Online Companion to "Agency Costs in a Supply Chain with Demand Uncertainty and Price Competition" (V. G. Narayanan, Ananth Raman, and Jasjit Singh)

Proof of Lemma 1: This retailer would see (y/L) of the total demand, hence the cumulative distribution function of its demand is H(s) = F(sL/y) where F( ) gives the cumulative distribution function for total market demand, x. Differentiating H(s) gives the required result.

Proof of Lemma 2:

$$E_s\left(\max(q-s,0)\right)=\int_0^q (q-z)h(z)dz=\int_0^q (q-z)\frac{L}{y}f(\frac{zL}{y})dz=\int_0^{\frac{Lq}{y}}(q-\frac{ys}{L})f(s)ds$$

$$=\int_0^{\frac{Lq}{y}}\frac{y}{L}(\frac{Lq}{y}-s)f(s)ds=\frac{y}{L}\int_0^{\frac{Lq}{y}}(\frac{Lq}{y}-s)f(s)ds=\frac{y}{L}\int_0^{\frac{Lq}{y}}F(s)ds \text{ (integration by parts)}$$

Proof of Lemma 3: Expected profits that retailer 1 maximizes are given by

$\Pi_1=(p_1-p_w-o)q_1-(p_1-j)E_{s_1}\left(\max(q_1-s_1,0)\right)=(p_1-p_w-o)q_1-(p_1-j)\frac{y_1}{L}\int_0^{\frac{Lq_1}{y_1}}F(s)ds$. Let

$Q_1=(L/y)q_1$. Then

$\Pi_1=\frac{y_1}{L}\left[(p_1-p_w-o)Q_1-(p_1-j)\int_0^{Q_1}F(s)ds\right]$. $\frac{d^2\Pi_1}{dQ_1^2}=-\frac{y_1}{L}(\text{p}_1-\text{j})\text{f}(\text{Q}_1)<0.$

For any p_1, there exists a unique value of Q_1 that maximizes Π_1. The first-order condition with respect to Q_1 (for given p_1) is

$\frac{y_1}{L}\left[(p_1-p_w-o)-(p_1-j)F(Q_1^*)\right]=0 \Rightarrow Q_1^*=F^{-1}(\frac{p_1-p_w-o}{p_1-j})$. Hence,

$\Pi_1=\frac{y_1}{L}\left[(p_1-p_w-o)Q_1^*-(p_1-j)\int_0^{Q_1^*}F(s)ds\right]$. An analogous equation exists for Π_2.

Proof of Lemma 4: The distance y_i of the customer who gets zero expected utility from shopping at retailer i is given by $\beta(v-p_i-y_i)=0$, where β is the conjectured service level for each retailer. This gives y_i=v-p_i. The condition that the customer at distance y_1 will not prefer to shop at the other retailer is $y_1 \leq$ L-y_2, i.e., v-$p_1 \leq$ L-(v-p_2), i.e. $p_1+p_2 \geq 2v-L$. If this condition does not hold, the market size of the two retailers can be found by determining the customer indifferent between shopping at either retailer. Since this customer is located at a distance y_i from R_i and L-y_i from R_{-i}, it follows that $\beta(v - p_i - y_i) = \beta(v - p_{-i} - (L-y_i)) \Rightarrow y_i=(p_{-i}-p_i+L)/2$.

Proof of observation 1(i): Let demand be x for certain.

First-best: We will prove that, in first best case, $p_1= p_2$=v-L/2, so that each retailers serves exactly half of the total market - the consumer in the middle is indifferent between buying from either retailer and gets zero utility from doing so. We prove this by considering three cases.

Case 1: $p_1+p_2>2v-L$ (the whole market is not covered and neither retailer serves customers in the interval (v-p_1, L+p_2-v)). Without loss of generality, assume $p_1>v-L/2$. For this to be first best, it should not be possible to increase profits by changing prices. A slight decrease in p_1 does not affect profits from the second retailers for this case, so all we need to show is that profits from sales at the first retailer do increase by lowering p_1 slightly. Profits arising from sales made at the first retailer are given by x(v-p_1)(p_1-c), with the derivative for p_1 being x(v-2p_1+c+o) < x(L+c+o-v) since $p_1>v-L/2$. But, from A7, this is strictly negative, i.e. the profits can be improved by cutting price. So case 1 cannot be first best.

Case 2: $p_1+p_2<2v-L$ (the whole market is covered with all customers getting strictly positive utility). It is easy to see that the consumer located at a distance (L+p_2-p_1)/2 from the first retailer is indifferent between which retailer it shops at. The total profits, obtained as the sum of profits from sales at the two retailers, are given by x(p_1-c-o)(L+p_2-p_1)/2 + x(p_2-c-o)(L+p_1-p_2)/2. Observe that increasing both p_1 and p_2 by an equal amount increases profits. So case 2 cannot be first best.

Case 3: $p_1+p_2=2v-L$ (the whole market is just covered with the indifferent consumer getting zero utility). The profits are given by $x(p_1-c-o)(L+p_2-p_1)/2 + x(p_2-c-o)(L+p_1-p_2)/2 = x(p1-c-o)(v-p1) + x(2v-L-p1-c-o)(L+p_1-v)$ since $p_1+p_2=2v-L$. This function is strictly concave in p_1. From first-order conditions we get $p_1=v-L/2$, which also gives $p_2=v-L/2$ as $p_1+p_2=2v-L$

Second-best: We will prove that, by setting $p_w=v-3L/2-o$, the manufacturer can replicate the first best prices as an equilibrium in the price-setting subgame involving the two retailers. We prove that this is indeed the unique equilibrium by considering three cases. The manufacturer can then simply extract all the surplus (which will equal the profits in the first best case) by adjusting the transfer fee to give the retailers exactly zero profits.

Case 1: $p_1+p_2>2v-L$ (the whole market is not covered and neither retailer serves customers in the interval $(v-p_1, L+p_2-v)$). Without loss of generality, assume $p_1>v-L/2$. For this to be equilibrium the first retailer should not be able to improve profits by deviating. But profits of the first retailer are given by $x(v-p_1)(p_1-p_w-o)$, with marginal effect of price change being $x(v-2p_1+p_w + o)< -L/2$ (since $p_1>v-L/2$ and $p_w=v-3L/2-o$). Since this marginal effect is strictly negative, retailer 1 has an incentive to increase profits by cutting price. So case 1 cannot be equilibrium.

Case 2: $p_1+p_2<2v-L$ (the whole market is covered with all consumers getting strictly positive utility). Retailer 1's profits are given by $x(p_1- p_w -o)(L+p_2-p_1)/2$. The first order necessary condition for p_1 to be optimal given p_2 is $p_1= ½ (L+p_2+p_w+o)$. Likewise, the first order necessary condition for p_2 to be optimal given p_1 is $p_2= ½ (L+p_1+p_w+o)$. Solving the two gives $p_1=p_2=L+ p_w +o=v-L/2$ (since $p_w=v-3L/2-o$). But this means that $p_1+p_2=2v-L$, which violates the condition that $p_1+p_2<2v-L$, so case 2 cannot be equilibrium.

Case 3: $p_1+p_2=2v-L$ (the whole market is just covered with the indifferent consumer getting zero utility). Without loss of generality, assume $p_1\geq v-L/2$. For this to be equilibrium the first retailer should not be able to improve profits by deviating. But profits of the first retailer if it *lowers* price are given by $x(p_1-p_w-o)(L+p_2-p_1)/2$, with marginal effect of price change in the neighborhood of p_1 (from below) being $x(L+p_2-2p_1+p_w+o) = 3(v-L/2-p_1)$ (since $p_1+p_2=2v-L$ and $p_w=v-o-3L/2$). This equals 0 if and only if $p_1=v-L/2$, so anything other than $p_1=v-L/2$ (and analogously $p_2=v-L/2$) cannot be an equilibrium. But for $p_1=v-L/2$ and $p_2=v-L/2$ to be an equilibrium, the retailers should not have an incentive to *raise* price either. But that follows through the argument in case 1 since that is the objective function from deviating towards higher price is same as in case 1. Thus the unique equilibrium is indeed $p_1=v-L/2$ and $p_2=v-L/2$, which is exactly the first best prices. Substituting for p_1,p_2, and p_w, we get $x(L)(L)/2 = L^2(x/2)$. Hence, setting $T_i= L^2(x/2)$ ensures that both retailers choose first-best price and earn their reservation level of profits. The manufacturer earns $Lx*(v-3L/2-o-c)+ L^2(x)=Lx(v-L/2-o-c)$, which is the first-best level of profits.

Proof of observation 1(ii): Let the exogenous prices be p_1^* and p_2^*. Let the demand for the two retailers be d_1 and d_2 respectively. The demand at the two retail locations is a function of the prices and total demand x, i.e., $d_1(p_1^*, p_2^*, x)$ and $d_1(p_2^*, p_1^*, x)$.

<u>First-best</u>: The profits at retail location 1 is $Min(q_1,d_1)\ (p_1^*) + (q_1- Min(q_1,d_1))j-q_1(c+o)$ and the profits at retail location 2 is $Min(q_2,d_2)\ (p_2^*) + (q_2- Min(q_2,d_2))j-q_2(c+o)$. The expectation is taken over x to determine expected profits. Since prices are exogenous and the profit functions are additively separable, maximizing the expected profits of each retail location is equivalent to maximizing the sum of the expected profits of the two retail locations. Let the optimal quantities be q_1^* and q_2^*, respectively.

<u>Second-best</u>: The profits at retail location 1 is $Min(q_1,d_1)\ (p_1^*) + (q_1- Min(q_1,d_1))j-q_1(p_w+o)-T_1$ and the profits at retail location 2 is $Min(q_2,d_2)\ (p_2^*) + (q_2- Min(q_2,d_2))j-q_2(p_w+o)-T_2$. The

expectation is taken over x to determine expected profits. Since prices are exogenous and identical to the first-best case, if p_w=c, the two retailers will choose quantities $q_1 = q_1^*$ and $q_2 = q_2^*$, respectively because the objective functions in the first-best and second-best case will be identical except for the fixed constant transfer price T. By setting the transfer price $T_i = \pi_i$, the manufacturer sets the expected profits of the retailer equal to their reservation profits of zero and the manufacturer achieves first-best profits.

Proof of Lemma 5: In proving this lemma, we drop the subscripts for notational convenience. In the absence of competitive effects, retailer i's price p and market share y are related by p=v-y as the indifferent customer gets 0 expected utility.

$$Q^*(y) = F^{-1}\left(\frac{p(y)-c-o}{p(y)-j}\right) = F^{-1}\left(\frac{v-y-c-o}{v-y-j}\right)$$

$$\Rightarrow \Pi_i = \frac{y}{L}\left[(v-y-c-o)Q^*(y) - (v-y-j)\int_0^{Q^*(y)} F(s)ds\right]$$

. Differentiating with respect to y and applying the envelope theorem (noting that $\partial\Pi_i / \partial Q = 0$ at Q=Q*)

$$\frac{d\Pi_i}{dy} = \frac{1}{L}\left[(v-y-c-o)Q^*(y) - (v-y-j)\int_0^{Q^*(y)} F(s)ds\right] + \frac{y}{L}\left[-Q^*(y) + \int_0^{Q^*(y)} F(s)ds\right]$$

Differentiating totally w.r.t. y again, and again noting that $\partial\Pi_i / \partial Q = 0$ at Q=Q*

$$\frac{d^2\Pi}{dy^2} = -\frac{2}{L}\left[Q^* - \int_0^{Q^*} F(s)ds\right] - \frac{y}{L}\left[1 - F(Q^*)\right]\frac{dQ^*}{dy}$$

. Substituting Q*= D(p - c - o)/(p - j) = D(v - y - c - o)/(v – y - j) and F(s)=s/D for the uniform distribution case gives

$$\frac{d^2\Pi}{dy^2} = -\frac{2}{L}\left[\frac{D(v-y-c-o)}{(v-y-j)} - \frac{D(v-y-c-o)^2}{2(v-y-j)^2}\right] - \frac{y}{L}\left[\frac{c+o-j}{v-y-j}\right]\left[\frac{-(c+o-j)D}{(v-y-j)^2}\right]$$

$$= -\frac{D}{L}\left[\frac{2(v-y-c-o)}{(v-y-j)} - \frac{(v-y-c-o)^2}{(v-y-j)^2} - \frac{y(c+o-j)^2}{(v-y-j)^3}\right]$$

Since c+o>j, (v-y-c-o)/(v-y-j) <1, so $[(v-y-c-o)/(v-y-j)]^2$ <(v-y-c-o)/(v-y-j). Therefore a sufficient condition for the above to be negative is that

$$\frac{(v-y-c-o)}{(v-y-j)} > \frac{\mathrm{y(c+o-j)}^2}{(\mathrm{v-y-j})^3} \quad i.e., \quad (v-y-c-o) > y\left(\frac{\mathrm{c+o-j}}{\mathrm{v-y-j}}\right)^2$$

Since v-y = p > c+o > j, a sufficient condition for this to hold is v-y-c-o > y, i.e., v>2y+c+o. However, this holds by assumption A7 as v > 3L+c+o+j > 2y+c+o (as y ≤ L). So we have shown that the profit function is concave in y. Additionally, we need to show that the profit function increases in y. But to show this, given concavity, we only need to show that the slope is still positive at the highest value in the domain of y, which is L. Substituting the uniform distribution assumption in the equation for the slope gives

$$\left.\frac{d\Pi}{dy}\right|_{y=L} = \frac{1}{L}\left[(v-2L-c-o)\frac{D(v-L-c-o)}{(v-L-j)} - (v-2L-j)\,\frac{D(v-L-c-o)^2}{2(v-L-j)^2}\right]$$

$$= \frac{1}{L}\frac{D(v-L-c-o)}{(v-L-j)}\left[(v-2L-c-o) - (v-2L-j)\,\frac{(v-L-c-o)}{2(v-L-j)}\right]$$

For this to be positive, we need $(v-2L-c-o) > (v-2L-j)\,\dfrac{(v-L-c-o)}{2(v-L-j)}$, i.e.

$$(v-L)^2 - (L+c+j)(v-L) + 2jL + jc - cL > 0$$

It is easy to verify that Assumption A7 is sufficient for v-L to be greater than its bigger root in the quadratic expression on the left hand side of the desired inequality, and hence the inequality does hold under Assumption A7. We have therefore shown that profits are increasing and concave in the choice of y.

Proof of Lemma 6: We will show that the whole market not being served is inconsistent with profit-maximizing choice of p_w by the manufacturer. If the whole market is not covered, the profits accruing to retailer 1 are given by $\Pi_1 = \dfrac{D(p_1 - p_w - o)^2}{2L(p_1 - j)}(\mathrm{v - p_1})$ (using lemma 4(i))

We now show that p_1 will be an increasing function of p_w. To show this, it is sufficient to show that the cross-partial of Π_1 w.r.t p_1 and p_w is positive. Differentiating Π_1 w.r.t. p_w first

$$\frac{\partial \Pi_1}{\partial p_w} = \frac{-D(p_1 - p_w - o)}{2L(p_1 - j)}(\mathrm{v} - \mathrm{p}_1) = \left[\frac{D(p_1 - p_w - o)^2}{2L(p_1 - j)}(\mathrm{v} - \mathrm{p}_1)\right]\left[\frac{-1}{(p_1 - p_w - o)}\right]$$

Now, differentiating this with respect to p_1 using chain rule, and noting that the derivative of the first term in the product is 0 from the first order condition for optimal p_1 (envelope theorem)

$$\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_w} = \left[\frac{D(p_1 - p_w - o)^2}{2L(p_1 - j)}(\mathrm{v} - \mathrm{p}_1)\right]\left[\frac{1}{(p_1 - p_w - o)^2}\right] = \frac{D(\mathrm{v} - \mathrm{p}_1)}{2L(p_1 - j)}, \text{ which is clearly positive.}$$

Thus p_1 will be monotonically increasing in p_w. A similar logic holds for p_2. Thus, in case the whole market is not covered, reducing p_w a little induces both the retailer to decrease their prices, hence increasing their market shares. But recall that, from lemma 5, the joint profits from sales made from each retailer are increasing in its market share in case the complete market is not covered. Since the retailers only have to be given their reservation utility (through appropriate choice of the fixed fees), the manufacturer can therefore increase its profits by decreasing p_w. Thus the entire market not being covered cannot be an equilibrium.

(ii) Suppose only one retailer had the complete market share, so it make positive profits while the other makes zero profits. The fact that this cannot be an equilibrium follows by noting that the second retailer can make positive profits (equal to half the profits that the other retailer was making) by setting its price equal to the other retailer (so that half the customers now come to it) and the quantity it stocks is exactly half of what the other retailer was originally stocking (which is the optimal news-vendor quantity since the demand is exactly half in all states of the world).

Proof of Lemma 7: (i) From lemma 6, the second best choice of p_w will ensure that the entire market is covered, so we can use the profit expression with y_i substituted in accordingly from lemma 4(ii). Differentiating the profit function

$$\frac{\partial \Pi^1(p_1,p_2)}{\partial p_2} = \frac{1}{2L}\left[(p_1 - p_w - o)Q_1^* - (p_1 - j)\int_0^{Q_1^*} F(s)ds\right]$$

$$\Rightarrow \frac{\partial^2 \Pi^1(p_1,p_2)}{\partial p_1 \partial p_2} = \frac{1}{2L}\left[Q_1^* - \int_0^{Q_1^*} F(s)ds + (p_1 - p_w - o)\frac{dQ_1^*}{dp_1} - (p_1 - j)F(Q_1^*)\frac{dQ_1^*}{dp_1}\right]$$ The third and fourth terms cancel out from definition of Q_1*. Since F(s) <1, the overall expression is therefore positive. Prices are therefore strategic complements. The maximization of the profit function with respect to p_1 for different values of p_2 would give the reaction correspondence $p_1(p_2)$ of the retailer 1 for different prices of retailer 2. In general, the profit function need not be globally concave. However, in order to show that the reaction correspondence is weakly upward sloping, it is sufficient to show that it is super-modular, which follows from lemma 7(i) where prices are shown to be strategic complements (Fudenberg and Tirole, 1991, theorem 12.7).

(ii) We now show that this monotonicity of the reaction correspondence is enough to rule out asymmetric equilibria in pure strategies. Suppose there did exist an asymmetric equilibrium (p_1=a, p_2=b) such that, without loss of generality, a > b. Then, from the symmetry of the problem, there must also exist another asymmetric equilibrium (p_1=b, p_2=a). But, for that to hold, the reaction correspondence $p_1(p_2)$ has to violate the monotonicity result somewhere in the range $a<p_1<b$ as there exists an element b in $p_1(a)$ that is smaller than an element a in $p_1(b)$. Thus there cannot exist an asymmetric equilibrium. Hence, only symmetric equilibria are possible in pure strategies.

References

Fudenberg, D. and J. Tirole (1991). Game Theory. The MIT press, Cambridge, MA.