs. I assume that the cost per message sent is the same for all senders and receivers. Let $c \geq 0$ be this cost per message, so that the cost of targeting types $B \in \mathcal{B}$ is $c\gamma(B)$. The net payoff of sender j given X and c is then

$$\pi_j(X; c) \equiv s_j \sigma_j(X) - c\gamma(X_j).$$

Each communication cost c thus defines a game Γ^c in normal form in which the players are the n senders, each sender's strategy set is $\mathcal{B}$, and sender j's payoff function is $\pi_j(\,\cdot\,; c)$. By an equilibrium for Γ^c, I mean a pure-strategy Nash equilibrium. My task is to examine how the equilibria of Γ^c depend on c.

4. Equilibrium

■ Each sender's surplus is linear in sales, the cost of communication is linear in the number of messages sent, and sales to a receiver depend only on the messages sent to that receiver. Therefore, the game Γ^c can be decomposed into independent single-receiver games. This is proved in Proposition 2 after developing necessary notation and definitions.

For $t \in T$, let $\Gamma^c(t)$ be the single-receiver game for type t. This is the game in normal form in which (a) there are n players; (b) each player's strategy set is $\{0, 1\}$, where 0 means "not send" and 1 means "send"; and (c) player j's payoff, given the strategy profile $x \equiv \langle x_1, \ldots, x_n \rangle \in \{0, 1\}^n$, is

$$u_j(x; c, t) \equiv \begin{cases} s_j t_j \left(\min\{1, m/\#x\}\right) - c & \text{if } x_j = 1, \\ 0 & \text{if } x_j = 0. \end{cases}$$

(Here $\#x$ denotes the number of messages sent, i.e., the cardinality of $\{i \mid x_i = 1\}$.) For example, if $n = 2$ and $m = 1$, then the payoff matrix for $\Gamma^c(t)$ is as shown in Table 1.

The game $\Gamma^c(t)$ is similar in structure to a costly-entry oligopoly game or to a commons game with only two actions: "exploit" and "not exploit." In particular, each player's payoff from sending a message (exploiting the resource) is decreasing in the number of other players who send a message and is independent of the identities of these players. Consequently, each single-receiver game has a pure-strategy equilibrium, as stated in Proposition 1.

Proposition 1. For all c and all $t \in T$, $\Gamma^c(t)$ has a pure-strategy equilibrium.

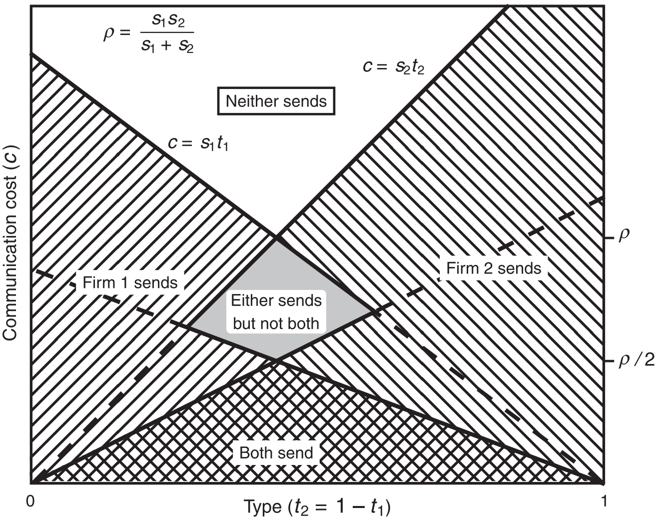
Figure 1 shows the equilibria of $\Gamma^c(t)$ as a function of c and t (for particular values of the

TABLE 1 Payoffs of a Single-Receiver Game

		Sender 2	
		Not send	Send
Sender 1	Not send	0, 0	$0, s_2t_2 - c$
	Send	$s_1t_1 - c, 0$	$(1/2)s_1t_1 - c, (1/2)s_2t_2 - c$

FIGURE 1

EQUILIBRIUM AS FUNCTION OF TYPE AND COMMUNICATION COST



other parameters). In the figure, $n = 2$, $m = 1$, supp($\gamma = 1$), and $s_1 = (3/4)s_2$. Each region is marked according to which sender(s) sends messages in equilibrium. In the gray region, there are two equilibria in which one and only one of the senders sends a message.

The aggregate game Γ^c is similar to an oligopoly game with costly entry and many independent markets or to a commons game with many independent resources to be exploited. A sender's payoff in Γ^c given a strategy profile $X \equiv \langle X_1, \ldots, X_n \rangle$ is equal to the average of his payoffs in the games $\Gamma^c(t)$ given $\langle X_1(t), \ldots, X_n(t) \rangle$. That is,

$$\pi_j(X; c) = \int_T u_j(X_1(t), \ldots, X_n(t); c, t)\, d\gamma(t).$$

Proposition 2 then follows easily.

Proposition 2. A strategy profile $\langle X_1, \ldots, X_n \rangle$ is an equilibrium for Γ^c if and only if, for γ-a.e. $t \in T$, $\langle X_1(t), \ldots, X_n(t) \rangle$ is a pure-strategy equilibrium for $\Gamma^c(t)$.

Corollary 1. Γ^c has an equilibrium.

For points in the boundaries dividing the regions in Figure 1, the set of equilibria of the game $\Gamma^c(t)$ consists of the equilibria for the bordering regions. It is convenient to be able to ignore these multiplicities. It would be sufficient to assume that the distribution of types has a density, but I state a weaker assumption that has two advantages: it is exactly what is needed in the proofs and it does not rule out the special case in which the support of γ is the simplex.[2]

Assumption 1. For all $i, j \in \{1, \ldots, n\}$ such that $i \neq j$, for all $a \in \mathbb{R}_+$, and for all $b \in \mathbb{R}$,

$$\gamma\{t \in T \mid t_j = at_i + b\} = 0.$$

Two strategies are said to be equivalent if their symmetric difference is γ-null. Two strategy profiles are said to be equivalent if, for each sender, the sender's strategies in the two profiles are equivalent. The term "unique"—whether applied to strategies, strategy profiles, or equilibria—means "unique up to equivalence."

[2] As usual, by the support of γ I mean the smallest closed set $B \subset T$ such that $\gamma(B) = 1$; I denote it by supp(γ). Any nonempty and relatively open subset of supp(γ) has strictly positive measure.

A useful implication of Assumption 1 is that each sender has a unique best response $X_j^*(X_{-j}; c)$ to any strategy profile X_{-j} of the other senders.

Proposition 3. Fix $j \in \{1, \dots, n\}$ and c. Let X_{-j} be a profile of strategies for senders other than j. Then sender j has a unique best response to X_{-j}. That is,

$$\max_{X_j \in \mathcal{B}} \pi_j(X_j, X_{-j}; c)$$

has a solution and it is unique up to equivalence. Denote this solution by $X_j^*(X_{-j}; c)$.

5. Strategies that maximize the senders' total payoffs

■ I can evaluate the outcomes of this communication game from the point of view of the senders, the receivers, or both. A typical reader is likely to identify with the receivers. However, senders are also hurt by information overload because the receivers' attention is not going to those senders who value it the most. In Sections 5–8, I focus on the senders. In Section 9, I bootstrap these results into statements about receiver welfare.

The following notation is useful. For $t \in T$ and $j \in \{1, \dots, n\}$, define sender j's valuation of t's attention to be $s_j t_j$ and denote it by $v_j(t)$. It equals j's expected gross payoff in the game $\Gamma^c(t)$ conditional on the receiver's processing his message. For $\ell \in \{1, \dots, n\}$, let $v^\ell(t)$ be the ℓth-highest valuation of t's attention; this is, if the senders are numbered such that $v_1(t) \geq \cdots \geq v_n(t)$, then $v^\ell(t) = v_\ell(t)$.

Observe that there is a unique strategy profile Y^c that maximizes the total net payoffs of the senders in the game Γ^c. I call Y^c the *efficient* strategy profile, keeping in mind that efficiency is with respect to only the senders' payoffs. The profile Y^c is defined as follows. It has no information overload; instead, for γ-a.e. $t \in T$, type t is targeted by the m senders with the highest valuations, except those whose valuations do not exceed c. This condition defines a unique strategy profile of the aggregate game because the n senders have distinct valuations for γ-a.e. $t \in T$ (due to Assumption 1). Sender j's strategy in this profile is

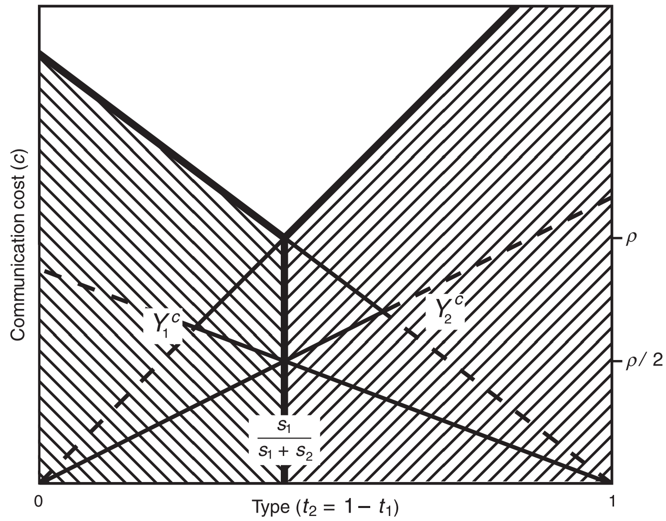
$$Y_j^c \equiv \{t \in T \mid v_j(t) \geq \max\{c, v^m(t)\}\}.$$

The boundaries of Y^c are drawn in Figure 2 for the example illustrated in Figure 1, in which $\operatorname{supp}(\gamma) = \Delta^1$. Again, $n = 2$, $m = 1$, and $s_1 = (3/4)s_2$. Sender 1 sends a message in the left-hand region Y_1^c delineated by the heavy line, and sender 2 sends a message in the right-hand region Y_2^c. When the communication cost c is less than or equal to $\rho = s_1 s_2/(s_1 + s_2)$, sender 2 optimally targets those receivers lying to the right of $s_1/(s_1 + s_2)$ in the figure and sender 1 optimally targets those lying to the left of $s_1/(s_1 + s_2)$. Compare this with the equilibrium shown in Figure 1. When the communication cost is zero, each sender communicates to all receivers (the dominant strategy) and all receivers are overloaded. As the communication cost increases from zero, the set of receivers who receive more messages than they can process decreases as each sender stops communicating to receivers who are unlikely to be interested in his message; however, there is still more communication than in Y^c for any communication cost less than $\rho/2$. When the communication cost reaches $\rho/2$, the two senders partition the set of receivers efficiently in equilibrium. When the communication cost is between $\rho/2$ and ρ, there are multiple equilibria, all of which have no information overload and one of which is Y^c. When the communication cost is greater than ρ, Y^c is the unique equilibrium.

The two main ideas (familiar from commons games) that this example illustrates are that, from the point of view of the senders, (a) there may be too much (but not too little) communication in equilibrium because the senders' messages have a negative externality on each other—each sender's message crowds the other messages; and (b) the communication cost is a natural rationing mechanism, so excessive communication arises in equilibrium only when the communication cost is too low. I summarize these ideas formally in Proposition 4. Part 1 says that the total communication in equilibrium is greater than or equal to the total communication given Y^c. Part

FIGURE 2

EFFICIENT COMMUNICATION (FROM POINT OF VIEW OF SENDERS) AS FUNCTION OF TYPE AND COMMUNICATION COST



2 says that if senders other than j adopt the strategies Y^c_{-j}, then sender j wants to target at least the receivers in Y^c_j and perhaps others. Part 3 says the strategy profile that maximizes the senders' profits is an equilibrium if and only if the communication cost is high enough.

Proposition 4.

(i) Let $\langle X_1, \ldots, X_n \rangle$ be an equilibrium for Γ^c. Then $\sum_{j=1}^n \gamma(X_j) \geq \sum_{j=1}^n \gamma(Y^c_j)$.
(ii) Let $j \in \{1, \ldots, n\}$. Then $Y^c_j \subset X^*_j(Y^c_{-j}; c)$.
(iii) Y^c is an equilibrium for Γ^c if and only if $c \geq [m/(m+1)](\text{ess sup}_{t \in T} v^{m+1}(t))$.[3]

6. Type-dependent mechanisms for allocating attention

■ If we can allocate attention using a mechanism that treats each receiver type independently, then we have a family of independent mechanism-design problems. Standard mechanisms for solving a tragedy of the commons can then be used to support Y^c.

For example, suppose the mechanism designer has the same information as the senders. Then the designer adds a surcharge that is just large enough to deter the sender with the $(m+1)$st highest valuation of t's attention from sending a message, given that m other senders send messages. However, there are two problems with such a mechanism. First, to determine the surcharges, the post office or network manager must know the descriptions of the senders and the marketing data. Second, it is burdensome to implement: to bill the senders it must keep track not only of the bulk quantity of each sender's messages but also of the identities or demographic characteristics of the receivers. With this information and accounting, the network could support Y^c simply by deciding which messages get delivered and to whom.

If we ignore the complexity of the mechanism and consider only the possibility that the senders' parameters and marketing data are their private information (contrary to the assumptions in my model), then we can use a simple bidding mechanism to allocate the receiver's attention. For a typical receiver, this is an m-unit private-value auction in which each sender wants at most one unit of the attention. A uniform-price auction as in Vickrey (1961) will work, as long as the

[3] Where "ess sup" is the supremum ignoring sets of measure zero, i.e., the lowest supremum on a set of full measure.

cost of the communication channels is zero (otherwise the decision of whether to bid depends on the anticipated bids of the other senders). Each sender writes a bid on each message it sends. For each receiver, the network delivers the messages addressed to her that have the m-highest strictly positive bids. It charges to each sender whose message is delivered the value of the $(m + 1)$st highest bid, or zero if no more than m messages are sent to the receiver.

If my model allowed for more structured screening by receivers, then an additional feature of a bidding mechanism is that receivers could use the bids as signals of the importance of messages. Such bidding with signalling and screening is a real phenomenon. For example, some advertisers offer free samples or sweepstakes if the receiver reads an advertisement.

7. Allocating attention through uniform surcharges

■ Both the type-dependent price mechanism and the bidding mechanism are complex to design and execute. Therefore, I consider what can be achieved using a surcharge on communication that is the same for all types of receivers. It is no longer possible to decompose the model into independent games for the different types. I say that a surcharge $p \geq 0$ supports Y^c if Y^c is an equilibrium of the game Γ^{c+p}.

Proposition 5 shows that typically there is no surcharge that supports Y^c when it is not an equilibrium for Γ^c. This happens because Y_j^c contains receivers who are "marginal" for sender j. That is, for arbitrarily small $\varepsilon > 0$, there is $t \in Y_j^c$ such that $s_j t_j - c < \varepsilon$. Any strictly positive surcharge on communication for sender j causes that sender to stop targeting some of these receivers. Thus, a surcharge cannot be used to support Y^c.

Proposition 5. Assume that Y^c is not an equilibrium for Γ^c. There is no surcharge that supports Y^c if either

(i) $\text{supp}(\gamma) = T$, or
(ii) $\Delta^{n-1} \subset \text{supp}(\gamma)$ and either $m > 1$ or $s_j \leq c$ for some j.

This is illustrated in Figure 3 for the case where $\text{supp}(\gamma) = T$. In the figure, $n = 2$, $m = 1$, $\text{supp}(\gamma) = [0, 1]^2$, $s_1 = s_2 = 1$, and $c = (2/5)$. In the efficient strategy, sender 1 sends to types in the region labelled Y_1^c and sender 2 sends to types in the region labelled Y_2^c. In equilibrium, both senders also send a message to types in the gray-shaded region. If this overloading is eliminated by a surcharge, the senders will no longer target receivers lying close to the lower-left-hand blank square. (Note that in the figure, the cost is fixed and the two dimensions are the two-dimensional type space; in Figures 1 and 2, the two dimensions are the one-dimensional type space and the communication cost.)

To understand what happens when $\text{supp}(\gamma) = \Delta^{n-1}$ and $m > 1$, suppose $c = 0$ and $s_1 = \cdots = s_n$. A receiver such as $(\varepsilon, 1 - \varepsilon, 0, \ldots, 0)$ is in Y_1^c because $t_j = 0$ for $j = 3, \ldots, n$, and yet is marginal for sender 1.[4]

8. Sales and payoffs when the communication cost is low

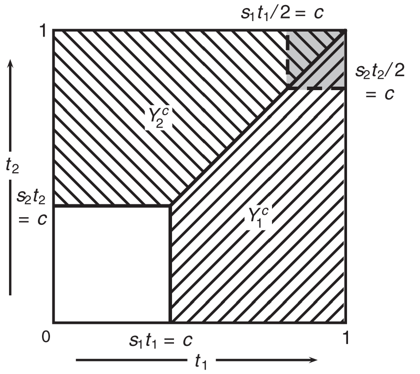
■ This section performs a comparative-statics exercise: How do the senders' sales and net payoffs depend on the cost of communication in a neighborhood of zero? The answer is summarized in Proposition 6, the most important result in this article.

This exercise has two interpretations. In the first, the cost of communication is a resource cost rather than an artificially imposed surcharge. I ask whether senders benefit from a fall in the cost of communication when this cost is already low. The answer is that the fall in the cost of communication actually reduces all senders' sales; furthermore, if the senders' marketing data

[4] When $\text{supp}(\gamma) = \Delta^{n-1}$ and $m = 1$, Y_j^c may not contain marginal receivers. Therefore, if small enough, a surcharge does not induce senders to drop receivers whom they should target in the efficient profile. The difficulty is that, in order to support Y^c, a surcharge must be large enough to eliminate information overload. Van Zandt (2003) delineates the limited set of parameter values for which these two requirements can be reconciled.

FIGURE 3

EFFICIENT VERSUS EQUILIBRIUM COMMUNICATION AS FUNCTION OF TYPE



are sufficiently accurate, it can even reduce all senders' net profits even though they save on communication costs. This highlights the role that the communication cost plays in rationing receivers' attention.

In the second interpretation, the cost of communication is mainly a surcharge and the resource cost of communication is negligible. I ask whether senders benefit from an increase in the surcharge. The answer is that an increase in the surcharge increases all senders' sales and hence all senders can benefit if the tax is redistributed as lump-sum payments; furthermore, if the senders' marketing data are good enough, all senders' net payoffs can increase even if the surcharge is not returned to them. Thus, although the message of the preceding section is that uniform surcharges on communication cannot generally support communication strategies that are efficient from the point of view of the senders, we show in this section that the optimal uniform surcharge is strictly positive if the resource cost of communication is low enough.

I begin by providing intuition for these results. Figure 4 illustrates the equilibrium for a small communication cost $c > 0$ when $n = 2$, $m = 1$, $s_1 = s_2$, and $\text{supp}(\gamma) = T$. Compare this with the equilibrium when $c = 0$, in which case both senders target all receivers. When the cost of communication increases from zero to c, sender 1 loses some sales because he stops targeting receivers in the region labelled "2." However, this loss in sales is negligible because these receivers are unlikely to be interested in his message anyway. On the other hand, sender 1's sales to receivers in the region labelled "1" increase because sender 2 stops targeting these receivers. Because many

FIGURE 4

WHY INCREASE IN COMMUNICATION COSTS FROM ZERO RAISES SALES

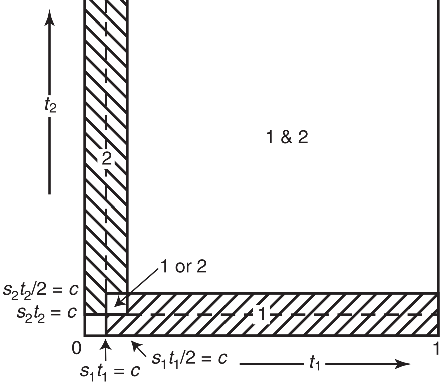
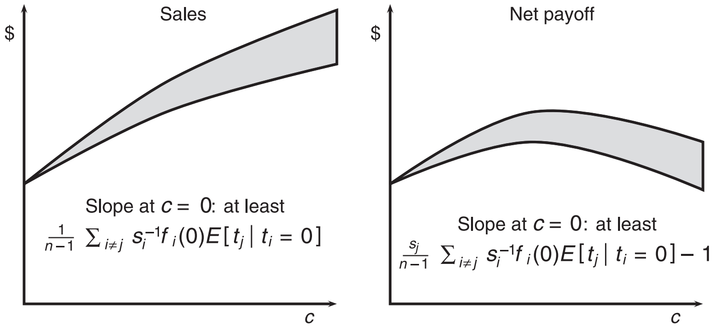


FIGURE 5

TOTAL SALES AND NET PAYOFF FOR SENDER AS FUNCTION OF COMMUNICATION COST



of these receivers are likely to be interested in sender 1's message, this increase in sales is not negligible and dominates the previously mentioned decrease.

Assumption 2. Either $\text{supp}(\gamma) = T$ or $\text{supp}(\gamma) = \Delta^{n-1}$, and γ has a continuous density, bounded above and away from zero, on the respective support.

For this intuition to hold up, the distribution of types has to have some mass in the shaded regions in Figure 4, and not all of this should be in the lower-left corner where $t_1, t_2 \approx 0$. Assumption 2 suffices.

Implications are that (a) the marginal distribution of each t_j has a continuous density f_j with $f_j(0) > 0$, and (b) for $i \neq j$, $E[t_i \mid t_j = 0] > 0$.

The meaning of Proposition 6 is that each sender's equilibrium sales are strictly increasing in the cost of communication in a neighborhood of zero and that if senders have accurate enough information about the receivers, then payoffs are also strictly increasing in a neighborhood of zero. However, because there may be multiple equilibria, I must characterize lower and upper bounds on payoffs. A pictorial version is given in Figure 5.

Proposition 6. Let $j \in \{1, \ldots, n\}$. For $c \geq 0$, let $\Sigma_j^+(c)$ be the maximum sales, let $\Sigma_j^-(c)$ be the minimum sales, let $\Pi_j^+(c)$ be the maximum net payoff, and let $\Pi_j^-(c)$ be the minimum net payoff for sender j in any equilibrium of Γ^c. Then the following statements hold.

(i) Σ_j^+, Σ_j^-, Π_j^+, and Π_j^- are well-defined continuous functions.

(ii) $\Sigma_j^+(0) = \Sigma_j^-(0)$ and $\Pi_j^+(0) = \Pi_j^-(0)$.

(iii) There is a continuously differentiable lower bound $B_j : \mathbb{R}_+ \to \mathbb{R}_+$ on Σ_j^- such that $B_j(0) = \Sigma_j^-(0)$ and

$$B_j'(0) = \frac{1}{n-1} \sum_{i \neq j} \frac{1}{s_i} f_i(0) E[t_j \mid t_i = 0] > 0.$$

(iv) There is a continuously differentiable lower bound $\hat{B}_j : \mathbb{R}_+ \to \mathbb{R}_+$ on Π_j^- such that $\hat{B}_j(0) = \Pi_j^-(0)$ and

$$\hat{B}_j'(0) = s_j \frac{1}{n-1} \sum_{i \neq j} \frac{1}{s_i} f_i(0) E[t_j \mid t_i = 0] - 1.$$

Hence, if

$$\frac{1}{n-1} \sum_{i \neq j} \frac{1}{s_i} f_i(0) E[t_j \mid t_i = 0] > \frac{1}{s_j}, \tag{2}$$

then $\hat{B}_j'(0) > 0$.

For the effects identified in Proposition 6 to be pronounced, two conditions are required. First, there should be enough receivers interested in some, but not all, of the messages so that it makes a difference for the effectiveness of communication which messages are sent. Second, the marketing data should be accurate, so that more mass is concentrated near the edges. Although senders know the contents of their messages, they are good screeners only to the extent that they have information about the receivers. Then each sender is pretty sure which receivers are unlikely to be interested in his message and should no longer be targeted once c rises.

9. Receiver welfare and the cost of communication

■ By focusing so far on the senders, we have highlighted that (a) information overload arises essentially as an outcome of strategic interaction between senders and (b) senders are also hurt from information overload. Nevertheless, the reader suffering from information overload in general and e-mail spamming in particular may be wondering: "What about me? What about the receivers' welfare?"

I now turn to these questions. The fact that a receiver gets positive surplus from an offer if and only if she accepts it—which is precisely the circumstance when the sender benefits—allows me to bootstrap the results in Sections 5–8 into statements about receiver welfare. I present these as corollaries, without further proofs.

The receiver behavior that underlies the model presented in Section 3 is as follows. A typical receiver obtains a surplus from accepting the offer in sender j's message. Thus, if that receiver gets and processes j's message, she accepts it if and only if this surplus is positive. Since search is costless up to the limit of m messages, her welfare is the sum of these surplus values over all the messages she receives, processes, and responds favorably to.

When a sender restricts his communication due to an increase in the communication cost, he stops targeting those receivers who, according to the marketing data, are least likely to be interested in his message. However, because the marketing data are imperfect, some of these receivers would be interested in the message; they end up worse off because they do not receive it. Therefore:

Corollary 2. Increasing the cost of communication never leads to a Pareto improvement for receivers.

For example, suppose that there is one record club for rock music and another for classical music. If direct-mail advertising is not free, then the classical club may not target 14-year-old males because most are interested only in rock music. Yet some of these receivers prefer classical to rock; they would be better off choosing randomly between the mailings for rock and classical music rather than receiving mail only from the rock club.

Consider what the logic behind Proposition 6 tells us about how receivers are affected by a small increase in the communication cost when it is initially low. I have already noted that some receivers are made worse off, but their numbers are very small—each sender stops targeting only receivers who are very unlikely to be interested in his message. In contrast, most receivers who receive fewer messages benefit because the messages they no longer receive did not interest them and were crowding out other messages. In our current example, the number of 14-year-old males made worse off by not receiving the mailing from the classical club is smaller than the number of those who benefit by being sure to process the mailing from the rock club. (This difference in numbers is more pronounced the better the senders' information about receivers.) We thus have the following.

Corollary 3. When the cost of communication is initially small and increases, many more receivers benefit than are hurt.

To quantify the total receiver surplus, we need the following notation. For simplicity, treat each receiver type as a separate person. Let y_{tj} be the surplus of receiver t for sender j's offer. The only implication my model has for how the distribution of y_{tj} depends on t is that $\text{Prob}[y_{tj} > 0] = t_j$.

Let $\bar{y}_{tj} = E[y_{tj} \mid y_{tj} > 0]$ be t's expected surplus from j's offer given that she is interested in it.[5] The expected surplus for receiver t generated by receiving and processing j's message is $\bar{y}_{tj}t_j$.

The simplest case is when (implausibly) $\bar{y}_{tj}$ is the same for all t. Denote the common value by ω_j. Then each sale by sender j generates receiver surplus ω_j and the total receiver surplus given a strategy profile X of the senders is

$$W(X) \equiv \sum_{j=1}^{n} \omega_j \sigma_j(X). \tag{3}$$

That is, it is the weighted sum of the senders' sales, with weights $(\omega_1, \ldots, \omega_n)$. According to Proposition 6, if the cost of communication is initially low and certain weak assumptions on γ are satisfied, then there is an increase in the cost of communication that leads to an increase in the sales of each sender and thus to an increase in receiver welfare.

More plausibly, $\bar{y}_{tj}$ is weakly increasing in t_j. Then the effect outlined in the previous paragraph is even more pronounced. When c increases from zero, sender j stops targeting receivers for whom t_j is low—and hence for whom $\bar{y}_{tj}t_j$ is low. Thus, even though some of these receivers are worse off because they are interested in j's message, not only are their numbers small but so is their lost surplus. Thus:

Corollary 4. Assume $\bar{y}_{tj}$ is weakly increasing in t_j. Then total receiver surplus is increasing in c in a neighborhood of zero.

Consider in more detail the single-receiver game $\Gamma^c(t)$. Receiver t's expected surplus given a strategy profile x is

$$\sum_{j=1}^{n} x_j \bar{y}_{tj} t_j \left(\min\{1, m/\#x\}\right) = \min\{m, \#x\} \frac{1}{\#x} \sum_{j:x_j=1} \bar{y}_{tj} t_j. \tag{4}$$

In particular, if t receives at least m messages, then her expected surplus is m times the average value of $\bar{y}_{tj}t_j$ over the senders from whom she receives a message. For a given sender j and strategy profile x_{-j} of the other senders, a message by j reduces the receiver surplus if and only if at least m of the other senders target t and $\bar{y}_{jt}t_j$ is lower than the average of $\bar{y}_{it}t_i$ over the other senders i who target t. This is when sender j's message exerts a negative externality on receiver t and this negative externality accounts for Corollaries 3 and 4.

On the other hand, when fewer than m of the other senders target receiver t, then a message by j raises t's surplus. Because the sender bears all the cost of communication and does not take into account this positive externality, he communicates too little to those receivers who are not overloaded.

Corollary 5. If the cost of communication is high enough that some receivers are not overloaded (but not so high that communication never raises total welfare), senders communicate too little to some receivers because they alone bear the cost of communication and they do not take receiver welfare into account.

This would be a reason to subsidize communication or to shift the cost of communication from senders to receivers when the cost of communication is high.

As I have noted and made use of, there is always the following minimal "coincidence of interests" between the senders and receivers: each gets surplus from a message's being processed if and only if the other does. In an extreme case, receiver welfare is given by equation (3), the weights $(\omega_1, \ldots, \omega_n)$ are proportional to $(s_1, \ldots, s_n)$, and the resource cost of communication is

[5] Alternatively, $\bar{y}_{tj}$ is the *average* surplus for j's offer among those receivers of type t for whom this surplus is positive.

zero. Then receiver welfare is proportional to the total payoffs of the senders, and any mechanism that maximizes these payoffs also maximizes receiver welfare and total welfare.

However, the receivers' and the senders' interests need not be nearly so aligned. This aggravates the consequences of information overload and can make it more difficult to raise the receivers' welfare by allocating their attention using the price mechanisms studied in this article. I take the liberty of moving outside the model of this article and limit myself to some suggestive remarks. Consider the targeting of a single receiver t. Suppose sender 1 is an e-mail spammer, sender 2 distributes public service information, and sender 3 is the receiver's boyfriend. Then s_1 may be larger than s_2, even though $\bar{y}_{t1}$ is smaller than $\bar{y}_{t2}$. Thus, with a nonzero communication cost, the receiver may be targeted by sender 1 but not sender 2, whereas she should be better off if the opposite were true. The senders may also have different communication costs; denote sender j's cost by c_j. Perhaps all the senders communicate by e-mail and so c_1 and c_2 are nearly zero, but c_3 can still be large because, even when communicating by e-mail, the boyfriend incurs a nonnegligible cost in composing the personalized message. It would be best to selectively increase the communication cost, rather than increase the cost for sender 3, who already bears a high communication cost. Nevertheless, an across-the-board increase in the communication cost may still increase the efficiency of communication if sender 1 is more responsive to the change in cost than is sender 3.

10. Remarks

■ The cost of transmission channels serves to allocate receiver input channels in communication networks. As these costs fall, the receiver input channels become relatively more scarce and receivers are overloaded with information. I showed in Sections 8 and 9 that in the absence of mechanisms for allocating the attention of receivers, all senders and many receivers in a network of targeted communication may become worse off when the cost of transmission channels falls.

This should not discourage efforts to reduce the cost of such channels; rather, it should encourage the design and adoption of mechanisms for allocating attention. I have studied how price and bidding mechanisms can increase the efficiency of communication. My results suggest, for example, that one way to reduce the burden of sorting through junk mail is to increase the postal rate for bulk mail. Advertisers will then target consumers more selectively, which may both increase their sales and benefit the consumers.

Of course, this article models only a specific class of networks under fairly severe restrictions. For example, in my model the messages were homogeneous. In practice, a mail network handles personal letters, solicited bulk mail, unsolicited commercial bulk mail, and unsolicited nonprofit bulk mail. Since any correspondence can pass for a personal letter, the price of bulk mail cannot exceed the first-class rate for personal letters. There is no reason to increase the cost of sending a personal letter, since the sender has already incurred a large cost in writing it. There is no reason to increase the cost of solicited communication, because the receiver has already made an informed decision to allocate his attention to the message. The inability of the network to differentiate perfectly among these types of mail constrains the feasibility of mechanisms.

Messages were not structured in my model. The allocation of attention when messages are structured is particularly interesting when senders can lie or use deceptive structuring, such as writing "Regional Weather Alert" on an advertisement for roofing or having the words "pay to the order of" appear in the address window. We need mechanisms that induce truthful, structured messages. The bidding mechanism discussed in Section 6 is an example.

Appendix

■ Proofs of Propositions 1–6 and Corollary 1 follow.

Proof of Proposition 1. I use the following structure of the game: (i) each player has a fixed payoff of zero from not sending a message and (ii) each player's payoff from sending a message is decreasing in the number of other players who also send a message but does not depend on the identities of those players.

Fix c and $t \in T$. For $j \in \{1, \ldots, n\}$, let $\ell_j \in \{0, 1, \ldots, n\}$ be such that sending a message is a best response for player j in $\Gamma^c(t)$ if and only if at most $\ell_j - 1$ other players send messages. Specifically,

$$\ell_j \equiv \max\{\ell = 0, 1, \ldots, n \mid \ell = 0 \text{ or } s_j t_j(\min\{1, m/\ell\}) - c \geq 0\}.$$

Renumber the players if necessary so that $\ell_1 \geq \cdots \geq \ell_n$. Let k be the highest-numbered player j for whom it is optimal to send a message given that players $1, \ldots, j-1$ also send a message. Specifically,

$$k \equiv \max\{j = 0, 1, \ldots, n \mid j = 0 \text{ or } \ell_j \geq j\}.$$

Let x be the strategy profile such that players $1, \ldots, k$ send messages and the remaining players do not. That is, $x_j = 1$ for $j = 1, \ldots, k$ and $x_j = 0$ for $j = k+1, \ldots, n$. For players $j = 1, \ldots, k$, we have $\ell_j - 1 \geq k - 1$; hence each finds it optimal to send a message given that $k - 1$ other players do so as well. For players $j = k+1, \ldots, n$, we have $\ell_j - 1 < k$, since k is defined such that $\ell_{k+1} < k + 1$; hence each finds it optimal to not send a message given that k other players do so. Therefore, x is an equilibrium of $\Gamma^c(t)$. *Q.E.D.*

Proof of Proposition 2. For all $j \in \{1, \ldots, n\}$,

$$\int_T u_j\,(X_1(t), \ldots, X_n(t); c, t)\; d\gamma(t) \geq \int_T u_j \left(X_1(t), \ldots, X_{j-1}(t), X_j'(t), X_{j+1}(t), \ldots, X_n(t); c, t\right) d\gamma(t) \qquad \text{(A1)}$$

for every $X_j' \in \mathcal{B}$ if and only if, for γ-a.e. $t \in T$, $X_j(t)$ solves

$$\max_{x_j \in \{0,1\}} u_j \left(X_1(t), \ldots, X_{j-1}(t), x_j, X_{j+1}(t), \ldots, X_n(t); c, t\right).$$

Q.E.D.

Proof of Corollary 1. Fix c. For all $t \in T$, let $\mathcal{E}^c(t) \subset \{0, 1\}^n$ be the set of equilibria for $\Gamma^c(t)$. By Proposition 1, $\mathcal{E}^c(t)$ is nonempty. Since the payoffs in $\Gamma^c(t)$ depend continuously on t, the graph of the correspondence $\mathcal{E}^c : T \to \{0, 1\}^n$ is closed. Therefore, $\mathcal{E}^c$ has a measurable selection $\langle X_1(\cdot), \ldots, X_n(\cdot)\rangle$. By Proposition 2, $\langle X_1, \ldots, X_n\rangle$ is an equilibrium. *Q.E.D.*

Proof of Proposition 3. Let $j \in \{1, \ldots, n\}$, let $c \geq 0$, and let $X_{-j} \equiv \langle X_1, \ldots, X_{j-1}, X_{j+1}, \ldots, X_n\rangle \in \mathcal{B}^{n-1}$ be a profile of strategies for senders other than j. Let X_j^- (respectively, X_j^+) be the set of types t for whom j has a strict (respectively, weak) incentive to send a message, given c and $X_{-j}(t)$. That is,

$$X_j^- \equiv \{t \in T \mid s_j t_j(\min\{1, m/(\#X_{-j}(t) + 1)\}) > c\}$$
$$X_j^+ \equiv \{t \in T \mid s_j t_j(\min\{1, m/(\#X_{-j}(t) + 1)\}) \geq c\}.$$

Then X_j^- and X_j^+ are both best responses. Furthermore, for any best response $X_j \in \mathcal{B}$, $X_j^- \underset{\text{a.s.}}{\subset} X_j \underset{\text{a.s.}}{\subset} X_j^+$ (where $B \underset{\text{a.s.}}{\subset} B'$ means that γ-a.e. element of B is in B'). The symmetric difference between X_j^- and X_j^+ is

$$X_j^+ \setminus X_j^- = \{t \in T \mid s_j\, t_j(\min\{1, m/(\#X_{-j}(t) + 1)\}) = c\}$$
$$\subset \bigcup_{\ell=m}^{n} \{t \in T \mid t_j = (\ell/m)(c/s_j)\}.$$

Assumption 1 implies that $\gamma\{t \in T | t_j = (\ell/m)(c/s_j)\} = 0$ for $\ell = m, \ldots, n$, so $\gamma(X_j^+ \setminus X_j^-) = 0$. Hence X_j^- and X_j^+, and any other best response, are equivalent. *Q.E.D.*

Lemma A1 and Corollary A1 are used in the proof of Propositions 4. Recall that, for $\ell \in \{1, \ldots, n\}$ and $t \in T$, $v^\ell(t)$ is the ℓth-highest valuation $s_j t_j$ of t's attention.

Lemma A1. Let $p \geq 0$ and $t \in T$. Then $Y^c(t)$ is an equilibrium of $\Gamma^{cp}(t)$ if and only if

$$v^{\#Y^c(t)}(t) - (c + p) \geq 0, \qquad \text{(A2)}$$

$$\frac{m}{m+1} v^{m+1}(t) - (c + p) \leq 0. \qquad \text{(A3)}$$

Proof. Equation (A2) means that the lowest-valuation sender who should send to type t according to $Y^c(t)$ finds it profitable to do so, given that his message will be processed for sure. Equation (A3) means that the highest-valuation sender who should not be sending to type t according to $Y^c(t)$ does not find it profitable to do so, given that m senders are already sending messages. These two conditions are necessary and sufficient when $\#Y^c(t) = m$. If instead $\#Y^c(t) < m$, then $v^m(t) \leq c$ and so equation (A3) is trivially satisfied while equation (A2) is necessary and sufficient. *Q.E.D.*

Corollary A1. $Y^c(t)$ is an equilibrium of $\Gamma^c(t)$ if and only if $[m/(m+1)]v^{m+1}(t) \leq c$.

Proof. Consider Lemma A1 for $p = 0$. Equation (A2) becomes $v^{\#Y^c(t)}(t) - c \geq 0$; this holds because of the efficiency of Y^c. The necessary and sufficient condition is thus equation (A3), which becomes $[m/(m+1)]v^{m+1}(t) \leq c$. *Q.E.D.*

Proof of Proposition 4.

(i) Let X be an equilibrium of Γ^c. I show that for γ-a.e. $t \in T, \#X(t) \geq \#Y^c(t)$. Let t be such that $\langle X_1(t), \ldots, X_n(t)\rangle$ is an equilibrium for $\Gamma^c(t)$; this holds γ-a.e. according to Proposition 2. If at least m messages are sent in this equilibrium (i.e., if $\#X(t) \geq m$), then $\#X(t) \geq \#Y^c(t)$, since $\#Y^c(t) \leq m$ (receivers are never overloaded in the efficient strategy profile). Suppose instead that $\#X(t) < m$. Then each sender's message is processed for sure, as would also be the case if one more sender sent a message. Assume that $s_j t_j \neq c$ for each j, which holds for γ-a.e. $t \in T$. Then sender j sends a message in this equilibrium if and only if $s_j t_j > c$. This is true also for the efficient strategy profile when this inequality holds for at most m senders. Hence, $\#X(t) = \#Y^c(t)$.

(ii) Let $t \in Y^c_j$. Then (a) $\#Y^c_{-j}(t) \leq m - 1$, and (b) $s_j t_j > c$. Condition (a) implies that j's message to t would certainly be processed if sent; condition (b) implies that j's best response is therefore to send t a message. Hence, $t \in X^*_j(Y^c_{-j}; c)$.

(iii) This follows directly from Corollary A1. *Q.E.D.*

Proof of Proposition 5. Suppose Y^c is not an equilibrium of Γ^c. Let $p > 0$. There is a sender j for which $s_j > c$, since otherwise Y^c is a profile of empty sets and is an equilibrium for Γ^c. Then define

$$U \equiv \{t \in T \mid p > v_j(t) - c > 0 \text{ and } v_j(t) > v^{m+1}(t)\}. \tag{A4}$$

The set U consists of types $t \in T$ such that $Y^c(t)$ is not an equilibrium of $\Gamma^{c+p}(t)$: the conditions $v_j(t) - c > 0$ and $v_j(t) > v^{m+1}(t)$ mean that $t \in Y^c_j$, but the condition $v_j(t) - (c+p) < 0$ means that j's dominant strategy in $\Gamma^{c+p}(t)$ is to not send a message. Observe that U is open because v_j and v^{m+1} are continuous. Hence, if I can show that U intersects $\text{supp}(\gamma)$, then $\gamma U > 0$ and Y^c is not an equilibrium of Γ^{c+p}.

Since $s_j > c$, there exists a $t'_j \in (0, 1)$ such that $p > s_j t'_j - c > 0$. The final step is to construct, from t'_j, an element $t \in U \cap \text{supp}(\gamma)$; I treat separately the two cases stated in the proposition.

(i) Suppose $\text{supp}(\gamma) = T$. Then $\langle 0, \ldots, t'_j, \ldots, 0\rangle \in U \cap \text{supp}(\gamma)$.

(ii) Suppose instead $\Delta^{n-1} \subset \text{supp}(\gamma)$. If $m > 1$, then choose any $i \neq j$. If instead there is i such that $s_i \leq c$, then let t be the element of Δ^{n-1} such that $t_j = t'_j$ and $t_i = 1 - t'_j$. Then $t \in U \cap \Delta^{n-1}$. *Q.E.D.*

Before beginning the proof of Proposition 6, I state the implications of Assumption 2 that are needed in the proof.

Lemma A2. The following are implications of Assumption 2.

(i) For all $j \in \{1, \ldots, n\}$, the marginal distribution of t_j has a continuous density f_j such that $f_j(0) > 0$.
(ii) For all $i, j \in \{1, \ldots, n\}$ such that $i \neq j$, there is a continuous version of $E[t_i \mid t_j]$ such that $E[t_i \mid t_j = 0] > 0$.
(iii) If $n > 2$, then there is some k such that, for all $i, j \in \{1, \ldots, n\}$ such that $i \neq j$ and for all $\bar{t} \geq 0$, $\gamma\{t \in T \mid t_i \leq \bar{t} \text{ and } t_j \leq \bar{t}\} \leq k\bar{t}^2$.

Proof. The existence of the marginal density for t_j follows immediately from the existence of a density for γ. $f_j(0) > 0$ follows from the fact that the density of γ is bounded from below.

If $\text{supp}(\gamma) = \Delta^{n-1}$ and $n = 2$, then $E[t_i \mid t_j] = 1 - t_j$ and part 2 therefore follows. Otherwise, the joint distribution of t_i, t_j also has a continuous density f_{ij} (on $[0, 1]^2$ if $\text{supp}(\gamma) = T$ and on $\{(t_i, t_j) \in \mathbb{R}^2_+ \mid t_i + t_j \leq 1\}$ if $\text{supp}(\gamma) = \Delta^{n-1}$) that is bounded from below and $E[t_i \mid t_j = 0] = \int_0^1 t_i f_{ij}(t_i, 0)/f_j(0)\, dt_i$. Since $f_{ij}(t_i, 0) > 0$ and $f_j(0) < \infty$, $E[t_i \mid t_j = 0] > 0$.

Let k be the upper bound on the density. Then $\gamma\{t \in T \mid t_i \leq \bar{t} \text{ and } t_j \leq \bar{t}\}$ is no more than k times the Lebesgue measure of this set. When $\text{supp}(\gamma) = T$, the Lebesgue measure of this set is exactly $\bar{t}^2$. When $\text{supp}(\gamma) = \Delta^{n-1}$, the Lebesgue measure is at most $4\bar{t}^2$. *Q.E.D.*

Proof of Proposition 6.

(i) First I show that these functions are well defined; that is, I show that there is an equilibrium with the highest sales, one with the lowest sales, one with the highest profit, and one with the lowest profit. Let $c \geq 0$. From the proof of Corollary 1, recall that the graph of the equilibrium correspondence $\mathcal{E}^c : T \to \{0, 1\}^n$ for the single-receiver games is closed. In particular, for each strategy profile of the single-receiver games, the set of types for which it is an equilibrium is closed. Consider Σ^+_j. I can rank the single-receiver strategy profiles $\{0, 1\}^n$ so that j's sales are weakly decreasing in the ranking for any single receiver game. (Profiles in which j does not send a message get the lowest ranking; other profiles are ranked inversely by the number of other senders who send a message.) For each receiver type, I select the highest-ranked equilibrium strategy profile. This selection is measurable and hence defines an equilibrium for Γ^c. Sender j's sales in this equilibrium are as high as for any other equilibrium. The proofs for Σ_j^-, Π_j^+, and Π_j^- are analogous.

Next I show continuity. Fix $c \geq 0$. For $\delta > 0$, let Θ^δ be the set of types $t \in T$ such that $\Gamma^{c'}(t)$ and $\Gamma^c(t)$ have the same equilibria for any c' such that $|c - c'| < \delta$. Let $\varepsilon > 0$. I show later that there is a $\delta > 0$ such that $\gamma(T \setminus \Theta^\delta) < \varepsilon$. If $|c - c'| < \delta$, then equilibria with the highest sales for a sender j in the games Γ^c and $\Gamma^{c'}$ can differ only with respect to types in $T \setminus \Theta^\delta$; hence, $|\Sigma_j^+(c) - \Sigma_j^+(c')| < \varepsilon$. Therefore, Σ_j^+ is continuous. The proofs for Σ_j^-, Π_j^+, and Π_j^- are analogous.

I still have to show the existence of a $\delta > 0$ such that $\gamma(T \setminus \Theta^\delta) < \varepsilon$. Consider a type $t \in T$ such that for all $j \in \{1, \dots, n\}$ and $\ell \in \{m, \dots, n\}$, $s_j t_j (m/\ell) \neq c$; Assumption 1 implies that the set of such types has full measure. Each of the equilibria in $\Gamma^c(t)$ is strict and persists for a small perturbation of δ. Hence, γ-a.e. $t \in T$ belongs to Θ^δ for some $\delta > 0$. Using the countable additivity of γ, I then have that $\gamma\Theta^\delta \uparrow \gamma T$ as $\delta \downarrow 0$. Hence, for every $\varepsilon > 0$ there is some $\delta > 0$ such that $\gamma(T \setminus \Theta^\delta) < \varepsilon$.

(ii) Game Γ^0 has a unique equilibrium in which senders target all receivers. Thus, $\Sigma_j^+(0) = \Sigma_j^-(0)$ and $\Pi_j^+(0) = \Pi_j^-(0)$.

(iii) I construct the lower bound $B_1 : \mathbb{R}_+ \to \mathbb{R}$ on Σ_1^-. By a change of indices, the result applies also to senders $j \neq 1$.

Let $c \in \mathbb{R}_+$. Denote by $\tilde{X}_1$ the set of types to whom it is a strictly dominant strategy for sender 1 to send a message; for each sender $j \neq 1$, denote by $\tilde{X}_j$ the types to whom j would not send a message if all other senders do so. That is,

$$\tilde{X}_1 = \{t \in T \mid (m/n)s_1 t_1 > c\},$$
$$\hat{X}_j = \{t \in T \mid (m/n)s_j t_j < c\}.$$

In any equilibrium, player 1 sends a message to at least the types in $\tilde{X}_1$ and these types receive at most n messages. For types in $\tilde{X}_1 \cap (\cup_{j=2}^n \hat{X}_j)$, not all senders can send messages in equilibrium, so these types receive at most $n - 1$ messages. Therefore, as a lower bound on $\Sigma_1^-(c)$, I have the sales $(m/n)\int_T t_1\, d\gamma(t)$ obtained when all senders target all receivers, with two adjustments: types in $\cup_{j>1}\hat{X}_j$ are targeted by at most $n - 1$ rather than n senders; and types in $\tilde{X}_1^\circ$ (the complement of $\tilde{X}_1$) might not be targeted by any sender 1—yet I have so far counted sales of up to $(m/(n-1))t_1$ for such types. I thus have

$$\sigma_1(X) \geq \frac{m}{n}\int_T t_1\, d\gamma(t) + \left(\frac{m}{n-1} - \frac{m}{n}\right)\int_{\cup_{j=2}^n \hat{X}_j} t_1\, d\gamma(t) - \frac{m}{n-1}\int_{\tilde{X}_1^\circ} t_1\, d\gamma(t).$$

I obtain another lower bound by breaking up the integral $\int_{\cup_{j=2}^n \hat{X}_j} t_1\, d\gamma(t)$ into one integral for each $\hat{X}_j$ and then subtracting potential overlap, replacing t_1 by its upper bound 1 as I do so:

$$\begin{aligned}\Sigma_1^-(c) \geq {} & \frac{m}{n}\int_T t_1\, d\gamma(t) + \left(\frac{m}{n-1} - \frac{m}{n}\right)\sum_{j=2}^n \int_{\hat{X}_j} t_1\, d\gamma(t) \\ & - \left(\frac{m}{n-1} - \frac{m}{n}\right)\left(\sum_{j=2}^n \sum_{\substack{i\geq 2\\ i\neq j}} \gamma(\hat{X}_j \cap \hat{X}_i)\right) - \frac{m}{n-1}\int_{\tilde{X}_1^\circ} t_1\, d\gamma(t). \qquad \text{(A5)}\end{aligned}$$

The double summation does not appear if $n = 2$. Otherwise, observe that

$$\hat{X}_j \cap \hat{X}_i = \{t \in T \mid t_i < (cn)/(ms_i) \quad \text{and} \quad t_j < (cn)/(ms_j)\}.$$

Therefore, according to Lemma A2, $\gamma(\hat{X}_j \cap \hat{X}_i) \leq \bar{k}c^2$, where $\bar{k} = k(n/(m \max\{s_j \mid j \neq 1\}))^2$ and k is the constant given in (iii) of the lemma. I rewrite the integrals in equation (A5) using the notation and assumptions in (i) and (ii) of Lemma A2. For example, since $\hat{X}_j = \{t \in T \mid (m/n)s_j t_j < c\}$, I have $\int_{\hat{X}_j} t_1\, d\gamma(t) = \int_0^{cn/ms_j} E[t_1 \mid t_j] f_j(t_j)\, dt_j$. I thereby obtain the following lower bound on $\Sigma_1^-(c)$:

$$\begin{aligned}B_1(c) \equiv {} & \frac{m}{n}E[t_1] + \left(\frac{m}{n-1} - \frac{m}{n}\right)\sum_{j=2}^n \int_0^{cn/ms_j} E[t_1 \mid t_j] f_j(t_j)\, dt_j \\ & - \left(\frac{m}{n-1} - \frac{m}{n}\right)(n-1)(n-2)\bar{k}c^2 - \frac{m}{n-1}\int_0^{cn/ms_1} t_1 f_1(t_1)\, dt_1.\end{aligned}$$

Note that B_1 is continuously differentiable. In particular, $t_j \mapsto E[t_1 \mid t_j] f_j(t_j)$ is continuous because (a) f_j is continuous and (b) $E[t_1 \mid t_j]$ is continuous when $f_j > 0$. The derivative of B_1 evaluated at $c = 0$ is

$$\frac{d}{dc}B_1(c)\bigg|_{c=0} = \left(\frac{m}{n-1} - \frac{m}{n}\right)\sum_{j=2}^{n}\frac{n}{ms_j}E[t_1 \mid t_j = cn/ms_j]f_j(cn/ms_j)\bigg|_{c=0}$$

$$= \frac{1}{n-1}\sum_{j=2}^{n}s_j^{-1}E[t_1 \mid t_j = 0]f_j(0).$$

This derivative is strictly positive if there is an $i \neq j$ such that $f_i(0) > 0$ and $E[t_j \mid t_i = 0] > 0$, so that $B_1(\cdot)$ is strictly increasing in a neighborhood of zero.

Finally, observe that $B_1(0) = \Sigma_1^-(0)$. Given the properties of Σ_1^+ and Σ_1^-, these must be strictly increasing in a neighborhood of zero if B_1 is.

(iv) Let $\hat{B}_1(c) = s_1B_1(c) - c$. Then $\hat{B}_1(c)$ is a lower bound on Π_1^- because $B_1(c)$ is a lower bound on $\Sigma_1^-(c)$. The function $\hat{B}_1(\cdot)$ is continuously differentiable and

$$\frac{d}{dc}\hat{B}_1(c)\bigg|_{c=0} = s_1\frac{1}{n-1}\sum_{j=2}^{n}s_j^{-1}E[t_1 \mid t_j = 0]f_j(0) - 1.$$

Equation (2) implies that $(d/dc)\hat{B}_1(\cdot)|_{c=0} > 0$, so that $\hat{B}_1(\cdot)$ is also strictly increasing in a neighborhood of zero.

We also see that $\hat{B}_1(0) = \Pi_1^-(0)$. Given the properties of Π_1^+ and Π_1^-, these must be strictly increasing in a neighborhood of zero if $\hat{B}_1$ is.

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