

HIDDEN INFORMATION ACQUISITION AND STATIC CHOICE

ABSTRACT. This note explores the consequence of hidden information acquisition for static choice theory. We show that any choice function in the observable problem can be consistent with some well-behaved choice function in a metaproblem with unobservable costly information acquisition. This illustrates how choices may not satisfy consistency conditions because a decision maker's decision process (in this case, information acquisition) depends on her feasible set. It also illustrates the importance of modeling the source of violations of consistency conditions, rather than simply weakening axioms on preferences.

KEY WORDS: Static choice, information acquisition, consistency, intransitivity.

1. INTRODUCTION

A finite static choice problem is a pair $\langle X, \mathcal{A} \rangle$, where X is a finite set of alternatives and \mathcal{A} is a set of non-empty subsets of X , called feasible sets. A decision maker's choice function $C : \mathcal{A} \rightarrow 2^X$ for $\langle X, \mathcal{A} \rangle$ associates with each $A \in \mathcal{A}$ the non-empty set $C(A) \subseteq A$ of alternatives the decision maker might choose if A is the set of feasible alternatives.

Assume that \mathcal{A} contains all subsets of X with one or two elements. Let \succsim be the binary relation on X defined by

$$x \succsim y \iff x \in C(\{x, y\}).$$

\succsim is called the (weak revealed) preference relation.

Since we have assumed that C has non-empty values, \succsim is complete and reflexive. Two basic assumptions of most normative and description decision theories, including "conventional" expected utilities theories such as Savage (1954) and some "non-conventional" theories such as Machina (1982), are:

Axiom 1. \succsim is transitive.

Axiom 2 (*Extendibility Principle or IIA.*) For $A \in \mathcal{A}$, $C(A)$ consists of the \succeq -maximal elements of A .

These are often referred to as ‘rationality’ or ‘consistency’ axioms. In this setting, they are equivalent to Houthakker’s axiom of revealed preference. Given Axiom 1, Axiom 2 is equivalent to the axiom of independence of irrelevant alternatives (IIA), also known as the ‘Chernoff condition’ or ‘property α ’ (Sen (1971)). In models with additional structure, such as choices among lotteries or state-contingent outcomes, the list of assumptions on the binary preferences may grow, but Axioms 1 and 2 are usually maintained.

In practice, experimenters observe that binary preferences do not satisfy transitivity, or other assumptions imposed in more structured models. An early example of a violation of transitivity is May (1954). See, for example, Thaler (1991) for a discussion of more recent results. It is often said that these inconsistencies have to do with bounded rationality or unobserved decision processes. For example, Slovik and Lichtenstein (1983) give such an explanation for the preference reversal phenomenon, and framing effects (Tversky and Kahneman (1981)) are certainly the result of unobserved decision processes.

The empirical violations of the standard models have led to a small industry of alternative models in which axioms on preferences are weakened, but the extendibility principle is maintained. See Fishburn (1991) for a review of some models in which preferences are intransitive, and see Machina (1987) for a review of some generalizations of Savage’s expected utility theory. Some of these alternate theories are put forth as normative theories, but they are nonetheless motivated by the empirical violations. For example, Machina (1989) states that an important task of researchers of non-expected utility models is “to show that [they] *fit the data better* than the standard expected utility model” (p. 1622).

The purpose of this paper is to explore what restrictions on choices in the observable problem can be generated for a highly structured and rational, but abstract, specification of decision processes. Suppose that the decision processes take the form of uncertainty with state-dependent preferences and costly information acquisition, and that the choices of information and alternatives in this extended framework satisfy Axioms 1 and 2 and have an additively separable

utility representation with a subadditive cost of information. In other words, there is hidden information acquisition, but the choices in this metaproblem satisfy very strong normative assumptions that arise in choice problems with uncertainty and information acquisition. What restrictions does this impose on the choice function in the observable problem?

The answer is *none*. The main result of this note is that, with hidden information acquisition as described, *any* choice function is possible. Not only are preferences not necessarily transitive, but preferences can be unrelated to choices from larger sets.

The phenomenon arises because the feasible set affects the value of information and thus what information is acquired. Hence, first and foremost, the result provides a simple and concrete *illustration* of how, and to what extent, *static choice axioms may be violated because feasible sets affect the decision maker's decision process*.

The result also illustrates how hidden decision processes affect not only the properties of preferences but also the extendibility principle. It thus indirectly supports the view that conventional normative decision theories cannot be made to accommodate empirical violations simply by weakening axioms on preferences, and that it is necessary instead to explicitly model the decision processes that lie behind the observed behavior. (This view is expressed, for example, by Tversky and Kahneman (1988).)

The result is not meant to *explain* what happens in experiments. It is obtained within a strictly static model. As will be seen, this gives the result a hypothetical nature, because the decision process would make preference elicitation experiments or expenditure surveys inherently dynamic.

We conclude this section with a comparison of our result with other theories and examples in which there are violations of IIA. Note first, though, that IIA is not violated whenever a choice problem has 'hidden' dimensions. For example, if the observer sees a decision maker choose footwear, but the decision maker is also choosing the rest of a wardrobe, then the observable choice function satisfies the two consistency axioms if the choice function in the hidden metaproblem does.¹ Instead, some hidden aspect of the decision problem must depend on the observable feasible set.

For example, IIA may be violated when tastes depend directly on the feasible set. This occurs in regret theory (Loomes and Sugden (1982); Sugden (1993)) and in an example by Sen (1993), in which a dinner guest may refuse, out of politeness, an extra helping when there is one portion left, but not if there are two. There are also examples and theories in which IIA is violated because information presented to the decision maker depends on the feasible set. Sen (1993) discusses the epistemic content of the feasible set, when it reveals information about the underlying situation, such as about another party in a signaling model. Framing effects, such as described in Tversky and Kahneman (1988), also have to do with the information presented to the decision maker. (However, framing effects require a richer model, in which two alternatives can be logically equivalent but not perceived as equivalent by the decision maker, because the decision maker does not make all possible logical inferences.)

In this note, it is also information and hence beliefs – rather than preferences – that depend on the feasible set, but it is the endogenous acquisition rather than exogenous availability of information that varies.

2. MAIN RESULT

Consider the following class of decision processes or metaproblems for a given choice problem $\langle X, \mathcal{A} \rangle$:

There is uncertainty that is not already incorporated into $\langle X, \mathcal{A} \rangle$, represented by a finite set Ω of states. Given a feasible set $A \in \mathcal{A}$ in the problem $\langle X, \mathcal{A} \rangle$, the decision maker chooses an information partition π of Ω , from a set Π of feasible partitions, and a state-contingent plan $f : \Omega \rightarrow A$ that depends on the realization of the information and hence is π -measurable.

Henceforth, the term *metaproblem* means a metaproblem from this class.

We note that a metaproblem is itself a static choice problem. Each feasible set A in the problem $\langle X, \mathcal{A} \rangle$ induces the feasible set

$$\hat{A} = \{ \langle \pi, f \rangle \mid \pi \in \Pi, f \in A^\Omega, f \text{ is } \pi\text{-measurable} \}$$

in the metaproblem.

We will use \hat{C} to denote the choice function for a metaproblem. The choice function may reflect implicit costs of acquiring

information. It may also have an additively separable utility representation. By this we mean that there is an information cost function $c : \Pi \rightarrow \mathbb{R}$ and a state-dependent utility function $u : \Omega \times X \rightarrow \mathbb{R}$ such that $\langle \pi, f \rangle \in \hat{C}(\hat{A})$ if and only if

$$\sum_{\omega \in \Omega} u(f(\omega), \omega) - c(\pi) \geq \sum_{\omega \in \Omega} u(f'(\omega), \omega) - c(\pi')$$

for all $\langle \pi', f' \rangle \in \hat{A}$. Such a choice function satisfies Axioms 1 and 2 for the metaproblem.

For such a representation, the information cost function c is said to be subadditive if $c(\{\Omega\}) = 0$ and if, for each π and π' in Π , the join $\pi \vee \pi'$ of π and π' is in Π and $c(\pi \vee \pi') \leq c(\pi) + c(\pi')$. The interpretation of subadditivity is that information can be combined at no cost.

Even with an additively separable utility representation, it is impossible to uniquely distinguish subjective beliefs from the state-dependent utility. (Utility must be state-dependent for information to have any value.) Therefore, probabilities are suppressed to simplify notation. However, there is never an implicit assumption that the states are equiprobable.

Given a metaproblem and a choice function \hat{C} for the metaproblem, let $\tilde{C}(A, \omega)$ be the set of alternatives that might be chosen when the feasible set is A and the state is ω . That is,

$$(1) \quad \tilde{C}(A, \omega) = \{f(\omega) | \exists \pi : \langle \pi, f \rangle \in \hat{C}(\hat{A})\}$$

$\tilde{C}(\cdot, \omega) : \mathcal{A} \rightarrow 2^X$ is called the choice function for $\langle X, \mathcal{A} \rangle$ in state ω .

We can now state the main result of this note:

THEOREM 1. *Let $\langle X, \mathcal{A} \rangle$ be a finite static choice problem. Let C be a choice function for $\langle X, \mathcal{A} \rangle$. Let $v : X \rightarrow \mathbb{R}$ be a utility function on X . Then there is a metaproblem $\langle \Pi \times X^\Omega, \hat{A} \rangle$ and a choice function \hat{C} for this metaproblem that has an additively separable utility representation $\langle u, c \rangle$ with subadditive c , such that, for some $\omega_0 \in \Omega$,*

$$u(\cdot, \omega_0) = v(\cdot) \text{ and } \tilde{C}(\cdot, \omega_0) : \mathcal{A} \rightarrow X \text{ is equal to } C.$$

Note that there are no restrictions on C or v , and that the choice function for the metaproblem is as well-behaved as one could ask

for. The metaproblem is such that, with full information, the utility function would be v in state ω_0 , and if the information partition were fixed, $\tilde{C}(\cdot, \omega)$ would satisfy Axioms (1) and (2) for all ω . However, with endogenous information acquisition, the choice function for $\langle X, \mathcal{A} \rangle$ in state ω_0 is C .

We have to keep the static nature of this model in mind when interpreting $\tilde{C}(\cdot, \omega_0) : \mathcal{A} \rightarrow X$. This is the observable choice function for the problem $\langle X, \mathcal{A} \rangle$ in state ω_0 , in the sense that, if the true state were ω_0 and the decision maker were offered $A \in \mathcal{A}$, then the decision maker would choose $\tilde{C}(A, \omega_0)$. However, static models of choice are often applied to sequences of decisions, even though the latter are inherently dynamic problems. This is especially inappropriate when there is hidden information acquisition. For example, if $A_1, A_2 \in \mathcal{A}$, and if the decision maker were offered A_1 and then A_2 , she would not necessarily choose $\tilde{C}(A_1, \omega_0)$ and then $\tilde{C}(A_2, \omega_0)$. That would occur only if the decision maker did not relate the choice from A_1 to the choice from A_2 . Otherwise, the decision maker might apply information acquired when choosing from A_1 to the choice from A_2 . If the decision maker is aware of the dynamic sequence of choices he or she will face, then this may also affect the choice of information at each stage.

We have only obtained $\tilde{C}(\cdot, \omega_0) = C$ for one state in the metaproblem. However, the theorem can be extended to *ex-ante* choice probabilities.² Because preferences are state-dependent in the metaproblem, any strictly positive probabilities on the set of states would be consistent with the metaproblem. In particular, for any ϵ , $0 < \epsilon < 1$, the probability of ω_0 can be $1 - \epsilon$, in which case the probability that the choice is in $C(A)$ for all $A \in \mathcal{A}$ is at least $1 - \epsilon$.

3. EXAMPLE

Here is an example that illustrates the result. For simplicity, we do not try to generate a specific utility function in state ω_0 and initially we do not require that the cost of information be subadditive. Let $X = \{a, b, c\}$, and let C have a choice cycle, with

$$\begin{array}{ll} C(\{a, b\}) = \{a\} & C(\{a, c\}) = \{c\} \\ C(\{b, c\}) = \{b\} & C(\{a, b, c\}) = \{a\} \end{array}$$

TABLE I

Example with $X = \{a, b, c\}$. Ω equals $\{\omega_0, \omega_a, \omega_b, \omega_c\}$. u is defined in the table on the left. π_a , π_b and π_c are the relevant information partitions.

<i>states</i>	ω_0	ω_a	ω_b	ω_c
$u(a, \omega)$.2	2	1	0
$u(b, \omega)$.1	0	2	1
$u(c, \omega)$	0	1	0	2

$\pi_a = \{\{\omega_0, \omega_a\}, \{\omega_b, \omega_c\}\}$

$\pi_b = \{\{\omega_0, \omega_b\}, \{\omega_a, \omega_c\}\}$

$\pi_c = \{\{\omega_0, \omega_c\}, \{\omega_a, \omega_b\}\}$

Note that the extendibility principle does not hold.

Table I defines Ω and u for the metaproblem. It also defines partitions π_a , π_b and π_c , which we momentarily assume to be the only feasible partitions and to have zero cost.

Observe:

- When the feasible set is $\{a, b\}$, the partition π_a is most valuable. State ω_0 lies in event $\{\omega_0, \omega_a\}$, and alternative a is chosen.
- Similarly, π_b and alternative b are chosen when the feasible set is $\{b, c\}$ and the state is ω_0 .
- π_c and alternative c are chosen when the feasible set is $\{a, c\}$ and the state is ω_0 .
- If $u(\cdot, \omega_0)$ were constant, then all three partitions would be of the same utility value when the feasible set is $\{a, b, c\}$. However, partition π_a is more valuable because $u(\cdot, \omega_0)$ and $u(\cdot, \omega_a)$ rank the alternatives in the same way. Thus, in state ω_0 , alternative a is chosen.

This example can be modified to make the cost subadditive. Suppose the partitions π_a , π_b and π_c each have a cost of 1.6. Since the join of any two of these yields the full information partition, full information must be feasible and must cost at most 3.2. Let this be the cost. Let the null partition have zero cost. The remaining partitions can still be infeasible. The value of π_a is 2 when the feasible set is $\{a, b\}$ or $\{a, b, c\}$, The value of π_b is 2 when the feasible set is $\{b, c\}$, and the value of π_c is 1.8 when the feasible set is $\{a, c\}$.

The value of full information is never more than 3. Hence, the choice of information is the same with this subadditive cost function.

4. PROOF

The proof of the theorem is constructive. We illustrate each step for the case of $X = \{a, b, c\}$. Here is an outline:

1. The state ω_0 generates the desired choice function and utility function.
2. For each $A \in \mathcal{A}$, there is a state ω_A such that, conditional on the event $\{\omega_0, \omega_A\}$, the decision maker chooses $C(A)$.
3. For each $A \in \mathcal{A}$, there is a partition π_A that contains $\{\omega_0, \omega_A\}$. This is the partition we want the decision maker to choose when the feasible set is A (so that, from Step 2, the decision maker chooses $C(A)$ in state ω_0).
4. The “carrot” for getting the decision maker to choose π_A is an event E_A whose individual states can be observed only in the partition π_A . Observing the states in E_A is particularly valuable when the feasible set is A .
5. We define the cost c_A of π_A so that the partition π_A gives the highest ex-ante utility, net of information costs, when the feasible set is A .
6. The cost function in Step 5 is not subadditive. The final step is to modify the construction so that it is.

Step 1: Since ω_0 generates the desired utility function, $u(x, \omega_0) = v(x)$ for each $x \in X$. This is illustrated in the left column of the top part of Table II.

Step 2: The utility in state ω_A is defined as follows:

$$u(x, \omega_A) = \begin{cases} 1 - v(x) & \text{if } x \in C(A) \\ -v(x) & \text{if } x \in A \setminus C(A) \\ 0 & \text{if } x \notin A. \end{cases}$$

This is illustrated at the top of Table II. Thus, conditional on the event $\{\omega_0, \omega_A\}$, the utility of x is 1 if $x \in C(A)$ and is 0 if $x \in A \setminus C(A)$.

TABLE II

The construction for $X = \{a, b, c\}$, with $C(\{a, b\}) = \{a\}$, $C(\{b, c\}) = \{b\}$, $C(\{a, c\}) = \{c\}$, and $C(\{a, b, c\}) = \{a, b\}$.

state	ω_0	$\omega_{\{a,b\}}$	$\omega_{\{a,c\}}$	$\omega_{\{b,c\}}$	$\omega_{\{a,b,c\}}$
$u(a, \omega)$	$v(a)$	$1 - v(a)$	$-v(a)$	$-v(a)$	$1 - v(a)$
$u(b, \omega)$	$v(b)$	$-v(b)$	$1 - v(b)$	$-v(b)$	$1 - v(b)$
$u(c, \omega)$	$v(c)$	$-v(c)$	$-v(c)$	$1 - v(c)$	$-v(c)$

event state	$E_{\{a,b\}}$		$E_{\{a,c\}}$		$E_{\{b,c\}}$		$E_{\{a,b,c\}}$		
	$\{a, b\}_a$	$\{a, b\}_b$	$\{a, c\}_a$	$\{a, c\}_c$	$\{b, c\}_b$	$\{b, c\}_c$	$\{a, b, c\}_a$	$\{a, b, c\}_b$	$\{a, b, c\}_c$
$u(a, \omega)$	λ	$-\lambda$	λ	$-\lambda$	0	0	λ	$-\lambda/2$	$-\lambda/2$
$u(b, \omega)$	$-\lambda$	λ	0	0	λ	$-\lambda$	$-\lambda/2$	λ	$-\lambda/2$
$u(c, \omega)$	0	0	$-\lambda$	λ	$-\lambda$	λ	$-\lambda/2$	$-\lambda/2$	λ

Step 3: For each $A \in \mathcal{A}$, there is an event $E_A = \{\omega_A^x \mid x \in A\}$, to be described further below. The partition π_A lets the decision maker observe the event $\{\omega_0, \omega_A\}$, plus each of the states in the event E_A , plus the other states ω_B and the other events E_B for each $B \in \mathcal{A} \setminus \{A\}$. That is,

$$\pi_A = \{\{\omega_0, \omega_A\}\} \cup \{\{\omega_B\} \mid B \in \mathcal{A} \setminus \{A\}\} \cup \{\{\omega_A^x\} \mid x \in A\} \cup \{E_B \mid B \in \mathcal{A} \setminus \{A\}\} .$$

Step 4: u is defined on the states ω_A^x , for $A \in \mathcal{A}$ and $x \in A$, as follows:

$$u(y, \omega_A^x) = \begin{cases} \lambda & y = x \\ -\frac{\lambda}{|A| - 1} & y \neq x, y \in A \\ 0 & y \notin A . \end{cases}$$

This is illustrated at the bottom of Table II.

Let $V(A, B)$ be the maximum ex-ante utility over the event E_A (i.e., summing the utility over the states in E_A) when the feasible set is B and the decision maker can observe the states in E_A . Then

$$V(A, B) = \lambda|A| \quad \text{if } A \subset B \\ V(A, B) \leq \lambda(|A| - 1) \quad \text{otherwise.}$$

Here is why. If the decision maker can observe the states in E_A , then:

- if $A \subset B$, then for each state ω_A^x in E_A , the decision maker can choose x and get utility λ , for a total of $\lambda|A|$;
- otherwise, for each $x \in A \setminus B$, the decision maker can get at most 0 in state ω_A^x .

On the other hand, if the decision maker cannot observe the states in E_A , she gets utility 0 in each such event.

Let $W(A, B)$ be the utility over the event $\{\omega_0\} \cup \{\omega_A \mid A \in \mathcal{A}\}$ when the information partition is π_A and the feasible set is B . Then, the maximum *ex-ante* utility over all states when information π_A is observed and the feasible set is B is

$$V(A, B) + W(A, B).$$

Step 5: Let c_A be the cost of partition π_A . Then the net *ex-ante* utility with information π_A when the feasible set is B is

$$NV(A, B) = V(A, B) + W(A, B) - c_A.$$

We next want to define $c_{(\cdot)}$ so that B strictly maximizes $NV(\cdot, B)$ for each B . For each $A \in \mathcal{A}$, let

$$c_A = \lambda(|A| - 1 + |A|^{-1}).$$

Then

$$\begin{aligned} NV(A, B) &= \lambda(1 - |A|^{-1}) + W(A, B) \text{ if } A \subset B \\ NV(A, B) &\leq \lambda(-|A|^{-1}) + W(A, B) \text{ otherwise.} \end{aligned}$$

If $W(A, B)$ is the same for all A and B , then B strictly maximizes $NV(\cdot, B)$ for each B . Furthermore, the gap between $NV(B, B)$ and $NV(A, B)$ for $A \neq B$ is proportional to λ . If $W(\cdot, \cdot)$ is not constant, then we simply have to choose λ large enough so that B strictly maximizes $NV(\cdot, B)$ for each B .

Step 6: This concludes the proof, except that the cost of information is not subadditive. Subadditivity implies that the partition $\pi_A \vee \pi_B$, for $A \neq B$, is feasible and the cost is at most $c_A + c_B$. Without further modification, if the decision maker's feasible set is B , the decision maker will want to acquire $\bigvee_{A \subset B} \pi_A$.

We now show how to modify the metaproblem so that the cost of information is subadditive. The detailed proof is tedious, even though this part of the theorem is not as important. Therefore, we give a less formal treatment.

We would like to inflate the costs of each π_A so that the join of two partitions is sufficiently costly. However, we are constrained by the fact that the cost of the trivial partition must be 0, and π_A must be preferred to the trivial partition when the feasible set is A . We therefore, modify the problem so that there is valuable information that is duplicated in the π_A 's, and that the decision maker would not be willing to 'pay for' twice. We must do this without perturbing the relative values of the partitions $\{\pi_A \mid A \in \mathcal{A}\}$.

The duplicate information is being able to distinguish between the events $\{E_A \mid A \in \mathcal{A}\}$. We can make this valuable without perturbing the rest of the proof by substituting $-\gamma$ for all the 0's in the bottom of Table II, or similarly for other cases. This only strengthens the relative value of π_A over π_B when the feasible set is A and $B \neq A$. This makes all the information structures more valuable, depending on the magnitude of γ , for all feasible sets. By making γ large enough, we can make the incremental cost of $\pi_A \vee \pi_B$ arbitrarily large. However, the incremental information continues to the ability able to observe the individual states in E_B , and the value of this does not depend on γ .

Now we extend the cost of information so that it is subadditive. Let Π be the set of partitions obtained by combining the partitions in $\{\pi_A \mid A \in \mathcal{A}\}$. That is,

$$\Pi = \{\vee_{A \in \mathcal{B}} \pi_A \mid \mathcal{B} \subset \mathcal{A}\} .$$

For each $\mathcal{B} \subset \mathcal{A}$, let

$$c(\vee_{A \in \mathcal{B}} \pi_A) = \sum_{A \in \mathcal{B}} c_A .$$

The function $c : \Pi \rightarrow \mathbb{R}$ is well defined because $\vee_{A \in \mathcal{B}} \pi_A$ is different for each $\mathcal{B} \subset \mathcal{A}$. It is also subadditive. From the above discussion, if γ is large enough, the decision maker chooses π_A given the feasible set A , when the set of partitions is Π and the cost function is c .

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NOTES

¹ See Malishevski (1993) for a general treatment of consistency conditions with hidden alternatives.

² See, for example, Suppes *et al.* (1989), Chapter 17. Note that, at least for these authors, this is not an intended application of choice probabilities. On page 384 they write: “Choice probabilities most commonly arise in the context of a choice experiment in which an individual makes repeated choices under essentially identical conditions.”

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