

Aspiration Level, Probability of Success and Failure, and Expected Utility*

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Abstract

Several studies have shown that aspiration levels are a relevant aspect of decision making. We develop a model that takes this into account. We include the overall probabilities of success and failure, i.e. the probabilities of reaching and not reaching the aspiration level, into an expected utility representation. This turns out to be equivalent, in a mathematical sense, to expected utility with a discontinuous utility function. We give a behavioral foundation to the proposed model and provide conditions to determine the relative weights of the overall probabilities of success and failure. The model leads to several predictions that fit empirical evidence. Most notably, an aspiration level rein-

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forces loss aversion, can account for simultaneous risk-averse and risk-seeking behavior, and can explain choices violating the mean-variance approach.

Key Words: expected utility, aspiration level, probability of success and failure, decision under risk.

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1 Introduction

Aspiration levels play an important role in everyday decision making. New York cab drivers are motivated to earn a daily target return (Camerer, Babcock, Loewenstein, & Thaler, 1997). On rainy, busy days, their earnings per hour are high and they go home early. On hot summer days, when many New Yorkers prefer to take a walk, they earn less per hour and make long hours to reach their target. When managers have to decide which projects or portfolios to invest in, they are keen on achieving a target rate of return as well. They disregard investment possibilities that are likely to result in a rate of return below their target (Payne, Laughhunn, & Crum, 1980; 1981). Some farmers also appear to have a minimal level of revenues that they want to achieve. Up to the subsistence level, they choose to cultivate ‘safe’ crops with a stable return. The remainder of their land is allocated to ‘risky’ crops (Lopes, 1987).

Common to all these examples is the presence of an aspiration level. The cab driver, the manager, and the farmer are confronted with risky choices on a daily basis. Some of them are willing to take risks, but often they also want to ‘win at least something,’ earn a daily target income, or prevent themselves from falling below the subsistence level. Therefore, in making their risky choices, subjects focus on reaching a special outcome, the aspiration level.

In this paper, we present a simple model of choice under risk that includes an aspiration level. Once an aspiration level is included, the probabilities of success and

failure are naturally identified: these are the overall probabilities that the aspiration will be reached or will fall short of. Our model entails that subjects are sensitive to the overall probabilities of success and failure. Payne (2005) tests several traditional models of decision making, including reference point based theories. His results provide clear support for the use of probability of success in decision making. He concludes that traditional models such as expected utility and prospect theory cannot account for his evidence, and if a model is to match the results from his experiments, such a feature is even indispensable.

We combine an aspiration level with expected utility (von Neumann & Morgenstern 1944). Given that it is intuitive to think about an aspiration level in terms of the overall probabilities of success and failure, it is remarkable that this model turns out to be mathematically equivalent to expected utility with discontinuities. Despite its simplicity it can accommodate and predict several behavioral regularities. The presence of an aspiration level reinforces loss aversion, it explains the simultaneous purchase of lottery tickets and insurance policies, and it clarifies choices violating the mean-variance approach in finance.

The contribution of the paper is as follows. We give a behavioral foundation for our model. We show that the focus on the overall probabilities of success and failure induces risk-seeking behavior for prospects close to the aspiration level. Finally, we present necessary and sufficient conditions under which the overall probability of success is more important than the overall probability of failure.

We believe that our results are relevant to understand the impact of the overall probabilities of success and failure on risk-taking behavior close to the aspiration level. Although not exclusively, this might be of particular importance for the interpretation of subjects' choices made in laboratory experiments. We therefore propose a way to elicit experimentally the relative weights given to the overall probabilities of success

and failure.

2 Aspiration levels

2.1 Motivations

Imagine a subject having to choose among risky prospects. We interpret an aspiration level as an outcome that takes a special position in the decision process. Subjects code outcomes above the aspiration level as successes and outcomes below the aspiration level as failures. They place value on the overall probability of success and the overall probability of failure. While in decision theory and psychology the focus is on the probability of success, in finance the emphasis is on avoiding falling short of a target rate of return on investments (Roy, 1952), hence on the overall probability of failure. In the modeling section, we focus on both the overall probabilities of success and failure.

When subjects focus on the overall probabilities of success and failure, they clearly ignore information and might ignore dominance issues. The natural question is why subjects would, deliberately or not, do so. One important reason might be to simplify decision problems. Because cognitive abilities of subjects are limited, focusing on an aspiration level can help to reduce the complexity of a problem (Brandstätter, Gigerenzer, & Hertwig 2006; Langer & Weber 2001 p. 731; Mezias, Chen, & Murphy 2002; Roy 1952; Simon 1955). Particularly in complex decision problems, an aspiration level can be helpful as a screening device (Manski 1988). The Value-at-Risk (VaR) (Dowd 1998) is formally equivalent to an aspiration level as a constraint: intrinsic in the VaR method is the probability (called shortfall probability) that a project will fall short of a certain target return. When the VaR of a certain project is lower than the desired return, it will not be considered as a potential option.

An aspiration level is linked to probabilities and it is different from a reference point. A reference point is not related to probabilities. It is used in prospect theory as the point where the value function changes curvature and slope (kink), and divides outcomes into gains and losses (Kahneman & Tversky, 1979). We show that the presence of an aspiration level, while inducing the overall probabilities of success and failure, creates a discontinuity of the utility function at the point of the aspiration level, which may or may not coincide with the reference point. Below we discuss experiments that discriminate between the use of an aspiration level and prospect theory.

2.2 Evidence

Payne (2005) provides clear experimental evidence supporting a focus on the overall probability of success. This evidence is in contrast with expected utility and prospect theory (Tversky & Kahneman 1992). We discuss this experiment in detail because its results explicitly call for the inclusion of an aspiration level in a model. Subjects were shown a base mixed prospect, with some positive outcomes, a zero outcome, and some negative outcomes. One of the base mixed prospects gave outcomes \$100, \$50, \$0, -\$25, -\$50 with equal probability. Thus, $X = (0.2, 100; 0.2, 50; 0.2, 0; 0.2, -25; 0.2, -50)$. The subjects were then given several choices. In each case, they were told that they could add a sum of money (\$38) to either one of two possible outcomes and were asked which one they preferred to add it to.

In the first case, they could add the money either to the outcome that paid \$100 or to the one that paid \$0. The majority of the subjects indicated a preference to add it to the zero outcome. In the second case the money could again be added to the outcome that paid \$100 or to the outcome that paid \$50. Now there was no longer a majority for either possibility. What is interesting is that in the former case there was

the possibility to increase the overall probability of a gain and the majority preferred to do so. In the latter case there was no such a possibility and there was no longer a clear pattern in choices.

There is a wealth of other evidence supportive of the relevance of the overall probabilities of success and failure. Closely related evidence is from a series of experiments by Payne et al. (1980, 1981). In these experiments, subjects had to choose between pairs of prospects, for example between $X' = (0.5, 74; 0.1, 30; 0.4, -25)$ and $Y' = (0.3, 40; 0.5, 30; .2, 15)$. Note that both prospects have an identical expected value of 30 but that prospect X' entails a higher risk because the spread of the prizes is larger. Naturally, many subjects prefer Y' to X' . The same decision problem was given but with all outcomes reduced by 60. This gives $X'' = (0.5, 14; 0.1, -30; 0.4, -85)$ and $Y'' = (0.3, -20; 0.5, -30; .2, -45)$. Out of these two prospects, many subjects prefer X'' , the one with the larger spread. These preferences are not easily compatible with expected utility and they may seem surprising, but note that both Y' and X'' have a higher overall probability of a gain than their alternatives X' and Y'' . The behavior is therefore easily explained by an aspiration level.

Some recent experimental evidence is also provided by Lopes & Oden (1999). Their experiment has similar features as that of Payne et al. (1980, 1981). Subjects are asked to choose between pairs of prospects. Two operations were performed on the base prospects: a shift that added the same payoff to all outcomes, and a scaling that multiplied all outcomes by the same factor. Interestingly, the prospects that before the shift had a high overall probability of a zero outcome became relatively much more attractive to the subjects after the shift than the prospects with a small probability on a zero outcome. After the shift, prospects that formerly had a high probability of a zero outcome also had a high overall probability of a gain. The prospects with a small probability on a zero outcome did not become much more attractive because

they already had a high overall probability of a gain. The scaling did not influence choices much, which is understandable because the overall probability of a gain is unaffected by this operation. In their study, they also report protocols from their experiments (Lopes & Oden, 1999 p. 304). Subjects, asked to comment on their choices motivate them by statements as “Have a greater chance of winning at least some money” and “There is a better chance to win money.” Very similar reactions by subjects are reported in Schneider and Lopes (1986).

Other experimental studies lead to similar conclusions. Baucells & Heukamp (forthcoming) attribute violations of loss aversion to subjects focusing on the overall probability of success. Edwards (1954, p.396) shows that subjects do not like to lose, thus prefer low probabilities of losing large amounts of money, to high probabilities of losing small amounts. Siegel (1957) presents early psychological evidence for paying attention to the probability of being above aspiration (success) when choosing between prospects. Langer & Weber (2001) present empirical evidence that subjects, when choosing between prospects, pay special attention to the overall probabilities of gains and losses.

Next, we turn to evidence from finance and risk management. The focus is on the probability of *not* reaching the aspiration level. The response of one manager makes it very clear that an aspiration level plays a role when he says that:

“Risk is the prospect of not meeting the target of return” (Mao 1970, p. 353).

In like manner, another manager puts it like this:

“I never worry about the project going above the return. Risk is what might happen when the return is going to be less” (Mao 1970, p. 354).

Both statements unambiguously point to the focus on an aspiration level. Overall, Mao concludes from his study that “risk is primarily considered to be the prospect

of not meeting some target rate of return” (Mao 1970, p. 354), i.e., the probability of failure. And the managers he interviewed are no exception in that respect: Petty & Scott (1980) found this idea of a target return among *most* managers in a more extensive study. Furthermore March & Shapira (1987) conclude that “... the primary focus is on avoiding actions that might place one below [the target level]. The dangers of falling below the target dominate attention; the opportunities for gains are less salient... since it is the dangers that are noticed, the opportunities are less important (page 1043).” Roy (1952, p. 432) argues that agents, when holding financial assets, will seek to reduce the overall probability of being below a level, i.e., that the gross return is not less than some predetermined quantity. However, also in the domain of finance, there are motivations to study the overall probabilities of success and failure. The behavior of paying particular attention to accomplishing the aspiration level is observed, especially among decision makers in investments (Fishburn 1977 and the references thereafter, Markowitz 1959). Mezias (1988 p. 389) reports evidence from real choices in the stock market. “The predictions (...) imply that choice under uncertainty will be a different process when performance is below aspiration than when performance is above aspiration.” Davies (forthcoming) considers the probability of success as a measure of risk.

In practice, however, decisions are not based on an aspiration level alone. The aspiration level and the overall probabilities of success and failure may receive special attention, but subjects will not be completely insensitive to difference within the classes of gains and losses (Libby & Fishburn 1977, p. 289; Lopes 1996; Lopes & Oden 1999; Luce 1996; Payne et al. 1980, p. 1047 and p. 1053). Aspiration levels alone capture a significant part of experimental data, and do so surprisingly well, but are too restricted to fit all the data (Lopes & Oden, 1999). Libby & Fishburn (1977, p. 285-286) suggest that the probability of meeting a target return is traded off with

expected value. Hence, a model solely based on an aspiration level is too crude to be normatively or descriptively relevant. This brings us to the next section, where we integrate the aspiration level into a more refined model of choice behavior.

3 A model with an aspiration level

Imagine a subject having to choose among risky lotteries. These objects will be called *prospects* and denoted $X = (p_1, x_1; \dots; p_n, x_n)$. The prospect X yields x_j with probability p_j , $j = 1, \dots, n$. Probabilities are nonnegative and sum to one. Without loss of generality assume that outcomes are rank ordered from best to worst, i.e. prospect $X = (p_1, x_1; \dots; p_n, x_n)$ satisfies $x_1 \geq \dots \geq x_n$. The set of outcomes of a prospect strictly above (below) the aspiration level is indicated by x^+ (x^-). The aspiration level is exogenously fixed and, without loss of generality, set to zero. Preferences over prospects are denoted by \succsim , with \succ (strict preference) and \sim (indifference) as usual.

For the prospect X , $P(x^+)$ is the *overall probability of success*, and $P(x^-)$ is the *overall probability of failure*. As people do not go exclusively by the overall probabilities of success and failure, we introduce a refined model in which these probabilities are inserted into expected utility. We then proceed by showing that the functional is mathematically equivalent to discontinuous expected utility and provide a set of necessary and sufficient conditions to determine the relative importance of the overall probabilities.

Background on expected utility Expected utility constitutes a key model of individual decision making under risk. This should come as no surprise, since its assumptions are normatively appealing for a wide range of choice problems (von Neumann & Morgenstern 1944). Descriptively, however, expected utility proved to have its limitations (Allais 1953, Ellsberg 1961). This has stimulated researchers to

formulate alternative decision theories that can account for the observed behavior. In the last decades several models have been proposed that explain the descriptive violations. All these models have been generally labeled as *nonexpected utility* (for surveys, see Camerer & Weber 1992, Schmidt 2004, and Starmer 2000).

Let u denote the *utility* function. u is a real valued function defined on the set of outcomes. In this paper outcomes are monetary. Then, the *expected utility* (EU) representation states that an agent evaluates a prospect X through the following formula:

$$X \mapsto \sum_{i=1}^n p_i u(x_i), \quad (1)$$

which says that the value of a prospect $X = (p_1, x_1; \dots; p_n, x_n)$ is equal to the sum of the utilities weighted by the probabilities p_1, \dots, p_n attached to the respective outcomes.

An EU functional with aspiration level We propose the following real-valued functional V to evaluate the prospect X :

$$X \mapsto V(X) = \sum_{j=1}^n p_j u(x_j) + \mu P(x^+) - \lambda P(x^-), \quad \mu, \lambda \in \mathbb{R}^+. \quad (2)$$

That is, we propose a sum of expected utility, overall probability of success, and overall probability of failure. The importance of the overall probability of success is weighted by μ , and that of the overall probability of failure is weighted by λ . Both evaluations play an important role in the psychology of risk. We propose to combine them in an additive way. u , μ , and λ in (2) are unique up to a joint scaling factor. We may normalize $u(0) = 0$, $u(1) = 1$, after which λ and μ are uniquely determined. If $\lambda > 0$, we can always set $\lambda = 1$, by replacing u by u/λ and μ by μ/λ . More details are provided in Theorem 5.

We do not see the aspiration level as primarily part of the intrinsic values of

outcomes, but as a consequence of a decision heuristic of decision makers to simplify decisions. The latter is more naturally modeled as the probability of a chosen target event. These two notions are distinct. It is therefore remarkable that Eq. (2) turns out to be mathematically equivalent to discontinuous expected utility. To see this, rearrange Eq. (2) and w.l.o.g. use $u(0) = 0$ to get:

$$V(X) = \sum_{j:x_j \in x^+} [p_j u(x_j) + \mu p_j] + \sum_{j:x_j \in x^-} [p_j u(x_j) - \lambda p_j] = \sum_{j:x_j \in x^+} p_j [u(x_j) + \mu] + \sum_{j:x_j \in x^-} p_j [u(x_j) - \lambda] \quad (3)$$

and define $v(x_j) = u(x_j) + \mu$ for $x_j \in x^+$ and $v(x_j) = u(x_j) - \lambda$ for $x_j \in x^-$. This gives:

$$X \mapsto \sum_{j=1}^n p_j v(x_j). \quad (4)$$

Thus, we are back to expected utility but with a utility function v that is discontinuous around the aspiration level zero: $\lim_{x_j \uparrow 0} v(x_j) \neq \lim_{x_j \downarrow 0} v(x_j)$. This result raises three points. First, the evidence by Payne (2005) can be accounted for by expected utility, provided that the utility function is discontinuous. Second, it is not a kink in the utility function as in the value function of prospect theory, but a discontinuity that reinforces loss aversion. Third, it shows that the existence of an aspiration level might be regarded as providing a behavioral foundation for discontinuous expected utility. The next section elaborates on this point.

An equivalence theorem The aim is to characterize a jump at the aspiration level zero. First of all, expected utility is derived in a standard manner using Fishburn (1970) or Jensen (1967). For this, we use the following axioms. These are all standard and we present them for the sake of completeness.

A1) *Weak Order* The preference relation \succsim is a weak order if it is complete (for all

prospects X, Y , $X \succcurlyeq Y$ or $Y \succcurlyeq X$) and transitive (for all prospects X, Y, Z , if $X \succcurlyeq Y$ and $Y \succcurlyeq Z$, then $X \succcurlyeq Z$).

A2) *Continuity in probability (Archimedean axiom)* For all prospects X, Y, Z , if $X \succcurlyeq Y \succcurlyeq Z$, then there exist γ and $\gamma' \in (0, 1)$, such that $\gamma X + (1 - \gamma)Z \succcurlyeq Y \succcurlyeq \gamma' X + (1 - \gamma')Z$.

A3) *(vN-M) Independence* For all prospects X, Y, Z , and $\gamma \in (0, 1)$, if $X \succcurlyeq Y$ then $\gamma X + (1 - \gamma)Z \succcurlyeq \gamma Y + (1 - \gamma)Z$.

The following axiom on stochastic dominance will allow us to restrict the domains of the relative weights given to the overall probabilities of success and failure.

A4) *Stochastic dominance* For any two prospects $X = (p, x_1; 1 - p, x_2)$ and $Y = (q, x_1; 1 - q, x_2)$, if $x_1 \geq x_2$ and $p > q$, then $X \succcurlyeq Y$.

To characterize the discontinuity at the aspiration level, we introduce a weakened definition of continuity in outcomes. This is done in axioms A5 and A6. The following well known observation serves as a preparation.

Observation 1 *If the utility function u is nondecreasing, then it can be discontinuous at countably many points at most.*

A5) *Lower continuity outside zero* The utility function u is said to be lower continuous at α if for every sequence $\{\alpha^j\}$ converging from below to α , $u(\alpha^j)$ converges from below to $u(\alpha)$, with $\alpha \neq 0$.

A6) *Upper continuity outside zero* The utility function u is said to be upper continuous at α if for every sequence $\{\alpha^j\}$ converging from above to α , $u(\alpha^j)$ converges from above to $u(\alpha)$, with $\alpha \neq 0$.

As an intermediate step, the following lemma shows that the latter two axioms characterize the discontinuity of utility at zero. We consider prospects of the following type: $(0.5, x; 0.5, \alpha)$. We often write it simply as $(x; \alpha)$.

Lemma 2 *Lower and upper continuity of utility* 1) For all $\alpha \neq 0, \beta \neq 0$, there exist sequences $\{\alpha^j\}$ and $\{\beta^j\}$ converging to α and β from below/above, respectively, with $(\alpha^j; \beta) \sim (\alpha; \beta^j)$ for all j if and only if utility is lower/upper continuous except possibly at zero. 2) Assume that lower/upper continuity holds everywhere outside zero.

- a. Then, for $\alpha = 0$, and for all $\beta \neq 0$, there exist sequences $\{\alpha^j\}, \{\beta^j\} \uparrow$ and $\{\beta^j\} \downarrow$ converging to α and β from below/above, respectively, with $(\alpha^j; \beta) \sim (\alpha; \beta^j)$ for all j if and only if utility is lower/upper continuous at zero. Then utility is lower/upper continuous also at zero ($\mu = \lambda = 0$).
- b. If such a β and the sequences $\{\beta^j\} \uparrow$ or $\{\beta^j\} \downarrow$ do not exist s.t (a) above holds, then utility is discontinuous at zero (and the reverse also holds), and Eq. 2 holds with $\mu, \lambda \geq 0$ and at least one strictly larger than zero.

Proof: see Appendix.

Axioms A1 – A6 will be used to characterize the shape of the function V in Eq. 2. As it turns out, we can also characterize the relative importance of the overall probabilities of success and failure. The following two conditions will prove to provide necessary and sufficient conditions to determine the relative weights μ and λ . Consider the outcomes $x, y > 0$.

Condition 3 $\exists x$ s.t. $\forall y (0.5, x; 0.5, -y) \preceq 0$.

Hence, under this condition there is at least one outcome x for which the subject will always prefer the sure outcome that gives zero to the risky prospect $(0.5, x; 0.5, -y)$. The following condition reverses the role of x and $-y$.

Condition 4 $\exists y$ s.t. $\forall x (0.5, x; 0.5, -y) \preceq 0$.

In the next theorem we formally state the central result. It essentially tells that Axioms A1 to A6 are equivalent to represent preferences by V , and it provides the conditions under which the weight μ given to the overall probability of success is greater, equal, or smaller than the weight λ given to the overall probability of failure.

Theorem 5 *The preference relation \succsim satisfies Axioms A1-A6 iff there exist a continuous nondecreasing $u : R \rightarrow R$ and $\mu, \lambda \in R^+$ such that $X \mapsto V(X) = \sum_{j=1}^n p_j u(x_j) + \mu P(x^+) - \lambda P(x^-)$ represents preferences. Furthermore:*

- a) *if lower and upper continuity hold at zero, then $\mu = \lambda = 0$; if only upper continuity holds, then $\lambda > 0$ and $\mu = 0$; if only lower continuity holds, then $\mu > 0$ and $\lambda = 0$.*
- b) *if condition 3 holds, then $\lambda > \mu$, if condition 4 holds, then $\lambda < \mu$, if neither condition 3 or 4 holds, then $\lambda = \mu$.*
- c) *u is unique up to location and a positive scaling factor, and μ, λ are unique up to the same scaling factor as u .*

Proof: see Appendix.

This behavioral foundation can be extended to the case where the aspiration level is not necessarily zero. Furthermore, Theorem 5 allows to characterize an endogenous aspiration level, i.e. to infer an endogenous aspiration level from preferences: Whenever there is a jump in the utility function, there may be an aspiration level. Besides the aspiration level, an exogenous reference point could be included without much complication. The utility function would be defined in terms of gains and losses from the reference point and then definitions and axioms for the aspiration and the

overall probabilities will still apply. To keep the notation simple and the model focused, we do not elaborate on this and we leave it for further research. The prospects discussed in the empirical part fit the case where the reference point coincides with the aspiration level.

Comments on related mathematical results Theorem 5 characterizes a functional V representing preferences. V is equivalent to the mathematical expectation of a prospect with respect to a discontinuous von Neumann-Morgenstern utility function. Our model could also be characterized by imposing Foldes' (1972) and Grandmont's (1972) weak continuity at all but one outcome. We prefer an alternative approach to continuity, one that is simpler in not involving variation of probability, is more general and flexible, and has a higher intuitive content.

Other approaches are present in the mathematical literature. Arzac & Bawa (1977) and Markowitz (1959) characterized a decision maker maximizing preferences over expected value of wealth and probability of ruin. They derived equilibrium conditions in a capital asset price model. Their functional is somewhat more restrictive because of the assumption of linear utility (Shefrin & Statman, 2000) and can be considered a special case of Eq. 2. Meginniss (1976) presented axioms to represent preferences through a combination of expected utility and entropy (a measure of uncertainty) for a prospect. Castagnoli & LiCalzi (1996) reinterpreted EU without a cardinal utility function: the EU of a lottery is the probability that the lottery outperforms a given independent benchmark-lottery. The probability of surpassing the benchmark replaces utility, whereas in our model it comes in addition to utility. Our approach has the advantage that we can, for example, account for the data of Payne (2005). Bordley & LiCalzi (2000) showed the equivalence between Savage's (1954) maximization of subjective expected utility and maximization of the probability of meeting an uncertain target, thus extending Castagnoli & LiCalzi (1996) to

the domain of uncertainty.

Estimating μ and λ An important advantage of our model is that the empirical elicitation of μ and λ is straightforward. The first step is to fix an outcome $x > 0$, and then to elicit the probability p that makes the subject indifferent between the prospect $(p, 1; 1 - p, 0)$ and the sure outcome x . If we normalize the utility such that $u(0) = 0$, $u(1) = 1$, then, as x converges to zero from above, p converges to $\mu/(1 + \mu)$.

We can use a similar procedure to estimate λ . We now elicit the probability p that makes the subject indifferent between the prospect $(p, -1; 1 - p, 0)$ and the sure outcome $-x$. Then, as $-x$ converges to zero from below, p converges to $\lambda/(\lambda - u(-1))$. It is necessary to elicit $u(-1)$, because we can normalize the utility of two outcomes only. To do so, one can elicit the probability q that makes the subject indifferent between the prospect $(q, 1; 1 - q, -1)$ and the sure outcome 0. Substituting for the resulting $u(-1)$ we then get that, as $-x$ converges to zero from below, p converges to $\lambda/(\lambda + \frac{q}{1+q})$.

This elicitation works as long as the aspiration level is known, but we conjecture that in experimental settings it is safe to assume it equal to zero (Payne 2005). Although not exclusively, this elicitation might be of particular importance for the interpretation of subjects' choices made in laboratory experiments. The evidence and the results on risk attitude may be confused by the presence of a subjective aspiration level, the overall probabilities of success and failure, and their respective weights. The next section elaborates on this.

4 Discussion and predictions

The prospect evaluation $X \mapsto \sum_{j=1}^n p_j u(x_j) + \mu P(x^+) - \lambda P(x^-)$ gives rise to a rich body of behavioral predictions. To illustrate this point, we now present several of them.

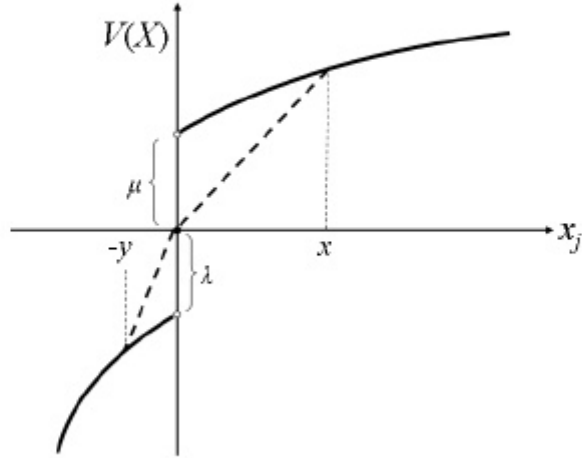


Figure 1:

Some implications of Eq. 2 are confirmed by previous empirical and experimental studies, others lend themselves to be tested.

A discontinuity around the aspiration level The most notable feature of our model is the discontinuity in outcomes. Continuity is not satisfied at the aspiration level. When an outcome that is at the aspiration level is increased/decreased to slightly above/below the aspiration level, it receives an extra weight μ ($/ - \lambda$). An arbitrarily small increase/decrease in an outcome x_i can therefore lead to a discrete jump in the valuation of a specific prospect (see the solid lines in Figure 1).¹

The discontinuity is a feature that is still to be tested. Jeffrey & Larrick (2002)

¹Empirical evidence suggests that utility for losses can be convex. We conform to the convention of a utility function that is concave everywhere, in the tradition of EU, but our model is not restricted to the latter case.

allude, nevertheless, to its presence in goal-related experiments. In the health domain, Stalmeier et al. (2004) provides empirical data showing evidence for discontinuity of utility around death. Fishburn (1977, p. 122) reports that the behavior of subjects exhibiting a particular appreciation for the outcomes above the aspiration level is often found in the literature and it can be represented by a “pronounced change in the shape of their utility function.” Swalm (1966) reports similar evidence and shows that this point is approximately zero (break-even point on a project). Holthausen (1981) when reviewing the literature reports similar findings. Mezias (1988) finds evidence of the effect of an aspiration levels in the pricing of securities, i.e. an abrupt jump in the valuation at a fixed and predetermined point.

Aspiration levels as extreme loss aversion In the case of EU the utility function for final wealth is commonly assumed to be differentiable. Under loss aversion the utility function for gains has a different slope than the utility for losses: loss aversion, entailing a drastic change at zero, is characterized by a (non differentiable) kink in the utility function. A standard definition of loss aversion (Wakker & Tversky 1993) for a differentiable function is: $x > 0 : u'(x) < u'(-x)$. Köbberling & Wakker (2005) proposed a behaviorally founded index of loss aversion. The case becomes even more extreme if we assume a power utility, as in Kahneman & Tversky (1979) and Tversky & Kahneman (1992). Then the first derivative of utility at zero is infinite, so that utility is extremely steep at zero. The extreme version of loss aversion that we present in this paper is a natural next step: the discontinuity in the utility function reinforces loss aversion. Our model is the first to characterize loss aversion as discontinuous utility. The psychology of loss aversion not only applies in the domain of failure, but also in the domain of success as we take the limit from above: $\lim_{x_j \downarrow 0} V(X)$. We still call it loss aversion because the underlying psychology of the phenomenon is the same as in the domain of failure.

Empirical evidence for loss aversion may be partly driven by a sensitivity to the overall probabilities of success and failure in addition to a bigger sensitivity to loss outcomes.

Risk attitude and μ and λ The functional V evaluating prospects, is discontinuous at the aspiration level for strictly positive values of μ and λ . The corresponding jumps determine the risk-attitude that subjects have for prospects close to the aspiration level (taken to be the zero outcome here). For two-outcome prospects that involve the aspiration level, we now show that subjects are always *risk-averse from above* and *risk-seeking from below*. This result holds independent of the shape of the function $u(\cdot)$. The dashed lines in Figure 1 illustrate this fact. They represent the expected values of prospects of the form $(p, x; 1 - p, 0)$ and $(p, -y; 1 - p, 0)$.

Consider the outcomes $x, y > 0$ and let z_i be the degenerate prospect that gives the expected value of prospect X_i with certainty.

Proposition 6 *Suppose preferences can be represented by V . Then, (a) if $\mu > 0$, subjects are risk-averse from above: $\forall p \exists \delta_1 > 0$ such that $X_1 = (p, x; 1 - p, 0) \prec z_1 \forall x < \delta_1$ and δ_1 is increasing in μ , and (b) if $\lambda > 0$, subjects are risk-seeking from below: $\forall p \exists \delta_2 > 0$ s.t. $X_2 = (p, -y; 1 - p, 0) \succ z_2 \forall y < \delta_2$ and δ_2 is increasing in λ .*

The type of prospects for which this result holds is often used in prospect theory to establish that subjects are risk-averse for gains and risk-seeking for losses. Provided that the aspiration level coincides with the reference points, these choices are also consistent with subjects paying attention to the overall probabilities of success and failure. Our model and prospect theory differ strongly in predictions however, for prospects not involving the zero outcome. For example, if prospect theory is correct in that subjects are risk-averse for gains, then subjects should also be risk-averse if a constant c is added to all outcomes in X_1 and z_1 , i.e. $X'_1 = (p, x + c; 1 - p, c) \prec$

$z'_1 = z_1 + c$. If instead subjects pay attention to the overall probabilities of success and failure and if for instance $u(\cdot)$ is convex everywhere, then subjects would prefer the risky prospect after the shift $X'_1 \succ z'_1$. Lopes & Oden (1999) find indeed that the risky prospect in the gain domain (their prospect ‘LS’) becomes more attractive to subjects after such a shift. A more direct test involving only two-outcome prospects is desirable. Relatedly, Fennema & van Assen (1999) claim that the results from several studies that find risk seeking in the domain of losses, do not necessarily imply a convex utility function.

Incidentally, the above result also implies that subjects may be simultaneously engaged in risk-averse behavior and risk-seeking behavior, for instance buying insurance and lottery tickets. Friedman & Savage (1948) explain such behavior by considering local convexities in the utility function. This turned out to be inconsistent with empirical data and raised criticism (Markowitz 1952, Quiggin 1991, Thaler & Ziemba 1988, Yaari 1965). The behavior can also be explained by subjects paying attention to the overall probabilities of success and failure. Indeed, Hirshleifer & Riley (1992, p. 28) advance that the behavior is caused by a ‘threshold phenomenon,’ i.e., ‘a single discrete step to a higher utility level’ which we show is a direct consequence of the aspiration level.

The mean-variance trade-off In standard portfolio theory, it is argued that managers trade off higher returns against higher variances (Markowitz 1952). Risk is usually identified with variance. Hence, among portfolios with the same mean, the one with the lowest variance is to be preferred, and vice versa. More specifically, it is often assumed that a prospect X is evaluated as:

$$E(X) - k\sigma^2, \tag{5}$$

where $E(X)$ is the expected value, σ^2 the variance, and the constant k a measure of risk aversion. A closely related version that is sometimes proposed, replaces the variance by a semivariance. The semivariance is calculated with respect to a target outcome rather than the expected value, and is restricted to outcomes below the target (Eeckhoudt & Gollier 1995).

This mean-variance trade-off is not implied by Eq. 2. A portfolio with the same mean but a higher variance may give a higher probability of reaching the aspiration level. Conversely, lowering the variance of a given portfolio (keeping the mean constant) may result in a lower valuation if some probability mass is shifted away from above the aspiration level to below (Hakansson & Ziemba 1995, p.80). The same applies to the version based on semi-variance.

Such a phenomenon was found in the experimental results of Payne et al. (1980, 1981). Recall from section 2.2 that many subjects prefer prospect $X'' = (0.5, 14; 0.1, -30; 0.4, -85)$ to $Y'' = (0.3, -20; 0.5, -30; .2, -45)$. The prospects have the same mean but the variance of X'' is higher. At the same time, X'' gives a higher overall probability of a gain. This choice cannot be explained by Eq. 5 irrespective of the degree of risk-aversion k . By contrast, it can be explained by Eq. 2 if the weight given to the overall probability of success is sufficiently high.

5 Conclusions and Future

There is evidence that subjects are particularly sensitive to success and failure. There is a reluctance to take prospects that may entail the possibility of failure measured with respect to an aspiration. This reluctance cannot be fully explained by risk aversion. A great deal of evidence describes that subjects perceive prospects in terms of the probability of being above or below an aspiration level. It seems that very small changes in outcomes that increase the overall probabilities of success and failure

can lead to significant changes in preference over prospects (Payne 2005). We have discussed and investigated the psychological intuition underlying the aspiration levels, presented a model, characterized the importance of the overall probabilities of success and failure, and explored some of the decision theoretical implications.

Expected utility has since its emergence been a classical benchmark in decision under risk and decision analysis. Parsimony, elegance, and broad applicability are its strong points. In this paper we present a model of decision under risk that includes the overall probabilities of success and failure into expected utility. Building on the psychologically appealing probabilities of success and failure, we then come up with a model that is equivalent to a discontinuous EU. The model allows for a number of predictions that can better describe the behavior of individuals. However, we don't aim at explaining all the possible behavioral irregularities in decision making under risk: Other complicating, but behaviorally relevant, factors (i.e. probability transformation) have been abstracted from for the sake of conceptual clarity. We have shown that insights into the significance of an aspiration level deriving from cognitive psychology can be integrated in decision theory.

One first obvious extension is to study decision under uncertainty. In this paper we have assumed that probabilities are known to the decision maker. In reality, this may often not be the case and it is interesting to see how agents deal with such uncertainty. We conjecture that the importance of the overall probabilities of success and failure will be enhanced by such uncertainties (Fellner 1961, Kahneman & Tversky 1979, Kahn & Sarin 1988, Weber 1994). Another natural extension is to characterize a model where the aspiration level is combined with elements from prospect theory such as probability transformations (Tversky & Kahneman 1992). The interest there is to investigate how the model changes once the probability transformation is considered. Then, both the model with expected utility and prospect theory can be generalized to

any finite number of aspiration levels. Finally, we conjecture that the more complex the problem is to a subject, the more attention the overall probabilities of success and failure receive.

Appendix (proofs)

Proof. Lemma 2. Necessity of the preference condition easily follows. Sufficiency: By observation 1, one can take a β where utility is continuous. Then lower/upper continuity of utility at each α follows by simple substitution of EU. Part *a* and *b* similar to above. ■

Proof. Theorem 5. The implication that Eq. 2 satisfies A1-A6 is straightforward. For the implication in the opposite direction any of the aforementioned EU axiomatization provides a constructive proof for the existence of a utility function \tilde{u} (unique up to location and a positive scaling factor) such that the EU form holds true, i.e., $X \mapsto \sum_{j=1}^n p_j \tilde{u}(x_j)$ represents preferences. By A5 and A6 and by Lemma 2, \tilde{u} is lower/upper continuous everywhere outside zero and, thus, $u(0) - \lim_{x \uparrow 0} \tilde{u}(x) = \lambda \geq 0$, and $u(0) - \lim_{x \downarrow 0} \tilde{u}(x) = -\mu \leq 0$ with at least one of the inequalities being strict. By stochastic dominance μ and λ are nonnegative. Define $u = \tilde{u} - \mu$ for positive outcomes and $u = \tilde{u} + \lambda$ for negative outcomes.

Part *a* is a special case of the above and its proof is similar. To prove part *b* of the Theorem:

\Rightarrow Suppose condition 3 holds: $\exists x$ s.t. $\forall y$ $(.5, x; .5, -y) \preccurlyeq 0$. This implies

$$.5u(x) + .5u(-y) \leq \lambda - \mu. \quad (6)$$

Let $u(x) = c$. By continuity and nondrecreasingness of $u(\cdot)$ and with $u(0) = 0$, we know that for $\varepsilon, \delta > 0$, $\forall \varepsilon$ there exists a δ s.t. $-u(-y) < \varepsilon$ for $y < \delta$. In particular, if $c = \varepsilon$, then $-u(-y) < u(x)$ for $y < \delta$. Then (6) above can only be satisfied for $\lambda > \mu$.

\Leftarrow Let $\lambda > \mu$. Then the RHS of (6) is strictly positive. Note that the least upper bound of the LHS is given by $u(x)$. Hence, by continuity of $u(x)$, for small enough values of x the LHS never exceeds the RHS for any y . This implies condition 3 is satisfied.

Similar arguments can be used to show that condition 4 is satisfied if and only

if $\lambda < \mu$. It then follows in a straightforward manner that if neither condition 3 nor condition 4 is satisfied, it must be the case that $\lambda = \mu$. ■

Proof. Proposition 6. Part (a). The utility of the sure outcome z is given by $u(px) + \mu$. The utility of the prospect is given by $pu(x) + p\mu$. The risky prospect is preferred if $pu(x) - u(px) \geq (1 - p)\mu$. As $x \rightarrow 0$, $pu(x) - u(px) \rightarrow 0$. Since the RHS is negative, the inequality is always satisfied for sufficiently small x . Part (b) can be proved along similar lines. ■

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