

Agency Costs in a Supply Chain with Demand Uncertainty and Price Competition

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In this paper, we model a manufacturer that contracts with two retailers, who then choose retail prices and stocking quantities endogenously in a Bayesian Nash equilibrium. If the manufacturer designs a contract that is accepted by both retailers, it sets the wholesale price as a compromise between two conflicting roles: reducing intrabrand retail price competition and inducing retailers to stock closer to first-best levels (that is, optimum for the supply chain as a whole). In equilibrium, fill rates are less than first best. If, on the other hand, the manufacturer eliminates retail competition by designing a contract accepted by only one retailer, the assignment of consumers to retailers is inefficient. In either equilibrium, the performance of the supply chain is strictly less than first best. However, the manufacturer achieves first-best retail prices and fill rates if it can subsidize the retailers' leftover inventory. Absent such subsidies, the two-retailer equilibrium arises when the two retailers compete less intensively. In that equilibrium, numerical results indicate that the value of subsidizing unsold inventory is increasing in demand uncertainty, intensity of retail competition, and salvage value of inventory, and is decreasing in manufacturing cost and opportunity cost of shelf space.

Key words: supply chain management; contracts; incentives; safety stock; buyback; inventory subsidies; Hotelling models; Linear City

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1. Introduction

This paper models a manufacturer-retailer contract design problem. The setting is as follows. The manufacturer produces a product and sells it to two retailers that sell the product to consumers. Thus, the manufacturer must contract with the retailers that carry the product in the face of uncertain retail demand. Each retailer chooses the retail price it charges, as well as its stocking quantity. The manufacturer cannot coerce the retailers to carry the product or precommit to a retail price (i.e., resale price maintenance is either illegal or impossible). The contract can therefore only specify a wholesale price and a fixed fee. We show that, in the absence of inventory subsidies, such a supply chain cannot simultaneously achieve first-best prices and fill rates when demand is uncertain and retail prices are endogenous, even though first-best results could be achieved if demand were deterministic or if retail prices were exogenously fixed.

However, under the possibility of subsidizing the retailer's leftover inventory, we show that a manufacturer can achieve first-best performance (i.e., on both price and fill rates) even when retail prices are endogenous and demand is uncertain. Thus, this paper adds to the literature that demonstrates

the value of inventory subsidies. The practice of subsidizing leftover inventory has been present for a long time in book and music retailing and, as multiple authors (e.g., Taylor 2001, Padmanabhan and Png 1995) have noted, is spreading to other industries such as personal computers (PCs), shoes, and apparel. The subsidies are implemented slightly differently in the different industries. In books and music, the publisher or the record company typically asks the retailer to return unsold merchandise for credit. In PCs, apparel, and shoes, the manufacturer pays the retailer a certain amount of money for each unit of unsold inventory at the retailer, but asks the retailer to liquidate the unsold units for salvage value.

If subsidizing of inventory is not possible, we show that the equilibria of our model can be of one of two kinds (depending on the value of exogenous parameters), neither of which can replicate first-best channel profits. If the competition between the two retailers is intense, the manufacturer would prefer to offer a contract that is accepted by only one retailer. This, however, leads to lower than first-best prices and stocking quantities. The retailer lowers the price to induce some consumers to shop at their nonpreferred retail location. On the other hand, if the competition between the two retailers is relatively less intense,

the manufacturer prefers to offer a contract that is accepted by both retailers. In this case, the wholesale price serves two roles: (1) When high, it restrains intrabrand retail competition, and (2) when low, it induces retailers to stock more. Optimal wholesale price is thus a trade-off between these roles, which leads to lower than first-best fill rates. With either kind of equilibrium, manufacturers who can subsidize leftover retail inventory can mitigate these inefficiencies and are able to achieve the first-best channel profits.

This paper provides a rationale for empirically observed phenomena such as manufacturers subsidizing unsold inventory when demand is stochastic and retailers engage in price competition. Additionally, numerical results give intuition about how the value of subsidizing unsold inventory varies with demand uncertainty, intensity of retail competition, salvage value of inventory, manufacturing cost, and opportunity cost of shelf space.

A literature review is provided in the following section. In §3, we describe the model and assumptions. Section 4 assumes that demand is uniformly distributed and compares performance under three cases. Subsection 4.1 considers the vertically integrated (i.e., first-best) case, §4.2 examines the differentiated supply chain, and §4.3 examines the impact of subsidizing leftover (unsold) inventory in a differentiated supply chain. Section 5 uses numerical integration to examine how changes in demand uncertainty, retail competition, cost of shelf space, and salvage value affect the difference between first-best and second-best performance when demand is normally distributed. We present our conclusions in §6 and proof of propositions in the appendix.

2. Literature Review

Research in operations management, marketing, and economics is relevant to our paper. Operations management researchers have examined the impact of wholesale price on fill rate, while typically treating retail price as exogenous to the model. Researchers in economics and marketing, on the other hand, have focused on understanding the impact of wholesale price and return policies on retail price, while ignoring uncertainty and inventory issues.

Cachon (1998), Lariviere (1998), and Tsay et al. (1998) provide excellent reviews of different aspects of the operations management literature as it pertains to supply chain contracts. This literature examines a variety of mechanisms for improving coordination and efficiency in a supply chain. These mechanisms include buy-back agreements, quantity discounts, transfer payments, reallocation of decision rights, and quantity flexibility contracts. However, most papers

in this stream of literature, with a few notable exceptions discussed below, treat retail price as exogenous.

Padmanabhan and Png (1997) consider retail price to be endogenous to the supply chain. However, our model differs from the model considered in their paper in a number of ways. First, they restrict their attention to “full return” policies, while we identify the optimal buy-back price for the supply chain. Second, unlike their paper, we permit the manufacturer to offer or charge a fixed payment (e.g., slotting allowances or franchise fees). Third, in their model demand can be either high or low, while we consider uniform distribution and provide numerical analysis for the truncated normal distribution. Fourth, they assume that the retailer chooses price only after demand is known (i.e., uncertainty is resolved). We, on the other hand, assume that the retailer chooses stocking quantity and price when demand is uncertain.

Cachon (2004) shows that when retail prices are exogenous, a wholesale price contract with buy-back of unsold inventory (equivalent to subsidization of unsold inventory in our model) achieves first-best results. However, if retailer prices are endogenous, then a wholesale price contract with buyback of unsold inventory does not achieve first-best results. Marvel and Peck (1995) and Bernstein and Federgruen (2005) also demonstrate this result. In contrast, in our paper a wholesale price contract augmented with a fixed fee and buyback of unsold inventory does achieve first-best results even when retail prices are endogenous because the wholesale price and buy-back price for unsold inventory are used to maximize supply chain profits, while the fixed intercept is used to hold the retailers to their reservation utilities.

Taylor (2001) and Lee et al. (2000) have shown that price protection is valuable for channel coordination in markets such as PCs, where retail prices are declining over time. Our model differs from these papers because we take retail prices as endogenous and allow for fixed transfer payments in the supply chain. Hence, we extend the findings in these papers by showing that subsidizing unsold inventory can be valuable where there is intrabrand retail price competition.

Research in marketing and economics has also modeled the impact of wholesale price and slotting allowances on retail price. However, papers in economics do not consider uncertainty in demand and the impact of wholesale price on a retailer’s stocking level. The reader can get a good idea of the related economics literature from Shaffer (1991), Mathewson and Winter (1986), and Rey and Tirole (1986). Similar to the economics literature, the marketing literature on supply chains also typically assumes demand to be certain. Moorthy (1987) and McGuire and Staelin

(1983) show that channel coordination problems can be solved by two-part tariffs that allow for fixed transfer payments. This is because two-part tariffs separate the problem of maximizing channel profits from the problem of splitting the profits between the upstream and downstream players. While two-part tariffs play a similar role in our model, they are insufficient to induce optimal inventory levels because retail prices are endogenous and demand is uncertain.

Iyer (1998) analyzes circumstances under which retailers who are ex ante symmetric in their cost and other characteristics might end up choosing the same or different retail prices and service levels. He finds a symmetric equilibrium in markets with substantial horizontal differentiation relative to vertical differentiation, and an asymmetric equilibrium in markets with substantial vertical differentiation relative to horizontal differentiation. Because there is no vertical differentiation in our model, our results are consistent with Iyer for the case where both retailers accept the manufacturer’s contract and choose to compete. However, Iyer does not model demand uncertainty, inventory decisions, and the contractibility of inventory, all crucial features of our model.

3. Model

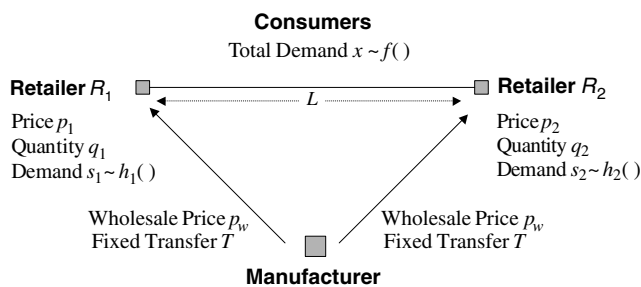
We consider a one-period model in which a risk-neutral manufacturer produces, at cost c per unit ($c > 0$), a consumer product that it can sell to two differentiated risk-neutral retailers, R_1 and R_2 . The model is represented in Figure 1. Consumers are uniformly distributed along a line of length L ; the retailers are located at the ends of the line. Each consumer may buy the product from one retailer or choose to not buy the product from either retailer. The value of a unit of the product for each consumer is v . The prices per unit chosen by the two retailers are p_1 and p_2 , respectively. Consumers are indexed by their distance d from R_1 , with the cost of shopping being d per unit to shop at R_1 and $L - d$ per unit to shop at R_2 . This cost is incurred in addition to the purchase price of the product.

Let the expected probability that consumers find the good that they want in stock (i.e., Type 2 service level) be β_i for R_i , $i \in \{1, 2\}$. Thus, the net

expected value to consumer d is $\max[\beta_1(v - d - p_1), \beta_2(v - (L - d) - p_2), 0]$.¹ We assume that if both retailers choose to participate, the consumers conjecture the expected service levels at the two retailers to be equal; i.e., $\beta_1 = \beta_2 = \beta$.² Note that assuming symmetric conjectures about service levels is *not* the same as simply assuming a symmetric first- and second-best outcome in prices and quantities. Instead, we will actually *prove* that these conjectures, for certain conditions on the exogenous parameters, turn out to be self-fulfilling in an equilibrium that is also symmetric in prices and quantities (i.e., a symmetric outcome is the Bayesian Nash equilibrium). The analysis also reveals that, under other conditions, we may have an asymmetric equilibrium instead, wherein only one retailer accepts the manufacturer’s contract.

Our formulation assumes that the “travel cost” (d or $L - d$) is incurred only when the item is found to be in stock at the retailer. This formulation captures the case when “distance” between a consumer and a retailer is viewed as deviation from the ideal combination of service and retailer characteristics for each consumer, rather than as physical distance.³ Consider the case where a consumer considers purchasing a particular item from one of two e-tailers. Clearly, consumers do not have to travel physical distance when shopping from an e-tailer. However, even e-tailers selling the same product often differ in services such as product information supplied, design of website, choice of packaging, order tracking, 24-hour customer service on the phone, points for frequent consumers, and return privileges. For example, it might be that e-tailer A offers consumers choice of packaging and order tracking, while e-tailer B offers 24-hour customer service and return privileges. Assuming that a consumer would like to have all the services offered by both e-tailers, shopping with either

Figure 1 The Supply Chain



¹ Our model differs from Lippman and McCardle (1997) and Anupindi and Bassok (1999), where consumers visit a second store if they encounter a stockout at the first store, in that prices are endogenous in our model. As shown later, in our model a stockout at one store implies a stockout at the other as well. Moreover, the assumption of not visiting the second store is reasonable for trendy products. Most retailers tend to stock out of “hot” products at roughly the same time.

² This assumption need not hold if the retailers follow differentiation strategies based on their service levels. However, because our primary goal in this paper is to illustrate possible loss of efficiency from excessive retail competition, we abstract from retailer differentiation issues.

³ If distances are interpreted as physical distances, an alternative formulation might be to assume that the cost is incurred independent of whether the item is found in stock or not. That is, $\max[\beta_1(v - p_1) - d, \beta_2(v - p_2) - (L - d), 0]$ would be the expression for the customer’s expected utility calculation. We found that such a formulation makes the analysis much more cumbersome, without adding new insights.

e-tailer is costly compared to the consumer's ideal shopping experience.

The quantity of product stocked by the two retailers is q_1 and q_2 , respectively. If demand exceeds the total quantity stocked, the sale is foregone. If the quantity stocked exceeds demand, excess inventory is salvaged at price j . The retailers incur an opportunity cost of o per unit for retail shelf space. The overstocking cost is positive; that is, $o + c - j > 0$. We make the following additional assumptions:

ASSUMPTION 1. Total potential demand, x , is uncertain and distributed with support $[0, \bar{x}]$, density function $f(x)$, and distribution function $F(x)$. We assume that $F(x)$ is differentiable.

In a typical linear-city model (see Tirole 1988), each atomistic consumer located on the line between the two cities decides whether or not to buy the unit, based on whether the private valuation of the good or service is greater than travel cost plus retail price. The total potential demand, x , represents the demand that would be achieved in the supply chain if every consumer interested in purchasing the product bought it from one of the two retailers. Thus, treating x as a random variable captures shifts in demand caused by changes in exogenous factors such as fashion, the overall economy, and weather. Each retailer derives a share of x based on the retail prices it charges. Realized market demand could be lower than x if the retail prices are high and some consumers decide to not buy the product at either retailer. Because variance in x is the only source of demand uncertainty, our model applies better for trendy products such as toys, apparel, music, PCs, and consumer electronics, where the primary source of uncertainty is in market acceptance of the product.

ASSUMPTION 2. The retail outlets are independent profit maximizers, and collusion between them is assumed to be illegal or impossible. Retail prices are not contractible.

Resale price maintenance has been per se illegal in the United States since 1975. Moreover, retail prices are unobservable if discounts off list price or services provided by the retailer are not observable to the manufacturer.

ASSUMPTION 3. The retailers observe no new information about demand between the instant they make the stocking-quantity decision and the stock price decision. They also do not observe each other's ordered quantity.

This assumption, which we defend based on practice in multiple supply chains and on related papers in operations management, allows us to model the setting as a game in which quantity and price decisions take place simultaneously. Consider, for example, the "publishing" supply chain, which we mean to

include books, magazines and newspapers, and music (CDs and tapes). In such a supply chain, price is fixed well in advance and is not altered on the basis of demand signals. In fact, the pricing decision often precedes the stocking decision in these supply chains—concurrent decision making is reasonable to assume because no (or very little) information is revealed between the two decisions. Similarly, in "produce" retailing (e.g., fresh fruit and vegetables), supermarkets typically are unable to alter prices after observing demand signals and before the unit "perishes" on the retail shelf. Given its practical importance in many supply chains, the one-period stochastic inventory problem with simultaneous selection of unit retail price has been studied widely in operations management. The earliest attempt to address this problem can be traced to Whiting (1955), while a recent review of this stream of research can be found in Petruzzi and Dada (1999).⁴

ASSUMPTION 4. The retailer's stocking quantity is not contractible.

It is harder for a manufacturer to observe a retailer's stocking quantity than to observe the retailer's purchase quantity. Retailers can often mislead manufacturers on the quantity stocked by "diverting" a portion or all of the purchased products to other retailers. See Tirole (1988) for a discussion of diversion and its impact on contractibility in supply chains.

*ASSUMPTION 5. The manufacturer offers a contract to the retailers on a take-it-or-leave-it basis.*⁵

We use an agency model, with the manufacturer as the principal and the retailers as the agents. The manufacturer conjectures whether the contract offered

⁴ In some supply chains (e.g., apparel and footwear), quantity commitments are made 6–12 months in advance of the selling season, and pricing decisions can be postponed. We have not considered this situation in our paper for two reasons. One, as we noted above, is that concurrent pricing and stocking decisions are typical of many other supply chains. Two, the model becomes analytically intractable when prices are set after demand information has been realized. However, we analyzed a model with a simple binary support for demand where the retailers set prices after observing total demand, and our results are unchanged.

⁵ Under the Robinson Patman Act, a manufacturer must offer all retailers the same prices as long as there is no difference in the cost of serving the different retailers. If one of the two retailers offers the contract to the manufacturer, the contract with the other retailer must be identical. Hence, if the retailer(s) rather than the manufacturer offers the contract, there will be no change in equilibrium retail price, wholesale price, or retail stocking quantities. The only difference will be in the fixed transfer payments, T_i , used to hold the contract recipients to their reservation utility. This result holds because (1) there are no information rents in our model and the participation constraint binds, and (2) all parties are risk neutral and there are no wealth effects.

would be accepted by both retailers or by only one retailer. In equilibrium, these conjectures will have to be borne out.

ASSUMPTION 6. *The retailers cannot price discriminate among their consumers.*

We assume that the manufacturer can impose a two-part tariff (a fixed franchise fee or slotting allowance, T , plus a per-unit wholesale price, p_w) upon observing whether the retailer carries the product in question. Tirole (1988, p. 189) provides justification for assumptions similar to Assumptions 3, 4, 5, and 6, and describes why these assumptions lead to a two-part tariff scheme. Given these assumptions, we can formulate the manufacturer’s optimization program. Let consumer demand at retailer R_i , given the two retail prices, be $s_i(p_i, p_{-i})$ with support $[0, \bar{s}(p_i, p_{-i})]$, density function $h(s_i | p_i, p_{-i})$, and distribution function $H(s_i | p_i, p_{-i})$ for $i, -i \in \{1, 2\}$, $i \neq -i$. The manufacturer solves the following program (P1):

$$\max_{p_w, T_1, T_2, p_1, p_2, q_1, q_2} \sum_{i=1}^2 ((p_w - c)q_i + T_i)$$

subject to

$$\int_0^{q_i} (p_i s_i + (q_i - s_i)j)h(s_i) ds_i + \int_{q_i}^{\bar{s}} p_i q_i h(s_i) ds_i - T_i - (o + p_w)q_i \geq 0 \quad \forall i \in \{1, 2\}, \quad (1)$$

$$p_i, q_i \in \arg \max_{p_i, q_i} \int_0^{q_i} (p_i s_i + (q_i - s_i)j)h(s_i) ds_i + \int_{q_i}^{\bar{s}} p_i q_i h(s_i) ds_i - (o + p_w)q_i \quad \forall i \in \{1, 2\}. \quad (2)$$

The manufacturer maximizes revenues less manufacturing costs plus fixed transfer payments from the retailers. The transfer payments can be negative, in which case we interpret them as slotting allowances paid to the retailers. The first constraint is the participation constraint for each retailer; the second is the incentive compatibility constraint for the retailers’ choice of quantity and price. If a particular retailer (retailer R_i) decides not to accept the contract offered by the manufacturer, that retailer stocks zero units (i.e., $q_i = 0$) and neither pays nor receives any transfer payment from the manufacturer (i.e., $T_i = 0$).

Because the expected utility of shopping at retailer i is strictly decreasing in the distance from it, there will be a portion of the linear city such that all consumers in this part of the linear city shop at retailer i , and no customer outside this part does. We define this portion y_i ($0 \leq y_i \leq L$) of the linear city as the market share of retailer i . Note that the customer at distance y_i from retailer i is indifferent between shopping

at i and its next-best option (which could be either shopping at the other retailer or not shopping at all). If all consumers shop at one or the other retailer, this indifferent customer is the same for the two retailers, and the sum of the market shares of the two retailers is exactly L . We shall refer to this situation as the whole market being covered, which only occurs if prices are low enough so that there is no customer who prefers not shopping at all to shopping at one of the two retailers. For what follows, we will find the following lemmas useful.⁶

LEMMA 1. *For a given choice of market share $y_i > 0$, the market demand that R_i sees is given by the density function $h(s) = (L/y_i)f(sL/y_i)$, where $f(\cdot)$ gives the probability density function for total market demand.*

LEMMA 2. *The expected leftover (unsold) quantity for a retailer with market share $y_i > 0$ is*

$$E_s(\max(q - s, 0)) = \int_0^q (q - z)h(z) dz = \frac{y_i}{L} \int_0^{Lq/y_i} F(s) ds.$$

LEMMA 3. *If $p_i > p_w + o > j$ and $0 < y_i < L$ is the length of the market served by retailer i , then*

(i) *The optimal stocking quantity is given by $q_i^* = (y_i/L)Q^*(p_i)$, where*

$$Q^*(p_i) = F^{-1}\left(\frac{p_i - p_w - o}{p_i - j}\right),$$

(ii) *The expected profits, gross of fixed payment T_i , are*

$$\Pi_i = \frac{y_i}{L} \left[(p_i - p_w - o)Q^*(p_i) - (p_i - j) \int_0^{Q^*(p_i)} F(s) ds \right].$$

Lemma 3 implies that once the retail price and market share are known, the optimal quantity and expected profits can be easily calculated. Note that this result holds not just for the second-best case, but also for the first-best case because, for any given y_1 and y_2 , profits of Retailer 1 do not depend on Q_2 and profits of Retailer 2 do not depend on Q_1 . Therefore, the individually optimal and jointly optimal values of Q_1 and Q_2 are the same.

ASSUMPTION 7. $v > 3L + c + o + j$.

This assumption can be interpreted as the intrinsic value of the good being high relative to the distance between the retailers. This condition will help us ensure that all the consumers between the two retailers will be served both in the first-best case and the second-best equilibrium, which will help keep the analysis tractable. We should note that this condition is only a sufficient condition and not a necessary

⁶ Because of space constraints, the proofs of the lemmas and Observation 1 are available in Narayanan et al. (2004). The proofs of propositions are included in the appendix to this paper.

condition for our proofs. In fact, in §5 we numerically calculate first-best and second-best equilibria under a wide range of parameters, including several that do not satisfy Assumption 7, and show that all major analytic results continue to hold.

We will make use of the following lemma in the analysis that follows:

LEMMA 4. *If both retailers choose to participate, then for any given $p_1, p_2 < v$, the market share of retailer i is given by:*

- (i) $y_i(p_1, p_2) = v - p_i$ if $p_1 + p_2 \geq 2v - L$,
- (ii) $y_i(p_1, p_2) = (p_{-i} - p_i + L)/2$ if $p_1 + p_2 \leq 2v - L$.

The lemma states that when both retailers are participating, the market shares of the two retailers are completely determined for any given values of p_1 and p_2 .

Before we analyze the setting where Assumptions 1–7 hold, we make the following observation pertaining to two special cases.

OBSERVATION 1. In the second-best case, the manufacturer can duplicate the performance of the first-best channel:

- (i) When demand is certain by choosing $p_w = v - 3L/2 - o$ and $T_i = L^2(x/2)$,
- (ii) When retail price is exogenous, by choosing $p_w = c$ and $T_i = \Pi_i$.

Observation 1(i) follows because when demand is certain, the unique pure-strategy equilibrium is $p_i = (v + p_w + o)/2$. Thus, increasing the wholesale price shifts both best-response functions upwards, thereby increasing the equilibrium retail price because prices are strategic complements in this game. Thus, increasing wholesale price has the strategic effect of dampening intrabrand retail competition. Without demand uncertainty, the retailer stocks actual demand. Because the manufacturer no longer has to worry about inducing the retailer to stock the right quantity, it is no surprise that a two-part tariff scheme achieves complete channel coordination. Likewise, if retail prices were exogenously the same in the first-best and second-best cases, the two-part tariff scheme achieves first best by setting the wholesale price to be equal to marginal cost.⁷

4. Uniformly Distributed Demand

For analytical tractability, this section assumes that aggregate demand is uniformly distributed between zero and D . Thus, $f(x) = 1/D$ and $F(x) = x/D$ for

⁷ Complete channel coordination is also feasible if the manufacturer could directly reduce retail competition through mechanisms other than wholesale price, so that the wholesale price can be used purely to induce retailers to stock the first-best quantity at the first-best price. Such mechanisms include resale price maintenance (RPM), exclusive territories, and retail monopoly.

$0 \leq x \leq D$. Subsection 4.1 analyzes the first-best case, §4.2 the differentiated supply chain, and §4.3 the impact of subsidizing unsold inventory. In §5, we will relax the uniform distribution assumption to numerically verify the robustness of our results.

4.1. First-Best Case

As a benchmark, we analyze the case in which there are no goal congruence problems between the manufacturer and the retailers so that the prices and quantities are chosen in the order that their joint surplus is maximized. Using the expression for expected profits from Lemma 3(ii), noting that y_i is a function of p_1 and p_2 from Lemma 4, and replacing p_w with c (because the supply chain as a whole faces the unit cost c), the first-best problem can be written as

$$\max_{p_1, p_2} \sum_{i=1}^2 \frac{y_i(p_i, p_{-i})}{L} \left[(p_i - c - o)Q^*(p_i) - (p_i - j) \int_0^{Q^*(p_i)} F(s) ds \right] \quad \text{with}$$

$$Q^*(p_i) = F^{-1} \left(\frac{p_i - c - o}{p_i - j} \right).$$

Note that the only choice variables we need to consider are p_1 and p_2 , because once they are fixed, the market shares are fixed and the optimal quantities are known (from Lemma 3).

LEMMA 5. *If the complete market is not covered (i.e., $y_1 + y_2 < L$), the profits generated through sales made at either retailer in the first-best supply chain are increasing and concave in the retailer's market share.*

PROPOSITION 1. *The first-best solution is unique and symmetric, with the whole market being served, each retailer covering exactly half of the market, and the customer in the center getting zero utility from shopping from either retailer. Further, $p_1 = p_2 = p_{FB} = v - L/2$ and $q_1 = q_2 = q_{FB} = (D/2)(v - L/2 - c - o)/(v - L/2 - j)$.*

The first-best channel charges the maximum price that it can to leave the consumer located midway between the two retail outlets indifferent between buying the product and not buying at all. The total inventory stocked is the familiar critical fractile of demand, where demand is $(v - p_{FB})x/L = x/2$.

4.2. Second-Best Case: Differentiated Supply Chain

Now we consider the case in which the retailers non-cooperatively choose prices and quantities to maximize their respective profits. The manufacturer can only influence these variables through appropriate choice of the wholesale price p_w and the transfer payments T_i . We show that, absent subsidies for unsold inventory, the supply chain cannot achieve first-best

price and stocking quantity, leading to a loss of economic efficiency.

In the second-best case, the manufacturer can induce two types of equilibria. In the first potential equilibrium, the manufacturer offers a contract such that the retailers would make negative profits if both of them participate, but a single participating retailer will make zero profits. The manufacturer randomly chooses the first retailer to whom to offer the contract, who accepts the contract, knowing that the second retailer will rationally choose not to accept once the first has accepted. We now formally characterize this asymmetric equilibrium.

PROPOSITION 2. *Any second-best equilibrium in which only one retailer accepts the manufacturer’s contract must satisfy the following:*

- (i) *The manufacturer chooses $p_w = c$ and $T = D(v - L - c - o)^2 / [2(v - L - j)]$.*
- (ii) *The participating retailer chooses $p = v - L$ and $q = D(v - L - c - o) / (v - L - j)$ and serves the whole market from one of the two retail locations.*

The second and more complicated equilibrium to consider is one where the manufacturer offers a contract that both retailers will choose to accept. We start by studying the subgame in which p_w has been set and the fixed transfer payments T_i are already sunk. Using Lemma 3, this subgame can be viewed as being a price competition game, with the market shares and the stocking quantities being completely determined by the choice of retail prices. In particular, for any choice of p_1 and p_2 , the profits Π_i of retailer i before T_i are given by

$$\frac{y_i(p_i, p_{-i})}{L} \left[(p_i - p_w - o)Q^*(p_i) - (p_i - j) \int_0^{Q^*(p_i)} F(s) ds \right],$$

where $Q^*(p_i) = F^{-1} \left(\frac{p_i - p_w - o}{p_i - j} \right)$.

Under assumption of uniformly distributed demand, the expression for Π_i simplifies to

$$\frac{y_i(p_1, p_2)}{L} \left[(p_i - p_w - o) \frac{D(p_i - p_w - o)}{(p_i - j)} - (p_i - j) \cdot \frac{D(p_i - p_w - o)^2}{2(p_i - j)^2} \right] = \frac{D(p_i - p_w - o)^2}{2L(p_i - j)} y_i(p_1, p_2).$$

Before analyzing the retail competition subgame further, we briefly revert to the bigger game, where the manufacturer selects the wholesale price p_w .

LEMMA 6. *In any equilibrium where the manufacturer offers a contract (i.e., wholesale price p_w and fixed fee T) that both retailers accept, the value of p_w chosen by the manufacturer will be such that the following hold for the*

retail competition subgame:

- (i) *The whole market is served, i.e., all consumers prefer shopping to not shopping, and*
- (ii) *both retailers have strictly positive market shares.*

We now resume our analysis of the retail competition subgame. From Lemma 6, we know that the only relevant equilibria we need to consider for the subgame are those with an interior solution. We can therefore substitute the expressions for y_1 and y_2 from Lemma 4(ii) in the profit function for the retailers to come up with each retailer’s profit as a function of p_1 and p_2 :

$$\Pi_i = \frac{p_{-i} - p_i + L}{2L} \left[(p_i - p_w - o)Q^*(p_i) - (p_i - j) \int_0^{Q^*(p_i)} F(s) ds \right].$$

LEMMA 7. *In any second-best equilibrium in which both retailers accept the manufacturer’s contract, the retail competition subgame has the following properties:*

- (i) *Retail prices are strategic complements. That is, the reaction correspondence $p_i(p_{-i})$ of retailer i for a price p_{-i} by retailer $-i$ with $i, -i \in \{1, 2\}$ and $i \neq -i$, is upward sloping.*
- (ii) *Any pure-strategy equilibrium is symmetric.*

Lemma 6 and Lemma 7 together tell us that we can look for all potential equilibria simply by determining the first-order conditions of the profit function and then solving for symmetric solutions to these first-order conditions. However, we also need to check two additional conditions: First, the second-order conditions for local optimality of strategy should hold for the two retailers at the equilibrium point. Second, neither of the two retailers should have an incentive to jump to another (nonlocal) strategy, including the strategy that leads to a boundary outcome (where it is the only one serving the entire market). Following this procedure leads us to the following result:

PROPOSITION 3. *Consider the equilibrium where both retailers accept a manufacturer’s contract that specifies wholesale price p_w . The equilibrium of this game is symmetric, with the full market being covered. Denoting each retailer’s equilibrium price by $p^*(p_w)$ and quantity by $q^*(p_w)$,*

- (i) $p^*(p_w) = \frac{1}{2}[L + p_w + o + j + \sqrt{(3L + p_w + o - j)^2 - 8L^2}]$,
- (ii) $q^*(p_w) = \frac{D(p_w)}{4L} \cdot \frac{1}{[3L + p_w + o - j - \sqrt{(3L + p_w + o - j)^2 - 8L^2}]}$,
- (iii) p^* is increasing in p_w ,
- (iv) q^* is decreasing in p_w ,
- (v) retailer profits (before T) are given by

$$\Pi^*(p_w) = \frac{D(p^*(p_w) - o - p_w)^2}{4(p^*(p_w) - j)}.$$

Proposition 3(i) presents an expression for retail price as a function of wholesale price, and Proposition 3(iii) shows that an increase in wholesale price induces the retailer to increase its resale price. The retailer increases its retail price with increase in p_w for two reasons: First, there is a direct effect caused by an increase in its marginal cost. However, there is also an indirect effect that the rival retailer also increases its price, dampening retail competition. Unfortunately, higher wholesale costs lead to higher overstocking costs, thus inducing the retailer to stock out more often. Consequently, the manufacturer faces a trade-off in setting wholesale prices: It cannot simultaneously achieve first-best price and first-best stocking quantity.

PROPOSITION 4. *If the manufacturer chooses an equilibrium in which both retailers participate,*

- (i) *The manufacturer selects the wholesale price p_w so that the retailers select the first-best price,*
- (ii) *The whole market is served, and*
- (iii) *The retailers stock less than the first-best quantity.*

From Proposition 4, the retail price $p_i = v - L/2$ equals the first-best price. Using $p_i = v - L/2$ and Proposition 3(i) to solve for the wholesale price, we find that $p_w = v + 4L^2/(L + 2v - 2j) - (5L + 2o)/2$. Proposition 4 implies that the differentiated channel does not match the performance of the first-best supply chain because it stocks less than first-best quantities.

From Proposition 2, the manufacturer's profits under the asymmetric equilibrium are $D(v - L - c - o)^2/[2(v - L - j)]$. From Proposition 4, the manufacturer's profits under the symmetric equilibrium are $2([D(p - o - p_w)^2/(4(p - j))] + (p_w - c)q)$, where $p = v - L/2$, $p_w = v + (4L^2)/(L + 2v - 2j) - (5L + 2o)/2$, and $q = D[3L + p_w + o - j - \sqrt{(3L + p_w + o - j)^2 - 8L^2}]/(4L)$. Thus, the manufacturer will choose the contract to induce the equilibrium that yields higher profits depending on the values of exogenous parameters. In §5, using numerical results we characterize conditions under which each of the two equilibria arises.

4.3. Efficiency Gains from Subsidizing Leftover Inventory

We now consider the case where the manufacturer can subsidize the retailers' leftover inventory.

PROPOSITION 5. *The manufacturer can achieve first-best results even in a competitive channel by subsidizing unsold inventory. In other words, if subsidizing unsold inventory is allowed, the second-best prices, quantities, and channel profits equal the first-best levels. This requires*

a per-unit subsidy z for unsold inventory and wholesale price p_w , such that

$$z = v - j - \left[\frac{3}{2} + \frac{2(c + o - j)}{v - L/2 - c - o} \right] L,$$

$$p_w = v - \frac{L}{2} - o - \frac{v - L/2 - c - o}{v - L/2 - j} \left(v - \frac{L}{2} - j - z \right).$$

Intuitively, the manufacturer can now fix the wholesale price to induce the retailer to charge the first-best retail price, while achieving first-best stocking quantities through appropriate choice of subsidy for leftover inventory. Thus, the availability of a new contractible instrument gives the manufacturer more flexibility in writing efficient contracts. Thus, in a two-retailer case, the manufacturer is able to implement first best because there is no longer a trade-off between implementing optimal retail price and optimal stocking quantity. Note that the chosen equilibrium will now never be the asymmetric one because the asymmetric one allocates the consumers inefficiently and can never match first best.

5. Numerical Results: Normally Distributed Demand

We now allow the demand to have a truncated normal distribution to ensure that our analytic results are not driven by the assumption of uniform demand distribution. We also relax Assumption 7 and numerically calculate first-best and second-best equilibria under a wide range of parameters, many of which do not satisfy Assumption 7. We find that despite these generalizations, our previous results still hold. Specifically: (1) Absent subsidizing of unsold inventory, second-best channel profits are lower than first-best profits, and (2) once we allow subsidizing of unsold inventory, second-best equilibrium is able to achieve first-best profits. We examine the difference between first-best and second-best performance as a function of intensity of retail competition, demand uncertainty, manufacturing cost, opportunity cost of shelf space, and salvage value to identify situations where efficiency losses resulting from retail competition are large, and hence potential gains from subsidizing unsold inventory are substantial.

We assume a truncated normal demand distribution with a mean of $D/2$ (where D is an exogenous parameter), with the density function truncated to be positive only on the region $[0, D]$. Note that the uniform distribution considered in the prior sections is the limiting case of this truncated normal with $\sigma = D/12$. We considered a wide range of exogenous parameters: intensity of competition (L), uncertainty of demand (σ), cost price (c), shelving cost (o), and salvage value (j). For every set of exogenous parameter choices, we numerically compute

the first-best and second-best prices, quantities, and profits. Naturally, the manufacturer would choose to offer the contract corresponding to the equilibrium that offers higher second-best profits. However, for better intuition and exposition, we report both equilibria. We start with exogenous parameter values $L = 2.0$, $\sigma = 0.24$, $c = 10.0$, $o = 2.0$, and $j = 5.0$ (customer valuation $v = 20.5$ and upper limit on demand $D = 1.0$ are maintained constant in all examples). Next, each of the graphs in Figures 2–6 is obtained by varying one of these parameters. Thus, for example, the graph for uncertainty is obtained by keeping $L = 2.0$, $c = 10.0$, $o = 2.0$, and $j = 5.0$ fixed, and varying σ from 0.04 (almost zero variance) to 0.2887 (i.e., $1/12$, which is the variance when the truncated normal has the same mean and variance as the uniform distribution considered earlier). We now discuss each of these in turn.

Intensity of Competition (L). See Figure 2. The first-best profits fall as L increases because the price has to be dropped so that consumers that are farther away from the retailers are compensated for the extra distance traveled; consequently, the consumers that are closer to the retailers get extra surplus at the expense of the channel.

In the two-retailer second-best case, there are two effects that interact and cause profits to initially increase, and later decrease, as L increases. At very small L , the possibility of intense price competition between retailers results in the manufacturer setting a wholesale price significantly higher than the marginal cost (to mitigate losses from competition) so that even though the retailers end up charging the first-best price in equilibrium, they are no longer stocking efficient inventory levels. However, as L increases, the threat of price competition subsides and the wholesale price moves closer to marginal cost, reducing the difference between first- and second-best levels that was arising because of double marginalization. At very large L , as in the first-best case, the retailers have to drop prices considerably to induce consumers located farther away from them to shop for the product, reducing the supply chain profits as well.

In the single-retailer second-best case, the retailer ends up charging a lower price than the first-best because there are gains from serving more than half the market. Note that the wholesale price will always equal marginal cost in this equilibrium, while the surplus is extracted through an appropriate choice of the fixed fees. Because the losses for extra travel costs are negligible for very low L , the single-retailer case is able to approach first best, and hence dominate the two-retailer second-best case. As L rises but is still small, the price is just low enough for the whole market to be served and the gap between second best and first best increases. However, beyond a certain point (at $L > 3$ in this example), the single retailer finds it profitable to serve only a fixed length of the market independent of the value of L . For very large L , this market size will fall to be less than $L/2$ and the one-retailer solution will definitely be dominated by the two-retailer solution, because the second retailer can always replicate the first, and yet make nonnegative profits.

Demand Uncertainty (σ). See Figure 3. When σ is close to 0, first-best and two-retailer second-best profits converge because there are no double-marginalization losses when demand is certain. As σ increases, both first-best and two-retailer second-best profits decrease because of understocking and overstocking costs, but two-retailer second-best profits fall at a higher rate than the first-best profits because of the additional inefficiency arising from double marginalization. The understocking cost for the supply chain in the first-best case is higher than the retailer’s understocking cost in the two-retailer second-best case because the wholesale price is set higher than cost price to mitigate price competition. Consequently, we observe lower stocking quantities in the two-retailer second-best case. The extent of understocking increases with the level of demand uncertainty, and hence the difference between first-best and two-retailer second-best profits increases with demand uncertainty.

The one-retailer case profits start off lower than the two-retailer for low σ because of the lower price

Figure 2 Channel Profits vs. Intensity of Competition

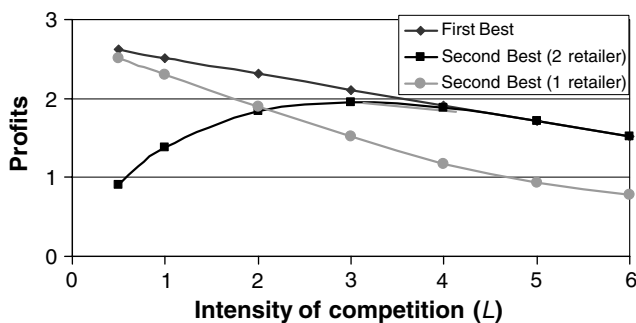


Figure 3 Channel Profits vs. Standard Deviation of Demand

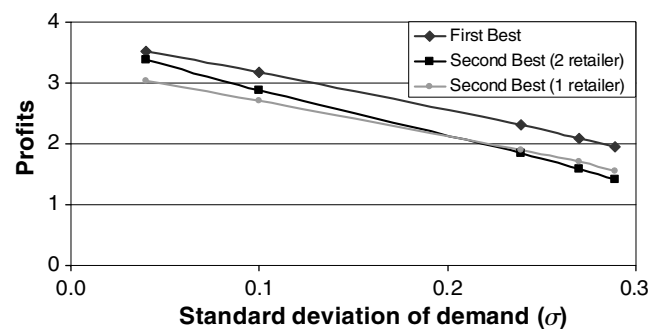
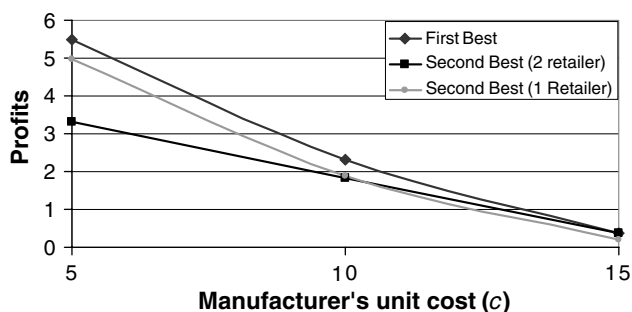


Figure 4 Channel Profits vs. Manufacturer's Unit Cost

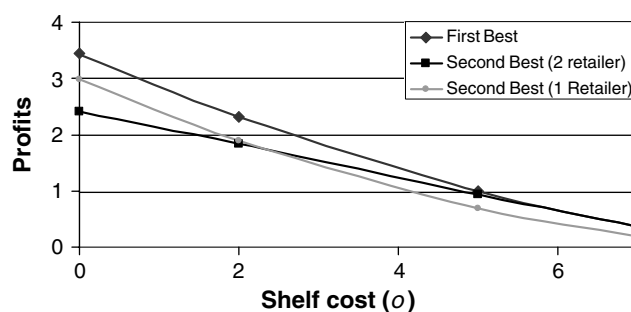


needed to support the whole market from a single location. However, as σ increases, profits in the one-retailer case do not fall as sharply as in the two-retailer case because there are no inefficiencies arising from double marginalization (because, for the single-retailer case, the wholesale price is always chosen to equal the cost price). In our example, this means that the one-retailer case dominates the two-retailer case once σ becomes large enough.

Manufacturer's Cost (c). See Figure 4. As c reaches the maximum feasible value $v - o$, first-best and second-best profits converge to zero. Both first-best and second-best profits rise as c becomes smaller. When c becomes smaller, holding v constant, the value of the product to society (i.e., distribution channel plus consumers) $v - c - o$, increases. In the first-best case, the channel is able to capture this increase in social surplus. Thus, as c decreases, first-best profits increase. However, the two-retailer second-best arrangement is not able to capture this increase in social surplus fully because the two-retailer second-best inventory levels are lower than first-best levels. Thus, the difference between first-best and the two-retailer second-best profits increases as c decreases. Double-marginalization issues, however, do not plague the one-retailer second best. Consequently, the inventory distortions are smaller for this case. Hence, the one-retailer profits exceed the two-retailer profits once c is low enough. However, it still does not reach first-best levels because the price chosen is lower than first-best levels to serve more than half of the market from a single location. For our parameter values, it turns out that the one-retailer case serves the entire market for $c = 5$ and $c = 10$, but only serves 68% of the market for $c = 15$.

Opportunity Cost of Shelf Space (o). See Figure 5. When o decreases, the insights are similar to those associated with a decrease in c . It is apparent that the impact of o and c on first-best profits would be similar because both are costs incurred on each unit stocked by the supply chain, and the first-best channel does not distinguish between unit costs incurred by the manufacturer and the retailer. In the second-best

Figure 5 Channel Profits vs. Shelf Cost



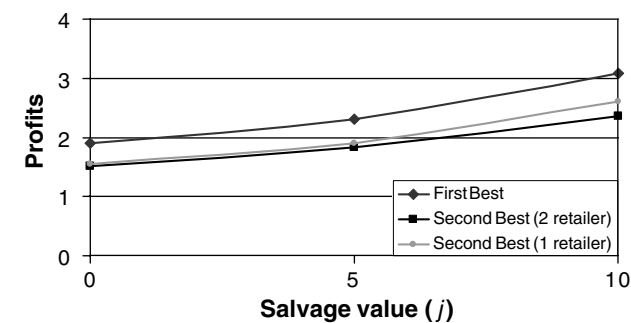
case, an increase in c causes an increase in $p_w \cdot p_w$ and o affect the retailer's pricing and stocking decisions in similar ways.

Salvage Value (j). See Figure 6. As j increases, the first-best inventory levels increase, as do the second-best inventory levels. However, the second-best inventory levels remain lower than inventory levels in the first-best case. (The difference is higher at higher levels of j .) When j is high, more safety stock is optimal, although the effect is different for the one-retailer case (where wholesale price equals cost price) and two-retailer case (where wholesale price exceeds cost price). We find that the gap between first-best and two-retailer second-best quantity increases with j , although the same does not happen for the one-retailer case because there are no double-marginalization losses.

6. Conclusion

In this paper, we model a supply chain with a single manufacturer supplying a product to two competing retailers located at the two ends of a linear city with an uncertain demand. The manufacturer can specify only a wholesale price and a fixed transfer amount in the contract that it offers to the retailers, with both retail prices and stocking quantities being determined endogenously in a Bayesian Nash equilibrium. If the manufacturer designs a contract that is accepted by both retailers, the wholesale price chosen is a compromise between two conflicting

Figure 6 Channel Profits vs. Salvage Value



roles: reducing intrabrand retail price competition and inducing retailers to stock closer to first-best levels. In equilibrium, fill rates are less than first best. If, on the other hand, the manufacturer tries to eliminate retail competition by designing a contract accepted by only one retailer in equilibrium, the assignment of consumers to retailers is inefficient. Either way, the performance of the supply chain is strictly less than first best. However, we show that the manufacturer can achieve first-best performance (in both retail prices and fill rates) if it has the possibility of subsidizing the retailers' leftover inventory. To verify that our results are not being driven by restrictive assumptions made for analytic tractability, we relax these assumptions in numerically computing the equilibrium outcome under more general conditions over a large range of parameter values. Numerical results suggest that the two-retailer equilibrium is likely to arise when the two retailers compete less intensively. In that equilibrium, the value of subsidizing unsold inventory is increasing in demand uncertainty, intensity of retail competition, and salvage value of inventory; and decreasing with increases in manufacturing cost and opportunity cost of shelf space.

The importance of inventory subsidies for achieving first-best outcome in our model is related to the more general point that access to multiple instruments can often help write better contracts (i.e., contracts that work better for channel coordination). As the model of a supply chain gets more complicated (e.g., as retail competition, demand uncertainty, and legal limitations like inability to contract on retail price are introduced), contractability on additional instruments (e.g., subsidies of unsold inventory, returns, or franchise fees) can help overcome some of the inefficiencies that would otherwise arise in the second-best equilibrium.

An electronic companion to this paper is available at <http://mansci.pubs.informs.org>.

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Appendix. Proofs of Propositions

PROOF OF PROPOSITION 1. If there is a range of consumers not served by either retailer, joint profits can be increased by lowering price at either retailer to increase market share (Lemma 5). Further, the indifferent customer gets zero utility because the retailers could otherwise do better by raising prices and still serving the whole market. The fact that we can only have a symmetric solution readily follows by noting that the profit function Π_i of each retailer is increasing but concave in its market served (Lemma 5), which implies that $2\Pi_i(L/2) > \Pi_i(y) + \Pi_i(L - y)$ for y not equal to $L/2$. Symmetry implies that the middle customer is indifferent

between shopping and not shopping, so $p_{FB} = v - L/2$. First-best quantity q_{FB} follows by applying Lemma 3 to the uniform distribution case. \square

PROOF OF PROPOSITION 2(i). From Lemma 5, it follows that the retailer serves the whole market ($y_i = L$). Hence, $p_i = v - L$ because a higher price will imply less than the full market is served, while a lower price will not increase demand. From the newsvendor formula, we can compute the retailer's optimal stocking quantity to be $q = D(v - L - c - o)/(v - L - j)$ and the retailer's profits before T (i.e., gross of transfer payments to manufacturer) to be $D(v - L - c - o)^2/[2(v - L - j)]$. The manufacturer can charge a wholesale price of $p_w = c$ and use the payment T to extract all retailer profits. \square

PROOF OF PROPOSITION 2(ii). We will prove that if the second retailer also accepted this contract ($p_w = c$, $T = D(v - L - c - o)^2/[2(v - L - j)]$), it will earn strictly negative profits. Let the price and quantity choices of the two retailers be $\{p_1, q_1, p_2, q_2\}$. Assume, contrary to the proposition, that the second retailer earns nonnegative profits. We will consider strictly positive profits and zero profits for the second retailer in turn. If the second retailer earns strictly positive profits by choosing $\{p_2, q_2\}$, the monopoly retailer in Proposition 2(i) could have also earned strictly positive profits by choosing the same $\{p_2, q_2\}$ instead of $p = v - L$ and $q = D(v - L - c - o)/(v - L - j)$, which is a contradiction of the proof of Proposition 2(i). Now assume that the second retailer earns zero net profits. From the concavity of the profit function proved in Lemma 5 and the fact that a firm's profits when competing in a duopoly cannot exceed its profits when it is a monopoly, holding costs constant; it follows that the second retailer can earn zero profits only if it serves the whole market and $p_2 = v - L$ and $q_2 = D(v - L - c - o)/(v - L - j)$, and the first retailer does not sell to any consumer and earns a net loss equal to $T = D(v - L - c - o)^2/[2(v - L - j)]$. In Lemma 7(ii) we rule out this case by proving that if both retailers accept the manufacturer's contract, the equilibrium has to be symmetric. \square

PROOF OF PROPOSITION 3(i). We start off by taking the first-order condition of Retailer 1's objective function with respect to its choice variable p_1 and applying the envelope theorem to make use of the fact that Q_1 is already chosen optimally:

$$\begin{aligned} & \frac{-1}{2L} \left[(p_1 - p_w - o)Q^*(p_1) - (p_1 - j) \int_0^{Q^*(p_1)} F(s) ds \right] \\ & + \frac{L + p_2 - p_1}{2L} \left[Q^*(p_1) - \int_0^{Q^*(p_1)} F(s) ds \right] = 0. \end{aligned}$$

Substituting $p_1 = p_2 = p$ to impose symmetry gives

$$(p - p_w - o - L)Q^*(p) - (p - j - L) \int_0^{Q^*(p)} F(s) ds = 0.$$

Substituting $F(s) = s/D$ and $Q^*(p) = D(p - p_w - o)/(p - j)$ for uniform distribution gives

$$\begin{aligned} & p^2 - (j + p_w + o + L)p + [2(p_w + o + L)j - (j + L)(p_w + o)] = 0 \\ \Rightarrow & p^* = \frac{L + p_w + o + j + \sqrt{(3L + p_w + o - j)^2 - 8L^2}}{2}. \end{aligned}$$

We have chosen the larger root of the quadratic equation because it is the only one where the second-order condition for profit maximization holds. Because the first-order

condition is quadratic in p , these second-order conditions are also sufficient for global optimality once we rule out the boundary case. In other words, to verify that $(p_1 = p^*, p_2 = p^*)$ is indeed an equilibrium, we only need to check that, given price p^* of a retailer, the other retailer does not have an incentive to deviate to price $p^* - L$ to capture the entire market. However, that is easily verifiable from the retailer's profit function. \square

PROOF OF PROPOSITION 3(ii). From Lemma 3, $q_i = y_i F^{-1}((p_i - p_w - o)/(p_i - j))/L$. Imposing symmetry, we get $y_i = L/2$. Thus, $q_i = \frac{1}{2} F^{-1}((p_i - p_w - o)/(p_i - j)) = \frac{1}{2} D(p_i - p_w - o)/(p_i - j)$. Substituting p_i from above gives the desired expression for q_i . \square

PROOF OF PROPOSITION 3(iii).

$$\frac{dp_i}{dp_w} = \frac{1}{2} + \frac{3L + p_w + o - j}{2\sqrt{(3L + p_w + o - j)^2 - 8L^2}} > 1. \quad \square$$

PROOF OF PROPOSITION 3(iv).

$$\frac{dq_i}{dp_w} = \left(1 - \frac{3L + p_w + o - j}{\sqrt{(3L + p_w + o - j)^2 - 8L^2}}\right) \frac{D}{4L} < 0. \quad \square$$

PROOF OF PROPOSITION 3(v). The expression of the equilibrium profits of each retailer can be written by substituting $y = L/2$ and p_i from above in the earlier profit expression. \square

PROOF OF PROPOSITION 4. Noting that each retailer stocks $q = (D/2)[(p - p_w - o)/(p - j)]$, channel profits are

$$\begin{aligned} \Pi &= 2 \left[\frac{D(p - o - p_w)^2}{4(p - j)} + (w - c)q \right] \\ &= \frac{D(p - o - p_w)}{2(p - j)} [p + p_w - o - 2c]. \end{aligned}$$

Totally differentiating with respect to p_w ,

$$\begin{aligned} \frac{d\Pi(p(p_w), p_w)}{dp_w} &= \frac{D}{2} \left[\frac{2(p - o - c)}{p - j} - \frac{(p - o - p_w)(p + p_w - o - 2c)}{(p - j)^2} \right] \frac{dp}{dp_w} \\ &\quad - \frac{D(2p_w - 2c)}{2(p - j)} \\ &= \frac{D}{2(p - j)^2} \left[[2(p - j)(p - o - c) - (p - o - p_w) \right. \\ &\quad \left. \cdot (p + p_w - o - 2c)] \frac{dp}{dp_w} - 2(p - j)(p_w - c) \right]. \end{aligned}$$

But we have already proven that $dp/dp_w > 1$. So, noting that $p - j \geq 0$, $p_w - c \geq 0$,

$$\begin{aligned} \frac{d\Pi(p(p_w), p_w)}{dp_w} &> \frac{D}{2(p - j)^2} [2(p - j)(p - o - c) - (p - o - p_w) \\ &\quad \cdot (p + p_w - o - 2c) - 2(p - j)(p_w - c)] \frac{dp}{dp_w} \\ &= \frac{D}{2(p - j)^2} [(p - p_w - o) + 2(c + o - j)](p - o - p_w) \frac{dp}{dp_w} \\ &> 0 \quad \text{because } p > p_w + o > j, c + o > j, \frac{dp}{dp_w} > 1. \end{aligned}$$

Thus, to maximize joint surplus (because the retailer's profits can then be extracted from the fixed transfer fee), the manufacturer would like to set p_w as high as possible, while still obeying the constraint that the solution is competitive. Hence, it selects p_w so that the whole market is just served, which means that the retailers are given the incentive to select first-best prices (which, in turn, lead to less than first-best quantities according to the newsvendor formula). \square

PROOF OF PROPOSITION 5. From Proposition 1, we know that $p_1 = p_2 = p_{FB} = v - L/2$ and $q_1 = q_2 = q_{FB} = (D/2) \cdot (v - L/2 - w - o)/(v - L/2 - j)$. If the supplier provides a subsidy z for unsold inventory, the supplier is in effect increasing the salvage price for the retailer to $(j + z)$. Thus, we can set the price and quantity to first-best levels in Proposition 3 to solve for z . We get $z = v - j - 3L/2 + (2(c + o - j)L)/(v - L/2 - c - o)$. The expression for p_w follows by noting that $p_w = F(2q^{FB})z - (p^{FB} - j)F(2q^{FB}) + p^{FB} - o$. \square

References

- Anupindi, R., Y. Bassok. 1999. Centralization of stocks: Retailers vs. manufacturer. *Management Sci.* 45(2) 178–191.
- Bernstein, F., A. Federgruen. 2005. Decentralizing supply chains with competing retailers under demand uncertainty. *Management Sci.* 51(1) 18–29.
- Cachon, G. P. 1998. Competitive supply chain inventory management. S. Tayur, R. Ganeshan, M. Magazine, eds. *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers, Boston, MA, 111–146.
- Cachon, G. P. 2004. Supply chain coordination with contracts. S. Graves, T. de Kok, eds. *Handbooks in Operation Research and Management Science*, Vol. 11. *Supply Chain Management*. Elsevier, Amsterdam, The Netherlands, Forthcoming.
- Iyer, G. 1998. Coordinating channels under price and nonprice competition. *Marketing Sci.* 17(4) 338–355.
- Lariviere, M. 1998. Supply chain contracting and coordination with stochastic demand. S. Tayur, R. Ganeshan, M. Magazine, eds. *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers, Boston, MA, 233–268.
- Lee, H. L., V. Padmanabhan, T. A. Taylor, S. Whang. 2000. Price protection in the personal computer industry. *Management Sci.* 46(4) 467–482.
- Lippman, S. A., K. F. McCardle. 1997. The competitive newsboy. *Oper. Res.* 45(1) 54–65.
- Marvel, H., J. Peck. 1995. Demand uncertainty and return policies. *Internat. Econom. Rev.* 36(3) 691–714.
- Mathewson, G. F., R. A. Winter. 1986. The economics of vertical restraints in distribution. J. Stiglitz, G. F. Mathewson, eds. *New Developments in the Analysis of Market Structures*. MIT Press, Cambridge, MA, 211–236.
- McGuire, T. W., R. Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. *Marketing Sci.* 2(2) 161–191.
- Moorthy, K. S. 1987. Managing channel profits: Comment. *Marketing Sci.* 6(4) 375–379.
- Narayanan, V. G., A. Raman, J. Singh. 2004. Agency costs in a supply chain with demand uncertainty and price competition. Working paper, Harvard Business School, Cambridge, MA.
- Padmanabhan, V., I. P. L. Png. 1995. Returns policies: Make money by making good. *Sloan Management Rev.* 37(1) 65–72.

- Padmanabhan, V., I. P. L. Png. 1997. Manufacturer's returns policies and retail competition. *Marketing Sci.* **16**(1) 81–94.
- Petruzzi, N. C., M. Dada. 1999. Pricing and the newsvendor problem: A review with extensions. *Oper. Res.* **47**(2) 183–194.
- Rey, P., J. Tirole. 1986. Vertical restraints from a principal-agent viewpoint. L. Pellegrini, S. Reddy, eds. *Marketing Channels: Relationships and Performance*. Lexington Books, Lexington, MA, 3–30.
- Shaffer, G. 1991. Slotting allowances and resale price maintenance: A comparison of facilitating practices. *RAND J. Econom.* **22**(1) 120–135.
- Taylor, T. A. 2001. Channel coordination under price protection, midlife returns, and end-of-life returns in dynamic markets. *Management Sci.* **47**(9) 1220–1234.
- Tirole, J. 1988. *The Theory of Industrial Organization*. MIT Press, Cambridge, MA.
- Tsay, A., S. Nahmias, N. Agrawal. 1998. Modeling supply chain contracts: A review. S. Tayur, R. Ganeshan, M. Magazine, eds. *Quantitative Models for Supply Chain Management*. Kluwer Academic Publishers, Boston, MA, 299–336.
- Whitin, T. M. 1955. Inventory control and price theory. *Management Sci.* **2**(1) 61–68.