Operational Flexibility and Financial Hedging:
Complements or Substitutes?

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Abstract

We consider a two-product firm that invests in capacity under demand uncertainty and thus faces two types of risk: mismatch between capacity and demand and profit uncertainty. While mismatch risk can be mitigated with greater flexibility, profit uncertainty can be reduced through financial hedging. We show that the relationship between these two risk mitigating strategies depends on the type of flexibility: Product (or mix) flexibility and financial hedging are complements (substitutes), i.e., product flexibility increases (decreases) the value of financial hedging and, vice versa, financial hedging increases (decreases) the value of product flexibility, when product demands are positively (negatively) correlated. The reason is that product flexibility increases (decreases) the operating profit variance when demands are positively (negatively) correlated. In contrast to product flexibility, postponement flexibility tends to be a substitute to financial hedging, as intuitively expected.

(Key Words: financial hedging, product flexibility, postponement, capacity, inventory, real options.)

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1 Introduction

This paper studies the relationship between financial hedging and operational flexibility when both are used to mitigate a firm’s exposure to demand uncertainty. Most operations are exposed to two related yet distinct risks. *Mismatch risk* is an operational risk that refers to the expected cost of supply differing from demand. It includes both cash cost such as excess capacity or inventory and opportunity costs such as lost sales. To mitigate mismatch risk, firms often invest in various flexibilities which enable them to better adapt to volatile market conditions. (For more on mismatch risk, see Cachon and Terwiesch 2005).

The other type of risk is related to *profit variability*. The finance literature identifies several rationales explaining why value-maximizing firms may benefit from reducing profit variability and thus should engage in financial hedging. Smith and Stulz (1985) show that financial hedging can reduce expected tax liabilities, bankruptcy costs, or compensation paid to risk-averse managers. Nance, Smith and Smithson (1993) provide empirical evidence for these arguments. Stulz (1990), Bessembinder (1991), and Froot, Scharfstein, and Stein (1993) show that financial hedging can increase firm value by leading to more efficient capital investment outcomes.

While there is considerable amount of literature on both flexibility and financial hedging, relatively little research examines their relationship and most of it considers financial hedging against currency exposure as opposed to product demand exposure. Our paper attempts to fill this gap in the literature by showing that, contrary to our initial intuition, flexibility and financial hedging can be complements (as well as substitutes) in the firm’s overall risk management strategy.

We consider a value-maximizing firm whose operating profit depends on an exogenous random shock, which we refer to as “demand vector.” Following the corporate finance literature (Smith and Stulz 1985), we assume that the firm value is a concave function of pre-tax profit as a result of corporate profit tax, bankruptcy cost and other contracting costs. To mitigate profit variability
that has a negative impact on the expected firm value, the firm engages in financial hedging.\textsuperscript{4} Specifically, the firm enters a financial hedging contract with a third party whose payoff depends on some commonly observable underlying variable(s) such as stock indices, commodity prices, or weather-related variables, that are (imperfectly) correlated with the demand vector.\textsuperscript{5} As is common in the finance literature (Froot, Scharfstein, and Stein 1993), we assume that the financial hedging contract is “fairly-priced,” i.e., its expected value is zero. Given that in reality financial hedging is likely to be costly, we also examine the maximum amount of money the firm should be willing to pay for it. We characterize the optimal hedging contract that maximizes the expected firm value for a generic functional form of the operating profit. While this contract is generally rather complex, it simplifies considerably in some special cases. For example, for a newsvendor whose demand is perfectly correlated with the underlying variable, the optimal hedging contract involves buying European-style call options.

Flexibility is the ability to adapt to change and may take many forms. This paper addresses two of them: product flexibility and postponement flexibility. Following Van Mieghem (1998), we consider a two-product firm that invests in a mix of one product-flexible and two product-dedicated resources while product demands are uncertain. The firm’s degree of product flexibility can be measured by the unit cost of flexible capacity relative to the unit cost of dedicated capacity. When this ratio exceeds 2, the firm acquires only dedicated capacity and thus has no product flexibility. Otherwise, as this ratio decreases (approaching 1), the firm substitutes dedicated capacity with flexible capacity and its product flexibility increases (approaching full flexibility).

We prove that when the firm value is moderately concave in pre-tax profit, full product flex-

\textsuperscript{4}We refrain from assuming risk-aversion to be able to use risk-neutral pricing of the hedging instrument which facilitates our analysis.

\textsuperscript{5}There exists empirical evidence that firms do use financial derivatives to hedge, although most non-financial firms hold derivatives positions that are small in magnitude relative to entity-level risks (Guay and Kothari 2003).
ibility and perfect financial hedging are complements – full product flexibility increases the value of perfect financial hedging and, vice versa, perfect financial hedging increases the value of full product flexibility – if and only if, demands are positively correlated. The reason is that full product flexibility increases (decreases) operating profit variability under positive (negative) demand correlation. Our numerical investigation suggests that the insight continues to hold when flexibility and hedging are less than perfect.

The other type of flexibility which we address is postponement flexibility. We consider a standard single-product newsvendor and measure its postponement flexibility by the fraction of the unit product cost that is incurred after demand realization. Higher postponement flexibility thus means a smaller unit capacity cost which (i) reduces the cost of excess capacity when demand is low and (ii) stimulates the firm to invest in a larger capacity level, which in turn mitigates capacity shortage when demand high. Full postponement flexibility eliminates the mismatch cost entirely.

In contrast to product flexibility, postponement flexibility follows intuition: postponement flexibility and financial hedging are always substitutes when the firm value is moderately concave in pre-tax profit. Therefore, a firm with greater postponement flexibility should be willing to spend less money on financial hedging and, vice versa, a firm engaged in financial hedging will benefit less from postponement flexibility. This result is not as obvious as it may appear because postponement flexibility also increases the upside profit variability in addition to mitigating the downside profit variability.

As an illustration of our model, consider the following example. A sportswear retailer sells at two ski resorts, each on one coast, where the amount of snowfall (and hence demand for its products) varies considerably each year. Most of the retailer’s products have short life-cycles but long leadtimes and must be ordered several months before the selling season. The retailer orders some “dedicated” units of a particular product that are shipped directly to each of the two retail
locations. In addition, she orders some “flexible” units which will be kept at a central warehouse and shipped to a particular store only after demand has been observed. The lower the cost of relying on the central warehouse, the more units will be stored centrally, and the higher the retailer’s allocation or product flexibility. (The assumption here is that inventory transshipment between the stores is not economical.)

Alternatively, the retailer may reduce the mismatch cost by increasing its postponement flexibility. Instead of ordering all inventory ahead of the selling season, the retailer only reserves quick-response capacity and orders after demand is known. (In reality, the retailer may order some units ahead of the selling season and use quick-response capacity to reorder once demand is known with high accuracy. In that case, our results may overstate the value of flexibility.) For a detailed example of the quick-response strategy in the ski apparel industry, see Sport Obermeyer by Hammond and Raman (1996).

Finally, to protect profit against the possibility of low snowfall which hurts sales, the retailer enters into a financial hedging contract whose payoff is a decreasing function of the amount of snowfall at the ski resort(s). Such financial hedging contract will mitigate uncertainty in the retailer’s profit to the extent that demand is correlated with the amount of snowfall. The other contract party may offset the accepted risk by selling a counterbalancing contract to a firm with opposite exposure, e.g., a municipality responsible for snow removal. In addition, it may diversify the risk by selling similar contracts in different world regions where weather tends to move in different directions (Sumitomo Group Special Report 2002).

Although not yet on a large scale, weather-related derivatives are something that firms are increasingly experimenting with. The modern market for weather derivatives was born in the mid 1990s in the United States with innovative but ultimately failed energy trader Enron in the vanguard. Japan followed several years later with Mitsui Sumitomo Insurance selling a contract to
a retail ski shop to hedge against low snowfall that could hurt its sales (Kao 2006). Other weather conditions sometimes specified in weather derivatives include temperature, rainfall, and wind. In 2002, Mitsui Sumitomo Insurance issued a weather derivative contract to a soft drink wholesaler based on the number of hours of sunshine. If the number of sunshine hours recorded in the July–September quarter fell below a certain predetermined threshold, Mitsui Sumitomo Insurance would pay the company a pre-determined amount (Sumitomo Group Special Report 2002). Since that time, the weather derivative market has expanded rapidly. The notional value of weather risk management contracts transacted globally from April 2005 through March 2006 increased nearly fivefold from last year’s $9.7 billion to $45.2 billion (WRMA Press release 2006). Whereas most of the volume comes from energy companies, the share of the retail sector in the overall amount of weather related hedging transactions increased up to 7% in 2006 (WRMA Survey 2006).

2 Relation to the literature

The operations literature on product flexibility is extensive and mostly assumes expected profit maximization (see e.g., Fine and Freund 1990, Jordan and Graves 1995, Van Mieghem 1998, and Chod and Rudi 2005). The literature addressing postponement flexibility is less abundant. Our model of postponement flexibility is similar to the one considered by Chod, Rudi and Van Mieghem (2006) who study its relationship with other types of flexibility. Also similar to our concept of postponement flexibility is “production postponement” examined by Van Mieghem and Dada (1999), and the “ability to reorder inventory” addressed by Fisher and Raman (1996). A notion closely related to flexibility is operational hedging, which refers to risk mitigation using operational instruments (Van Mieghem 2007). The operational hedging literature typically focuses on hedging against exchange rate fluctuations. A classical example is the study by Huchzermeier and Cohen (1996) of the value of the option to shift production among countries based on currency fluctua-
tions. A recent review of the operational hedging literature can be found in Boyabatli and Toktay (2004).

The seminal work on resource investment under risk aversion is Eeckhoudt, Gollier and Schlesinger (1995) who demonstrate that the optimal resource level of a single-resource newsvendor-type firm decreases in risk aversion for any concave utility function. Other papers studying single resource investment under risk aversion include Lau (1980), Anvari (1987), Bouakiz and Sobel (1992) and Agrawal and Seshadri (2000). Van Mieghem (2007) studies how risk-averse managers can use resource diversification, sharing and flexibility in newsvendor networks to mitigate risk. He shows that in networks, some resource levels and even the entire investment may increase in risk aversion. Specifically, risk aversion leads to substitution of dedicated resources with a flexible one, thus increasing the flexible resource level. Tomlin and Wang (2005) consider product flexibility and dual-sourcing in unreliable newsvendor networks under both loss aversion and conditional value-at-risk. They demonstrate that a dedicated strategy may be preferred to a flexible one even if the dedicated resources are costlier than the flexible resource because there is a resource-aggregation disadvantage to the flexible strategy that can dominate its demand pooling and contribution-margin benefits when resource investments are unreliable and the firm is risk-averse.

The operations management literature addressing financial hedging includes the following several papers. Gaur and Seshadri (2005) consider a risk-averse newsvendor that hedges its risk-exposure with financial options. They show for a wide range of utility functions and hedging strategies that the use of financial hedging results in a higher optimal inventory level. Caldentey and Haugh (2006) extend the work of Gaur and Seshadri by allowing continuous trading in the financial market and by considering alternative evolutions of the firm’s information. Furthermore, rather then focusing on the newsvendor problem, Caldentey and Haugh solve the general problem of simultaneously optimizing over the operating as well as hedging policy of a firm. Chen,
Sim, Simchi-Levi and Sun (2005) incorporate risk aversion and financial hedging in multi-period inventory and pricing models. They characterize the optimal policy for various measures of risk, demonstrating that the structure of the optimal inventory and pricing policy under the exponential utility is almost identical to the structure of the optimal risk-neutral policy.

Recently, several papers have considered the joint use of financial hedging and flexibility. Ding, Dong and Kouvelis (2004) study a risk-averse firm that uses a single production facility to serve the domestic and a foreign market. The firm has to choose production capacity under demand and exchange rate uncertainty, and may or may not have allocation (product) flexibility. To hedge against exchange rate risk, the firm may buy financial options. Ding et al. (2004) analyze the effect of allocation flexibility and financial hedging on the optimal capacity investment and the firm’s performance. They show, among others, that under negatively correlated sales in the two markets, increasing the capacity level may reduce the total profit variability and, therefore, a risk-averse firm may choose a higher capacity level than a risk-neutral firm.

Zhu and Kapuscinski (2006) consider a multinational risk-averse newsvendor who produces and sells at home as well as overseas and is exposed to demand and exchange rate uncertainties. In addition to using operational hedging (transshipment between domestic and foreign facility), the newsvendor hedges its exchange rate exposure in the financial market. Zhu and Kapuscinski show that the benefits of operational hedging are significantly greater than those of financial hedging. Our work differs from both Ding et al. (2004) and Zhu and Kapuscinski (2006) in that they consider financial hedging against exchange rate exposure whereas we study financial hedging against demand exposure. Furthermore, they do not explicitly address the question of complementarity/substitution between flexibility and financial hedging.

The issue of complementarity/substitution between product flexibility and financial risk management (FRM) has been addressed by Boyabatli and Toktay (2006a) who consider a firm that uses
FRM to influence the distribution of its budget available for investment in production technology, which may be dedicated or flexible. In their work, the link between FRM and flexibility stems from the fact that flexible technology is more expensive and thus requires more financing which can be generated externally at a cost and/or from an internal budget which is affected by FRM. The relationship between flexibility and FRM thus depends on factors such as the firm size. Specifically, flexibility and FRM tend to be complements for large firms. This is because with FRM, a large firm may be able to secure enough financing to purchase flexible technology without relying on costly external financing. In contrast, we consider a situation in which flexibility and financial hedging are used simultaneously to manage firm’s exposure to uncertain demand. Thus, the relationship between flexibility and financial hedging in our context depends on their joint impact on the operating profit variability, which we show to depend on demand correlation.

Boyabatli and Toktay (2006b) is an extension of Boyabatli and Toktay (2006a) which endogenizes the cost of external financing by modeling the interaction between the firm and the creditor as a Stackelberg game. Other research that investigates interactions between flexibility and financial decisions includes Lederer and Singhal (1994) who study the jointly optimal technology and financing decisions. In a recent paper, Zhou and Rudi (2007) study the pricing of financial hedging contracts used by firms to protect against demand risk by modelling the interaction between a hedging firm and the contract issuer.

Finally, there exists some finance and economics literature that integrates operational flexibility and financial hedging. Mello, Parsons and Triantis (1995) consider a model of a multinational firm with sourcing flexibility and with the ability to hedge exchange rate risk in the financial market. The firm’s financial hedging policy and its production decisions are linked through agency costs generated by the firm’s capital structure. Chowdry and Howe (1999) examine conditions under which global firms use financial and operational hedging against demand and exchange rate
uncertainties. Hommel (2003) studies incentives of a global firm to hedge currency risk with financial and operational means under a minimum profit constraint.

3 Optimal financial hedging and its value

Firm value. We consider a value-maximizing firm whose profit depends on uncertain demand and thus is itself uncertain. As discussed in the introduction, there exist several market imperfections such as taxes, the cost of financial distress, and other agency costs, as a result of which the firm value is a concave function of pre–tax profit (Smith and Stulz 1985). Rather than modeling these market imperfections explicitly, we assume, for tractability, that the firm value \( v \), as a function of pre-tax profit \( x \), has the following form:

\[
v(x) = \alpha \exp(-\gamma x) + \beta, \quad \text{where } \alpha \leq 0 \text{ and } \gamma \geq 0. \tag{1}
\]

Without loss of generality, we choose \( \alpha = -\beta = -1/\gamma \) so that \( v(x) \to x \) as \( \gamma \to 0 \), i.e.,

\[
v(x) = \gamma^{-1} - \gamma^{-1} \exp(-\gamma x). \tag{2}
\]

The concavity of firm value implies that there exists a positive risk premium \( r \) that the firm is willing to pay to eliminate (pre-tax) profit variability, i.e., \( \mathbb{E}v(x) = v(\mathbb{E}x - r) \). This risk premium

\[
r = \gamma^{-1} \ln \mathbb{E} \exp(-\gamma (x - \mathbb{E}x)) \tag{3}
\]

depends on the distribution of \( x \) as well as on the concavity of the value function captured in parameter \( \gamma = -v''(x)/v'(x) \). In our context, a higher value of \( \gamma \) corresponds to a stronger effect of taxes, bankruptcy and other contracting cost, etc.

Alternatively, \( v \) could be interpreted as the utility of the firm’s risk-averse owner, in which case \( \gamma \) would correspond to Arrow-Pratt coefficient of absolute risk aversion (see Pratt 1964). However,
interpreting $v$ as firm value will enable us to use risk-neutral pricing of the financial hedging contract, which will facilitate our analysis considerably.

**Financial hedging.** Let $\Pi_o(D)$ be the firm’s operating profit (of a general functional form) that depends on stochastic demand vector $D$. To mitigate the undesirable operating profit variability, the firm enters into a financial hedging contract that is designed to generate a positive cash flow when demand is low. In practice, demand depends on the firm’s effort which may create agency problems in financial contracting. Therefore, the hedging contract payoff is typically based on another “underlying” vector $\theta$ that is exogenous and commonly verifiable, such as the amount of snowfall at a certain location during a certain period of time, a commodity price or a stock index. As a result, financial hedging is typically imperfect: it counterbalances operating profit variability only partially.

**Optimal hedging contract.** Let $\Pi_h(\theta)$ denote the payoff of the financial hedging contract. Following Froot, Scharfstein, and Stein (1993), we assume that this contract is “fairly-priced,” i.e., its expected payoff $E\Pi_h(\theta)$ is zero. (Technically, this is the case when financial market is arbitrage-free and investors are risk-neutral.) In reality, financial hedging is likely to involve some transaction cost, and therefore, we will also examine how much a firm should be willing to pay for financial hedging. The firm chooses a hedging contract to maximize its expected value:

$$\max_{\Pi_h} E_v (\Pi_o(D) + \Pi_h(\theta)) \text{ subject to } E\Pi_h(\theta) = 0.$$ 

It is useful to define $\Pi = \Pi_o + \Pi_h^*$ as the total profit given the optimal hedging contract is being used. While the problem of finding the optimal hedging contract would typically involve calculus of variation, it has a surprisingly simple solution due to the exponential value function (2):

**Proposition 1** The optimal hedging contract is

$$\Pi_h^*(\theta) = f(\theta) - E f(\theta) \text{ where } f(\theta) = \gamma^{-1} \ln E [\exp(-\gamma \Pi_o(D)) | \theta]$$
and results in the following expected firm value:

\[ E_v(\Pi(D, \theta)) = \gamma^{-1} - \gamma^{-1} \exp(\gamma E_f(\theta)) \]  

(4)

**Proof:** Maximizing the expected firm value \( E_v(\Pi_o(D) + \Pi_h(\theta)) \) is equivalent to minimizing

\[
\mathbb{E} \exp(-\gamma (\Pi_o(D) + \Pi_h(\theta))) = \mathbb{E} [\exp(-\gamma \Pi_o(D)) \exp(-\gamma \Pi_h(\theta))]  
\]

\[
= \mathbb{E} \{ \mathbb{E} [\exp(-\gamma \Pi_o(D)) \exp(-\gamma \Pi_h(\theta)) | \theta] \}  
\]

\[
= \mathbb{E} \{ \mathbb{E} [\exp(-\gamma \Pi_o(D)) | \theta] \exp(-\gamma \Pi_h(\theta)) \}  
\]

\[
= \mathbb{E} \exp(\gamma f(\theta) - \gamma \Pi_h(\theta)). \quad (5)
\]

We show by contradiction that (5) is minimized by \( \Pi_h^*(\theta) = f(\theta) - E f(\theta) \) among all \( \Pi_h(\theta) \) such that \( \mathbb{E} \Pi_h(\theta) = 0 \). Suppose that there exists a better hedging payoff \( g(\theta) \) such that \( \mathbb{E} g(\theta) = 0 \) and

\[
\mathbb{E} \exp(\gamma f(\theta) - \gamma g(\theta)) < \mathbb{E} \exp(\gamma f(\theta) - \gamma \Pi_h^*(\theta))  
\]

\[
\iff \mathbb{E} \exp(\gamma f(\theta) - \gamma g(\theta)) < \exp(\mathbb{E} \gamma f(\theta))  
\]

\[
\iff \mathbb{E} \exp(X) < \exp(\mathbb{E} X) \text{ where } X = \gamma f(\theta) - \gamma g(\theta),
\]

which contradicts the Jensen’s inequality. Hence, there cannot be such \( g \), and \( \Pi_h^* \) is indeed optimal. □

The payoff of the optimal hedging contract depends on a conditional expectation and thus may be rather complex. However, as any continuous derivative, it can be arbitrarily closely approximated by a portfolio of simple options (Ross 1976). Obviously, the presence of financial hedging will impact the firm’s operations which we consider next.

**Optimal capacity.** Suppose that firm’s operating profit depends, in addition to the demand vector \( D \), on a vector \( K \) of resource or capacity levels that must be chosen prior to uncertainty resolution (together with the financial hedging contract). Reflecting decreasing marginal returns, we assume that operating profit \( \Pi_o(D, K) \) is jointly concave in capacity vector \( K \), as is the case
for any newsvendor network (Van Mieghem and Rudi 2002). The next proposition characterizes the optimal capacity vector.

**Proposition 2** The optimal capacity vector \( \mathbf{K}^* \) that maximizes the expected firm value (4) is uniquely characterized by the following first-order condition:

\[
\mathbb{E}\nabla_{\mathbf{K}} \Pi_o (\mathbf{D}, \mathbf{K}) + \mathbb{E} \left[ \text{Cov} \left( \nabla_{\mathbf{K}} \Pi_o (\mathbf{D}, \mathbf{K}), \frac{\sigma' (\Pi_o (\mathbf{D}, \mathbf{K}))}{\mathbb{E} (\sigma' (\Pi_o (\mathbf{D}, \mathbf{K})) | \theta)} \right) \right] = 0. \tag{6}
\]

**Proof:** We assumed that \( \Pi_o (\mathbf{D}, \mathbf{K}) \) is jointly concave in \( \mathbf{K} \) for any \( \mathbf{D} \), and it is well known that \( \ln \mathbb{E} \exp () \) is a convex function. Thus, the expected firm value (4) is strictly concave and the optimal capacity is the unique solution of the first-order necessary and sufficient condition,

\[\nabla_{\mathbf{K}} \mathbb{E} v (\mathbf{D}, \mathbf{K}, \theta) = 0, \] which can be rewritten as (6). \( \square \)

Note that the operating profit gradient \( \nabla_{\mathbf{K}} \Pi_o (\mathbf{D}, \mathbf{K}) \) is a vector, and thus, the covariance in (6) should be interpreted componentwise.

**Contract correlation.** In general, the efficacy of financial hedging depends on the entire joint distribution of \( \mathbf{D} \) and \( \theta \). However, we simplify our analysis by assuming that \( (\mathbf{D}, \theta) \) follows a multivariate normal distribution and, furthermore, that \( \mathbf{D} \) and \( \theta \) have the same number of components, i.e., there is an underlying variable corresponding to each demand class. We also assume that the correlation coefficient between \( D_i \) and \( \theta_i \), which we denote as \( \rho \), is the same for all \( i \). Thus, the efficacy of financial hedging depends on a single parameter \( \rho \), which we assume, without loss of generality, to be non-negative, and refer to as “contract correlation.” As contract correlation increases from 0 to 1, the efficacy of financial hedging increases from none (no financial hedging available) to perfect (financial hedging eliminates all profit variability). Finally, whenever we resort to numerical analysis, we assume the following structure of \( (\mathbf{D}, \theta) \):

\[
\mathbf{D} \sim \mathcal{N} (\mu_D, \Sigma_D), \quad \theta \sim \mathcal{N} (\mu_D, \rho^2 \Sigma_D) \quad \text{and} \quad \mathbf{D} | \theta \sim \mathcal{N} (\theta, (1 - \rho^2) \Sigma_D),
\]
where \((\Sigma_D)_{ii} = \sigma_D^2\) and \((\Sigma_D)_{ij} = \rho_D \sigma_D^2\), \(i, j = 1, 2\). (In this, \(D\) and \(\theta\) can be thought of as the states of a multidimensional Brownian motion at times 1 and \(\rho \leq 1\), respectively, i.e., \(\theta\) captures \(\rho^2 \times 100\%\) of the variability of \(D\).)

**An illustration: optimal hedging of newsvendor profit.** While the optimal hedging contract characterized in Proposition 1 does not assume any functional form of the operating profit, it is instructive to illustrate what this contract looks like when the firm’s operating profit has the standard newsvendor form,

\[
\Pi_o (K; D) = -cK + p \min (K, D),
\]

where \(K\) is capacity, \(c\) is the unit capacity cost and \(p\) is the unit contribution margin. There are two special cases in which the optimal hedging contract simplifies considerably: (i) when the firm has ample capacity \((c = 0)\) and (ii) under perfect contract correlation \((\rho = 1)\). We characterize these special cases in the following two corollaries to Proposition 1.

**Corollary 1** When the firm has ample capacity, i.e., the operating profit is \(\Pi_o (D) = pD\), the optimal hedging contract has a payoff that is linear in the underlying variable:

\[
\Pi_h (\theta) = -pp \frac{\sigma_D}{\sigma_\theta} (\theta - \mu_\theta).
\]

The contract characterized in Corollary 1 corresponds to linear contracts such as a simple forward, swap or futures contract.

**Corollary 2** When the operating profit is given by (7) and contract correlation is perfect, the optimal hedging contract involves buying \(p \frac{\sigma_D}{\sigma_\theta}\) European call options with spot price \(-\theta\) and strike price \(-\mu_\theta - \frac{\sigma_\theta}{\sigma_D} (K^* - \mu_D)\) where the optimal capacity \(K^* = \mu_D + \sigma_D \Phi^{-1} \left(\frac{p-c}{p}\right)\). The payoff of this hedging contract is

\[
\Pi_h (\theta) = -\Pi_o (D) + \mathbb{E} \Pi_o (D) = -p \min (K, D) + p \mathbb{E} \min (K, D)
\]
Figure 1: The payoff of the optimal hedging contract $\Pi_h(\theta)$ of a newsvendor with a given capacity $K$ at different contract correlations $\rho \in [0, 1]$.

where $D = \mu_D + \frac{\sigma_D}{\sigma_\theta} (\theta - \mu_\theta)$. 

When the contract correlation is less than perfect, the optimal hedging contract is more exotic as illustrated in Figure 1. This figure shows the payoff of the optimal hedging contract of a newsvendor with a given capacity at different contract correlations. With zero contract correlation, there is no financial hedging, i.e., $\Pi_h(\theta) = 0$. As contract correlation increases and approaches one, the optimal hedging contract approaches the option contract which becomes optimal when contract correlation equals one (as indicated by the bold graph).

The value of financial hedging. In reality, financial hedging is likely to come at a cost and so it is useful to examine how much a firm should be willing to pay for it. We define the value of financial hedging $\Delta(\rho, \gamma)$ as the maximum amount of money a firm is willing to pay for the optimal hedging contract, i.e.,

$$\max_{K} \mathbb{E}v(\Pi - \Delta) = \max_{K} \mathbb{E}v(\Pi_o). \quad (8)$$

The value of financial hedging is similar to the risk premium (3) with two distinctions. First, financial hedging reduces but generally does not eliminate all profit variability. Second, financial
hedging not only reduces profit variability but also affects the optimal capacity vector and thereby expected profit.

Some of our analytical results assume that the value function is “moderately concave” by which we mean that there exists \( \varepsilon > 0 \) such that the given result holds for any \( \gamma \in (0, \varepsilon) \). Practically, this means that the effects of taxes, bankruptcy cost, etc. are relatively small. The analytics derived for moderately concave value functions rely on the following result.

**Proposition 3** The value of financial hedging can be written as

\[
\Delta(\rho, \gamma) = \frac{1}{2} \gamma \left[ \text{Var}_{\Pi_o} (K^0) - \mathbb{E} \left( \text{Var}(\Pi_o (K^0) | \theta) \right) \right] + o(\gamma),
\]

where \( K^0 \) is the optimal capacity vector at \( \gamma = 0 \).

**Proof:** Using Proposition 1, the value of financial hedging defined by (8) can be expressed as

\[
\Delta(\rho, \gamma) = \gamma^{-1} \mathbb{E} \left[ \ln \frac{\mathbb{E} \exp (-\gamma \Pi_o (K^\ast (\rho)))}{\mathbb{E} [\exp (-\gamma \Pi_o (K^\ast (\rho)) | \theta)]} \right],
\]

where \( K^\ast (\rho) \) is the optimal capacity vector as a function of the contract correlation. Because \( \Delta(\rho, 0) = 0 \), the first-order Taylor expansion around \( \gamma = 0 \) gives

\[
\Delta(\rho, \gamma) = \frac{d \Delta(\rho, 0)}{d \gamma} \gamma + o(\gamma).
\]

Differentiating \( \Delta(\rho, \gamma) \) with respect to \( \gamma \) yields

\[
\frac{d \Delta(\rho, \gamma)}{d \gamma} = \frac{1}{\gamma^2} \left[ \mathbb{E} \left( \frac{\gamma \mathbb{E} [\Pi_o (K^\ast (\rho)) \exp (-\gamma \Pi_o (K^\ast (\rho))) | \theta]}{\mathbb{E} [\exp (-\gamma \Pi_o (K^\ast (\rho)) | \theta)]} - \mathbb{E} \left( \ln \frac{\mathbb{E} \exp (-\gamma \Pi_o (K^\ast (\rho)))}{\mathbb{E} [\exp (-\gamma \Pi_o (K^\ast (\rho)) | \theta)]} \right) \right) - \gamma \mathbb{E} [\Pi_o (K^\ast (\rho)) \exp (-\gamma \Pi_o (K^\ast (\rho))) | \theta] \right].
\]

Using L’Hospital rule to evaluate the derivative at \( \gamma = 0 \) gives

\[
\frac{d \Delta(\rho, 0)}{d \gamma} = \frac{1}{2} \left[ \text{Var}_{\Pi_o} (K^0) - \mathbb{E} \left( \text{Var}(\Pi_o (K^0) | \theta) \right) \right],
\]

and the result follows. \( \Box \)
For a moderately concave value function, the value of financial hedging depends on how much it reduces the operating profit variance.

**Operational flexibility and its value.** In addition to financial hedging, the firm’s ability to handle demand risk depends on its operational flexibility. We denote the degree of firm’s operational flexibility by a continuous parameter \( \phi \in [0, 1] \), and use subscripts \( N \) and \( F \) to refer to a non-flexible firm (\( \phi = 0 \)) and a flexible firm (\( \phi > 0 \)), respectively. (We give parameter \( \phi \) a specific meaning in the subsequent sections in which we consider two particular forms of flexibility: product flexibility and postponement flexibility.) We define the *value of flexibility* \( \Lambda (\rho, \gamma) \) as the maximum amount of money a non-flexible firm is willing to pay for flexibility, i.e.,

\[
\mathbb{E} v (\Pi_F (K^*_F) - \Lambda) = \mathbb{E} v (\Pi_N (K^*_N)). \tag{9}
\]

**Relationship between flexibility and financial hedging.** The goal of this paper is to study the relationship between financial hedging and flexibility. We do so by examining how firm’s flexibility affects the value of financial hedging, or, alternatively, how financial hedging affects the value of flexibility. In the case of perfect financial hedging (\( \rho = 1 \)), these are two sides of the same coin as formalized in the following lemma.

**Lemma 1** Perfect financial hedging increases the value of product flexibility if, and only if, product flexibility increases the value of perfect financial hedging, i.e.,

\[
\Lambda (1, \gamma) > \Lambda (0, \gamma) \iff \Delta_F (1, \gamma) > \Delta_N (1, \gamma).
\]

**Proof:** It follows from (9) that

\[
\Lambda (1, \gamma) = \Pi_F (K^0_F; 1, \gamma) - \Pi_N (K^0_N; 1, \gamma),
\]

and

\[
\Lambda (0, \gamma) = \gamma^{-1} \ln \mathbb{E} \exp (-\gamma \Pi_{oN} (K^*_N)) - \gamma^{-1} \ln \mathbb{E} \exp (-\gamma \Pi_{oF} (K^*_F)).
\]
It further follows from (8) that
\[
\Delta_F (1, \gamma) = \Pi_F (K_F^0; 1, \gamma) + \gamma^{-1} \ln \mathbb{E} \exp \left( -\gamma \Pi_{o,F} (K_F^*) \right),
\]
and \[
\Delta_N (1, \gamma) = \Pi_N (K_N^0; 1, \gamma) + \gamma^{-1} \ln \mathbb{E} \exp \left( -\gamma \Pi_{o,N} (K_N^*) \right).
\]

Combining these facts implies \( \Lambda (1, \gamma) - \Lambda (0, \gamma) = \Delta_F (1, \gamma) - \Delta_N (1, \gamma) \), and the result follows.\( \Box \)

In the next two sections, we show that the interplay between financial hedging and flexibility depends critically on the particular form of flexibility. Our results are derived analytically to the extent possible. When the expressions are not amenable to analytics, we rely on numerical optimization via simulation with 10,000 sample points.

4 Product flexibility

We consider a newsvendor-like firm which must choose capacity of three resources while facing uncertain demand for its two products, as analyzed in Van Mieghem (1998). Two resources are dedicated to the two products whereas the third resource is product-flexible. For simplicity, we assume all parameters to be equal for the two products. If we use \( K = (K_1, K_2, K_F)' \), \( c = (c_N, c_N, c_F)' \), and \( p \) to denote the capacity vector, the vector of unit capacity costs, and the unit contribution margin\(^6\), respectively, the firm’s operating profit equals

\[
\Pi_o (D, K) = \max_{x, y \in \mathbb{R}^2_+} p (x_1 + x_2 + y_1 + y_2) - c'K,
\]
subject to

\[
x_i + y_i < D_i, \ i = 1, 2,
\]
\[
x_i < K_i, \ i = 1, 2,
\]
and \( y_1 + y_2 < K_F \),

where \( x_i \) and \( y_i \) represent the amounts of non-flexible and flexible capacity, respectively, that are used to satisfy demand for product \( i, \ i = 1, 2 \). The operating profit (10) depends on the realized

\(^6\)The unit contribution margin is the unit revenue (price) minus the unit production cost.
demand as follows:

\[
\Pi_o (D, K) = -c'K = \begin{cases} 
  p(D_1 + D_2) & \text{if } D \in \Omega_0 (K) \\
  p(K_1 + K_F + D_2) & \text{if } D \in \Omega_1 (K) \\
  p(D_1 + K_2 + K_F) & \text{if } D \in \Omega_2 (K) \\
  p(K_1 + K_2 + K_F) & \text{if } D \in \Omega_3 (K)
\end{cases}
\]

where \( \Omega_0 (K) = \{ D \geq 0 : D_1 + D_2 \leq K_1 + K_2 + K_F, D_1 \leq K_i + K_F, i = 1, 2 \} \),

\( \Omega_1 (K) = \{ D \geq 0 : D_1 > K_1 + K_F, D_2 \leq K_2 \} \),

\( \Omega_2 (K) = \{ D \geq 0 : D_2 > K_2 + K_F, D_1 \leq K_1 \} \),

and \( \Omega_3 (K) = \{ D \geq 0 : D_1 + D_2 > K_1 + K_2 + K_F, D_i > K_i, i = 1, 2 \} \).

Because \( \Pi_o (D, K) \) is jointly concave in \( K \) (Van Mieghem and Rudi 2002), the optimal capacity vector is characterized by Proposition 2. The assumption of symmetrical product parameters together with the uniqueness of the solution imply that it is always optimal to set both non-flexible capacity levels equal, and thus, we can simplify notation by letting \( K_1 = K_2 \equiv K_N \).

**Level of flexibility.** To study the relationship between flexibility and financial hedging, we need an unambiguous measure of flexibility. Because the relative levels of flexible and non-flexible capacities depend on the relative costs of these two capacities, we can define the firm’s flexibility as

\[
\phi \equiv \frac{(2c_N - c_F)}{c_N}.
\]

If \( \phi = 0 \), a unit of flexible capacity has the same cost as two units of non-flexible capacity, \( c_F = 2c_N \), and, therefore, it is optimal to acquire only non-flexible capacity, \( K^* = (K_N^*, K_N^*, 0)^t \), and the firm has no product flexibility. If \( \phi = 1 \), flexible and non-flexible capacities are equally expensive, \( c_F = c_N \), and, therefore, it is optimal to acquire only flexible capacity, \( K^* = (0, 0, K_F^*)^t \), and the firm has full product flexibility. In general, as \( \phi \) increases from 0 to 1, the unit cost of flexible
Figure 2: Product flexibility increases the probability that both demands will be fully met, which happens when $\mathbf{D} \in \Omega_0$ (shaded region).

capacity $c_F$ decreases from $2c_N$ to $c_N$, and the firm substitutes non-flexible capacity with flexible one.

The effect of flexibility is illustrated in Figure 2 which shows the partitioning of demand state space into four events: if $\mathbf{D} \in \Omega_0$, both demands are fully satisfied; if $\mathbf{D} \in \Omega_1$ ($\Omega_2$) only demand for product 2 (1) is fully satisfied; and, finally, if $\mathbf{D} \in \Omega_3$, neither demand is fully satisfied (assuming flexible capacity is split between the two products proportionally). As shown in the figure, product flexibility increases sales when one demand is “high” while the other one is “low.” (Note that all panels in Figure 2 are plotted for the same total capacity $2K_N + K_F$ although flexibility may increase or decrease total capacity depending on problem parameters.)

**Product flexibility and financial hedging.** To assess the relationship between flexibility and financial hedging, we examine how flexibility affects the value of financial hedging. We first present an analytical result that compares the values of perfect financial hedging under no flexibility and under full flexibility for a moderately concave value function. In the following, we use subscripts $N$ and $F$ to denote the cases of no flexibility ($\phi = 0$) and full flexibility ($\phi = 1$), respectively.
Proposition 4 For sufficiently low $\gamma$, full product flexibility increases the value of perfect financial hedging ($\rho = 1$) if, and only if, demands are positively correlated, i.e.,

$$\Delta_F (1, \gamma) \geq \Delta_N (1, \gamma) \quad \iff \quad \rho_D \geq 0.$$ 

Proof: Proposition 3 implies that at sufficiently low $\gamma$ and $\rho = 1$, $\Delta_F \geq \Delta_N$ if, and only if,

$$\text{Var}\Pi_o F (K_F^0) \geq \text{Var}\Pi_o N (K_N^0)$$

$$\iff \text{Var} \min (D_1 + D_2, K_F^0) \geq \text{Var} \sum_{i=1}^{2} \min (D_i, K_N^0)$$

$$\iff \text{Var} \min \left(2\mu + Z_1\sigma_D \sqrt{2 + 2\rho_D}, 2\mu + z^0\sigma_D \sqrt{2 + 2\rho_D} \right) \geq \text{Var} \sum_{i=1}^{2} \min (\mu + Z_i\sigma_D, \mu + z^0\sigma_D)$$

$$\iff (2 + 2\rho_D) \text{Var} \min (Z_1, z^0) \geq 2\text{Var} \min (Z_1, z^0) + 2\text{Cov} \left[\min (Z_1, z^0), \min (Z_2, z^0)\right]$$

$$\iff \rho_D \geq \frac{\text{Cov} \left[\min (Z_1, z^0), \min (Z_2, z^0)\right]}{\text{Var} \min (Z_1, z^0)},$$

where $z^0 = \Phi^{-1} \left(\frac{\rho - \rho_D}{\rho} \right)$ and $Z = \frac{D - \mu_D}{\sigma_D}$ is the normalized demand vector with correlation coefficient $\rho_D$. Thus, the last inequality compares the correlation of two standard normal random variables with the correlation of the same standard normal random variables right-censored at $z^0$. (If $X$ is a random variable, we say that the distribution of $\min (X, x)$ is right-censored at $x$.) It is straightforward to verify numerically that the inequality holds if, and only if, $\rho_D > 0$ for any $z^0$. \(\square\)

It follows from Proposition 3 that for a moderately concave value function, the value of perfect financial hedging depends on the operating profit variance. Therefore, product flexibility increases the value of perfect financial hedging if, and only if, it increases the operating profit variance, i.e.,

$$\Delta_F (1, \gamma) \geq \Delta_N (1, \gamma) \quad \iff \quad \text{Var}\Pi_o F (K_F^0) \geq \text{Var}\Pi_o N (K_N^0).$$

Proposition 4 shows that product flexibility increases the operating profit variance if, and only if, demands are positively correlated. To gain some intuition behind this effect, consider Figures 3 and 4 which show the effect of flexibility on operating profit distribution under negative and positive demand correlation, respectively. When demands are negatively correlated, relative to
the dedicated capacity constraints, the flexible capacity constraint allows more uneven product sales (sales falling into shaded triangles in Figure 3b). This reduces operating profit variance (while increasing expected profit). Or, to put it differently, a key effect of product flexibility with negatively correlated demands is to reduce downside variability, i.e., the lower tail of the operating profit distribution (Figure 3c). This “operational hedging” effect of flexibility is strongest when demands are perfectly negatively correlated, in which case, full product flexibility eliminates operating profit variability entirely (given our symmetry assumption).

When demands are positively correlated, relative to the dedicated capacity constraints, the flexible capacity constraint allows more symmetrical product sales (sales falling into shaded triangles in Figure 4b). This increases operating profit variance (while also increasing expected profit). In other words, product flexibility with positively correlated demands increases the operating profit variance by increasing the upside variability (Figure 4c).

Our numerical examination suggests that the effect of flexibility on the value of financial hedging characterized in Proposition 4 continues to hold for partial flexibility and imperfect financial
hedging. This is shown in Figure 5 which plots the value of financial hedging as a function of product flexibility $\phi \in [0, 1]$ at different contract correlations $\rho \in [0, 1]$ and demand correlations $\rho_D \in \{-0.5, 0.5\}$. (This and all other figures are based on the following parametric values: $\mu_D = 1$, $\sigma_D = 0.5$, $p = 1$, $c_N = 0.5$, and $\gamma = 1$.) Figure 5 confirms that the value of financial hedging $\Delta$ decreases (increases) in product flexibility when demands are negatively (positively) correlated. This effect is strongest under perfect contract correlation when full product flexibility reduces (increases) the value of financial hedging by approx. 29% (6.5%) at demand correlation $\rho_D = -0.5 (+0.5)$.

Our results have the following managerial implications. When product demands are positively correlated, product flexibility increases operating profit variability and, therefore, a product-flexible firm should be willing to spend more money on financial hedging. Product flexibility and financial hedging are complements. Conversely, when product demands are negatively correlated, product flexibility decreases operating profit variability and, therefore, a product-flexible firm should be willing to spend less money on financial hedging. Product flexibility and financial hedging are substitutes.
Figure 5: The value of financial hedging $\Delta$ decreases (increases) in product flexibility when demands are negatively (positively) correlated.

An alternative approach to assessing the same relationship is to ask how financial hedging affects the value of flexibility. The next result simply mirrors Proposition 4.

**Corollary 3** For sufficiently low $\gamma$, perfect financial hedging increases the value of full product flexibility if, and only if, demands are positively correlated, i.e.,

$$\Lambda(1, \gamma) \geq \Lambda(0, \gamma) \iff \rho_D \geq 0.$$ 

**Proof:** The result follows directly from Lemma 1 and Proposition 4. \(\square\)
As we know from Lemma 1, perfect financial hedging increases the value of full product flexibility if, and only if, full product flexibility increases the value of perfect financial hedging. For a moderately concave value function, this is the case if, and only if, demands are positively correlated. Our numerical experiments suggest that the relationship continues to hold at different levels of flexibility and contract correlation.

This result has important managerial implications. When product demands are positively correlated, product flexibility increases profit variability. This reduces the value of product flexibility to the extent that the increased profit variability cannot be offset in the financial market. Therefore, in the presence of financial hedging, a firm should be willing to spend more money to acquire product flexibility. Financial hedging and flexibility are complements.

Conversely, when demands are negatively correlated, the value of product flexibility stems partially from its operational hedging aspect. In the presence of financial hedging, a firm will benefit less from this aspect of product flexibility and, thus, should be willing to spend less money to acquire such flexibility. Financial hedging and flexibility are substitutes.

**Strategic complementarity in economics.** Note that the economic theory defines strategic complementarity (substitutability) between two variables as supermodularity (submodularity) of the value function (Milgrom and Roberts 1990). Thus, product flexibility $\phi$ and contract correlation $\rho$ are strategic complements (substitutes) with respect to the value of financial hedging $\Delta(\rho, \phi; \gamma)$ in terms of Milgrom and Roberts (1990) if, and only if,

$$
\frac{d^2\Delta(\rho, \phi; \gamma)}{d\rho d\phi} \geq (\leq) 0.
$$

(11)

The result established in Proposition 4 is weaker because it considers discrete as opposed to infinitesimal increments in $\rho$ and $\phi$. In particular, Proposition 4 shows that when demands are positively (negatively) correlated, $\Delta(1,1; \gamma) - \Delta(0,1; \gamma) \geq (\leq) \Delta(1,0; \gamma) - \Delta(0,0; \gamma)$. However, our numerical study (Figure 5) suggests that the relationship holds for small increments as well,
i.e.,
\[
\frac{d^2 \Delta (\rho, \phi; \gamma)}{d\rho d\phi} \geq 0 \quad \iff \quad \rho_D \geq 0.
\] (12)

There are many other types of flexibility besides product flexibility. In the following section, we examine the relationship between financial hedging and postponement flexibility.

5 Postponement flexibility

Consider a single-product firm that acquires capacity level \( K \) at unit capacity cost \( c_K \) while facing uncertain demand \( D \). Later, after observing actual demand \( D \), the firm chooses output \( Q \) which is constrained by capacity as well as realized demand, i.e., \( Q = \min (K, D) \). The output is produced at a unit output cost \( c_Q \) and sold at a predetermined price \( p \). The firm’s operating profit is thus

\[
\Pi_o (K, D) = (p - c_Q) \min (K, D) - c_K K
\]

\[
= (p - c) D - G(D),
\]

where \( c = c_K + c_Q \) is the total unit cost, and \( G(D) = c_K (K - D)^+ + (p - c) (D - K)^+ \) is the well-known uncertainty or mismatch cost. Because the operating profit is concave in \( K \), the optimal capacity is uniquely determined by the first-order condition stated in Proposition 2.

**Level of flexibility.** We measure the firm’s postponement flexibility as the ability to postpone some of its decisions that impact cost until demand is known. Formally, we define postponement flexibility as the fraction of the total unit cost \( c \) that is incurred after demand is observed:

\[
\phi \equiv c_Q / c.
\]

To study the effect of flexibility, we vary parameter \( \phi \in [0, 1] \) while keeping the total product cost \( c \) fixed. With zero flexibility \( (\phi = 0) \), all costs are incurred before demand is known as in a pure make-to-stock production environment. With full flexibility \( (\phi = 1) \), the firm does not need to
reserve any capacity before observing demand and output is thus always equal to demand. This corresponds to pure *make-to-order* scenario in which the cost of capacity excess, $c_K(K - D)^+$, as well as the cost of capacity shortage, $(p - c)(D - K)^+$, are completely eliminated.

In general, greater flexibility corresponds to a lower unit cost of capacity, $c_K = (1 - \phi)c$, and thus to a lower expected cost of capacity excess $c_K\mathbb{E}[(K - D)^+]$. Furthermore, because greater flexibility means a lower unit cost of capacity excess $c_K$ without affecting the unit cost of capacity shortage, $p - c$, it results in a higher capacity level and thus a lower expected cost of capacity shortage $(p - c)\mathbb{E}[(D - K)^+]$. (Although greater flexibility always reduces the expected mismatch cost, it might result in a greater realized mismatch cost. This is because the lower unit cost of capacity may aggravate capacity overinvestment.)

**Relationship to other flexibilities.** As shown in Figure 6, postponement flexibility decreases the average total cost at low output levels while making large outputs feasible. This makes postponement flexibility closely related to volume flexibility typically defined as the ability to operate profitably at different output levels (Sethi and Sethi 1990). Postponement flexibility is also similar to how flexibility is often interpreted in the economics literature. In the seminal work on the topic, Stigler (1939) considers a plant to be flexible if it has a relatively flat average cost curve, and thus, incurs relatively smaller losses when deviating from the minimum average cost output.

**Postponement flexibility and financial hedging.** The relationship between flexibility and financial hedging depends, again, on how flexibility impacts operating profit variability. The impact of postponement flexibility on operating profit variability is a result of two opposing effects. The fact that greater postponement flexibility corresponds to a lower unit cost of excess capacity mitigates the downside variability. On the other hand, greater postponement flexibility results in a higher capacity level and thereby increases the upside variability. This is illustrated in Figure 7 which shows the operating profit distribution with zero and full postponement flexibility. Proposition 5
Figure 6: Postponement flexibility increases the range of feasible outputs on the upside while decreasing the average total cost on the downside.

c
K+c
K+c
Q
Q

characterizes the effect of full postponement flexibility on the value of perfect financial hedging for moderately concave value functions. Once again, the subscripts $N$ and $F$ denote the cases of no flexibility ($\phi = 0$) and full flexibility ($\phi = 1$), respectively.

**Proposition 5** For sufficiently low $\gamma$, full postponement flexibility decreases the value of perfect financial hedging, i.e.,

$$\Delta_F (1, \gamma) \leq \Delta_N (1, \gamma).$$

**Proof:** Proposition 3 implies that at sufficiently small $\gamma$ and $p = 1$, $\Delta_F \leq \Delta_N$ if, and only if,

$$\text{Var} \Pi_{oF} \leq \text{Var} \Pi_{oN} (K^0_N),$$

$$\Leftrightarrow (p - c)^2 \sigma^2_p \leq p^2 \text{Var} \min (K^0_N, D),$$

$$\Leftrightarrow \Phi^2 (z^0) \leq \text{Var} \min (z^0, Z),$$

where $z^0 = \Phi^{-1} \left( \frac{p-c}{p} \right)$ and $Z \sim \mathcal{N} (0, 1)$. It is straightforward to verify numerically that the last inequality holds for any $z^0$. $\square$
Figure 7: Postponement flexibility decreases downside variability (left tail of the profit distribution) while increasing upside variability.

Proposition 5 shows that for moderately concave value functions, the effect of full postponement flexibility on mitigating the downside risk always dominates its effect on increasing the upside variability. Thus, full postponement flexibility always reduces the operating profit variance and, as a result, the value of perfect financial hedging. As shown in Figure 8, this insight continues to hold at different levels of contract correlation $\rho \in [0, 1]$ and flexibility $\phi \in [0, 1]$.

To look at the same issue from another perspective, we can show that perfect financial hedging reduces the value of full postponement flexibility.

**Corollary 4** For sufficiently low $\gamma$, perfect financial hedging decreases the value of full postponement flexibility, i.e., $\Lambda(1, \gamma) \leq \Lambda(0, \gamma)$.

**Proof:** The result follows directly from Lemma 1 and Proposition 5. $\square$

Our results have the following managerial implications. Postponement flexibility decreases operating profit variability, and therefore, a postponement-flexible firm should be willing to spend less money on financial hedging. Furthermore, in the presence of financial hedging, the operational
hedging aspect of postponement flexibility is less important and, therefore, a firm that uses financial hedging should be willing to invest less in postponement flexibility. Financial hedging and postponement flexibility are substitutes.

6 Discussion and Limitations

The key contribution of this paper is to provide better understanding of the relationship between flexibility and financial hedging when both are used to mitigate demand risk. We show that the type of flexibility matters. Product flexibility and financial hedging are complements (substitutes) when product demands are positively (negatively) correlated. Thus, when demands are positively (negatively) correlated, (i) product-flexible firms should be willing to spend more (less) on financial hedging, and (ii) in the presence of financial hedging, firms should be willing to invest more (less) in product flexibility. In contrast to product flexibility, postponement flexibility follows intuition: postponement flexibility and financial hedging are substitutes.

Our model has several limitations. First, we assume that the firm value is an exponential function of profit, which is equivalent to assuming that risk-premium is independent of expected
profit. Second, we assume normally distributed demand and underlying variable of the hedging contract. It would be interesting yet nontrivial to examine the robustness of our results for other value functions and probability distributions. Finally, our model of product flexibility assumes symmetrical product parameters which undervalues the revenue maximizing option embedded in product flexibility.

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