Optimal Contracts for Outsourcing of Repair and Restoration Services

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Outsourcing of equipment repair and restoration is commonly practiced by firms in many industries. The operational performance of equipment is determined by joint decisions of the firm (client) and the service provider (vendor). Although some decisions are verifiable and thus directly contractible, many decisions are not. The result is a double-sided moral hazard environment in which each party has incentives to free ride on the other’s effort. A performance-based contract allows the client to align the incentives of the vendor, but it also exposes the vendor to stochastic earnings and thereby creates disincentives to make first-best decisions. To capture these issues, we develop a novel principal-agent model by integrating elements of the machine repairman model and a stochastic financial distress model within the double-sided moral hazard framework. We apply our model to solve the client’s problem of designing the optimal performance-based contract. We find that the client can attain the first-best profit by restricting the search space to only two classes of performance-based contract structures: linear and tiered. We show that the linear contract structure has limited ability in attaining the first-best outcome, contingent on the vendor’s exogenous characteristics. In contrast, the tiered contract structure enables the client to attain the first-best outcome regardless of vendor characteristics. Our results provide normative insights on the role of contract structures in eliminating any loss due to double-sided moral hazard or to the vendor’s financial concerns. These results also provide theoretical support for the extensive use of tiered contracts observed in practice.

Subject classifications: double-sided moral hazard; performance-based contracts; financial distress; machine repairman model.

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1. Introduction

Over the last two decades, firms have increasingly outsourced repair and restoration services of their equipment to external service providers. The advantages of hiring and training engineers to manage repair in-house are becoming limited for firms as a result of continuous advancement in equipment technology, decreasing margins due to competition, and the increasing preference for geographical diversification of business units. For example, the repair and restoration outsourcing contract by DBS Bank (Singapore) to IBM exemplifies all of these factors. To keep up with the latest technology, DBS Bank engaged IBM for 10 years to carry out repair and restoration of servers in its two data centers, which are located in Singapore and Hong Kong. DBS Bank expected to gain overall cost savings of about 20% by leveraging the specialized information technology know-how of IBM engineers.

Firms typically trade off the advantages of outsourcing against the agency problems that naturally arise in such settings. The inability of a firm to contract directly on the outsourced decisions creates moral hazard issues between the firm and the service provider. These issues may result in loss of efficiency of the service supply chain and loss of profit for the firm. The outsourcing of repair and restoration services poses additional challenge because the equipment’s operational performance, defined here as availability, is determined by the joint efforts of both the firm and the service provider. In such joint effort settings, each player may have incentives to free ride on the other player’s effort, and this may result in suboptimal system outcomes (Holmstrom 1982, Bhattacharyya and Lafontaine 1995).

The operational performance of equipment is determined by how frequently it fails (the failure rate) and how quickly it can be restored (the repair rate). Whereas various decisions of the service provider (e.g., the hiring and training of repair engineers) determine the repair rate, various decisions of the firm (e.g., training of operators and investments in the operating environment) determine the failure rate.

To illustrate the role of the firm in equipment failure rate, consider the following context. In the DBS-IBM outsourcing relationship, DBS Bank is responsible for decisions regarding investments in the operating environment of its data centers, which affects the failure rate of its servers. In case of such high-tech industrial equipment, which depends critically on electrical and electronic subsystems, the most common causes of failure are power spikes or surges, dust or dirt levels, humidity levels, and instances of extreme
temperature in the operating environment. For example, in a controlled experiment conducted by a data center client of Sun Microsystems, a 14% marginal increase in the failure rate of servers was observed when the operating conditions deviated by a single degree from the recommended range (Miller 2011). Similarly—but in a much different industrial setting—we learned through our discussions with the management of Raymond Corporation that the operating environment (in particular, the floor quality) at the client’s premises plays a large role in determining the failure rate of its forklift trucks. Moreover, the failure rate of equipment is also strongly affected by the technical competence of equipment operators. For example, the U.S. National Institute for Occupational Safety and Health estimates that more than three fourths of forklift repair and maintenance costs are due to operator abuse of the equipment (McCraney 2006). A similarly high incidence of operator-induced failure is observed for a wide class of equipment types ranging from medical equipment (Cummins et al. 1991) to high-tech servers in data centers (Ponemon 2010) to mechanical boilers. These examples illustrate the critical role of various decisions made by the firm in influencing the equipment’s failure rate.

The repair rate is largely determined by the technical competence of repair engineers staffed by the service provider, by its expenditures on training for these engineers, and by its capital investment in sophisticated diagnostic and repair equipment. In the DBS-IBM outsourcing relationship, for example, IBM is responsible for the hiring and training of its engineers. In settings that rely critically on electrical and electronic subsystems, the speed of repair is highly dependent on the technological sophistication of the diagnostic equipment and on the skill of the repair engineers. As mentioned previously, the rapid growth in focused business models and technological advancement have increasingly led client firms to outsource the entire repair process to specialized, third-party vendors.

The examples just discussed indicate that, in a variety of industrial settings, failure and repair rate are primarily determined by decisions of (respectively) the firm and the service provider. We analyze such settings in this study. Some of these decisions may be verifiable, but others (e.g., efforts in the training of equipment operators and repair engineers) are not. Because direct contracts cannot be written on unverifiable decisions, in this case we have a double-sided moral hazard problem.

If direct contracts are thus precluded, then the firm must design contracts that measure and appropriately reward or penalize the service provider based on metrics that provide information about its efforts. Such incentive contracts, or performance-based contracts (PBCs) are increasingly used in the service outsourcing arena because they mitigate the moral hazard problems that arise in outsourcing and also free the client of transaction costs associated with closely monitoring the vendor. For example, the U.S. Department of Defense issued a directive in 2003 requiring program managers to develop and implement performance-based contract strategies to optimize systems availability (Kim et al. 2007). Hasija et al. (2008) highlight the growing adoption of incentive contracts in the call center outsourcing industry.

However, performance-based contracts expose a service provider to financial uncertainties. As noted by Smith et al. (2004), in their article—“The promise and pitfalls of performance-based contracts”—a fundamental element of client-vendor tension “stems from the financial risks that vendors face when they enter into these contracts. Some service providers have ignored those risks; others, to their credit, are more realistic, factoring the risks into their decisions about whether to accept the contracts, and in some cases—particularly in today’s difficult economic climate—simply concluding that the stakes are too high. As a result, some city agencies have had the experience of receiving no agency proposals when they issue RFPs for PBCs” (p. 21). Moy (2010) likewise notes that vendor fear of financial uncertainty is one of the main obstacles to implementing performance-based contracting. Along these lines, industry experts remark as follows: “No vendor expects to achieve 100% of all service levels on a regular basis, thus they pad their profit margins to accommodate some performance issues. The most sophisticated vendors run probability models to determine likely impacts to their profitability.” For large vendors like IBM, contract-specific financial uncertainty may not jeopardize the firm’s overall financial health. From our conversations with managers of service support firms, however, we learned that they are extremely conscious of each contract’s financial viability because it influences not only their compensation but also their career trajectory within the firm. Hence, such financial concerns distort the incentives of these managers to make optimal project-specific decisions. One way of encouraging a service provider to make optimal decisions is to remit some of the surplus to that provider. Doing so ensures that the vendor’s financial concerns remain within acceptable levels, although this limits the firm’s ability to extract first-best optimal profits.

In this paper we study contract structures that address two key challenges faced by a firm (the client, “he”) that outsources the repair and restoration service of his equipment to a service provider (the vendor, “she”): (i) double-sided moral hazard, and (ii) the vendor’s induced exposure to financial distress on account of performance-based contracts. We assume that this outsourcing environment is exogenous and so do not study the client’s decisions concerning what should be outsourced (and to whom). In many industries, such as banking and healthcare, firms may choose to own and operate their own equipment because of concerns about confidentiality, quality of service, and other factors affecting the firm’s competitive advantage; hence such firms might outsource only the repair and restoration services. We focus on decisions that influence the repair and failure rates, that are made via long-term commitments,
and that cannot be altered over short periods. We abstract cost of all such decisions made by the client as monetary investments in operating technology and likewise the cost of decisions made by the vendor as monetary investments in repair technology.

We study the contract design problem from the client’s perspective and compare the effectiveness of the optimal contract structure against the first-best outcome achieved by the client in a centralized setting. Specifically, we ask the following questions: (i) What is the optimal contract structure that should be offered to the vendor? (ii) How much does the client lose (compared with the first-best outcome) under such an optimal contract? (iii) How does the optimal contract structure depend on the vendor’s tolerance for financial distress?

To answer these questions, we limit our focus to two classes of contract structures: linear and tiered. Our choice of linear contracts is motivated by the past literature (Bhattacharyya and Lafontaine 1995, Corbett et al. 2004, Roels et al. 2010). In our setting, linear contracts allow the customer to penalize the vendor with a proportional penalty contingent on her realized performance. In practice, however, we find that contracts are designed to levy a constant penalty on the vendor only if her performance deteriorates beyond a threshold level (see Table 1). Analogously, contracts in the call center outsourcing industry often stipulate service-level agreements (SLAs)—for instance, Pr(delay ≤ 60 sec) ≥ 0.8 for a day—along with an associated (constant) penalty for not meeting the SLA. See Hasija et al. (2008) for more examples of such constant-penalty SLA contracts used in practice. These observations motivated our choice of tiered contracts.

We find that, for a vendor with a high tolerance for financial distress and/or a high reservation value, the client can attain its first-best outcome with either a linear or a tiered contract. In contrast, for a vendor with a low tolerance for financial distress and/or a low reservation value, we find that linear contracts fail to attain the first-best outcome. It is interesting that the client can attain its first-best outcome in this case by using tiered contracts. Intuitively, the limited penalty structure of a tiered contract may have a countervailing effect: while reducing the vendor’s exposure to potential financial distress, it may also reduce her incentives to invest optimally in the repair technology. Yet surprisingly there exists a class of tiered contracts that allow the client to avoid any loss due to the vendor’s low tolerance for financial distress, thereby making it unnecessary for the client to provide any surplus to the vendor.

Our paper makes three important contributions. First, it is, to the best of our knowledge, the first work that studies double-sided moral hazard agency issues that naturally occurs in repair outsourcing settings. Second, we capture the vendor’s exposure to financial distress due to performance-based contracting, an aspect of PBC that is lately recognized by practitioners. These elements of our work allow us to examine analytically a repair outsourcing model that is a step closer to practice than past literature. Our findings complement earlier studies in many ways. We find that, in our setting, the client can attain its first-best outcome—despite the presence of double-sided moral hazard and the vendor’s financial concerns—by selecting the right contract structure. This highlights, like previous studies, how a carefully designed contract can have a considerable effect on the client’s profit. More importantly, we provide new insights by showing that a right contract structure can also alleviate vendor’s financial concerns without paying her any financial surplus. Finally, our findings provide theoretical support for the prevalence of tiered contracts in practice. From a modeling perspective, the paper offers a novel approach that combines technical elements from three different fields of research: operations (renewal theory), economics (contract theory), and actuarial science (ruin theory).

The rest of the paper is organized as follows. In §2 we review the related literature, and in §3 we describe the model setup. In §4, we analyze the comparative performance of the two contracts under our focal setting and then discuss the findings. We show the persistence of these findings under three alternative settings in §5. We conclude in §6 by summarizing the main contributions of our work and opportunities for future research.

2. Literature Review

Our work is related to three primary research streams: the operations management literature on service operations outsourcing, the economics and operations management literature on incentive alignment under double-sided moral hazard, and the growing body of literature at the operations–finance interface that studies operational decisions contingent on their impact on the financial health of a firm or service providers.

In the service operations outsourcing stream, our work is in the specific substream of after-sale service for equipment. Murthy and Asgharizadeh (1999) is among the early papers to study repair outsourcing for equipment. They model the repair outsourcing problem between multiple clients and a single vendor as a Stackelberg game in which the vendor is the leader and the clients (equipment operators) are followers. The vendor offers a fixed-price contract to the clients and makes decisions on the number of service channels to set up and the number of customers to
serve. Kim et al. (2007) is the first paper to study such after-sale services supply chain contracting issues within the principal-agent framework. They study a one-to-one relationship between client and vendor at the product component level. These authors address the single-sided moral hazard problem that arises when the vendor makes two key repair decisions—cost reduction and spare-parts inventory management—on the client’s behalf. They show in this case that, if both parties are risk neutral, then the first-best outcome can be attained with a performance-based contract. If either party is risk averse, then a contract with a fixed payment, a cost-sharing incentive, and a performance-based incentive can attain the second-best outcome. In a related paper, Kim et al. (2011) study the trade-offs in the decisions of a vendor (an original equipment manufacturer) that are made during two different phases of a product’s life cycle: development and exploitation. During the development phase, the vendor invests in R&D that improves product reliability; during the exploitation phase, the investment she makes in spare-parts inventory determines the after-sale repair service level. These two decisions jointly determine the total availability of product in the exploitation phase. The authors find that a performance-based contract is preferable to a material contract and that investment in product reliability is increasing in vendor’s ownership of the spare-parts inventory. Bakshi et al. (2011) study the role of performance-based incentives in attaining economically efficient outcomes when the equipment’s failure rate is exogenous and the manufacturer has private information on that failure rate. These authors show that, when spare-parts inventory is verifiable, performance-based contracts can be used as a signaling mechanism by the informed manufacturer and thus lead to a separating equilibrium that yields the first-best outcome.

This paper differs in several respects from the studies cited previously. First, we use a double-sided moral hazard framework to study the joint role of the client and the vendor in ensuring availability of the equipment. Second, we limit our focus to the exploitation phase of the product life cycle. Third, we abstract from specific decisions that determine the equipment’s failure rate (e.g., frequency of regular upkeep) and repair rate (e.g., spare-parts inventory level), focusing instead on the aggregated cost of decisions as investments made in operating and repair technologies by the client and vendor, respectively.

This paper is also related to the double-sided moral hazard literature in economics and operations management. In economics, different contract structures (e.g., revenue sharing, shared savings, buyout agreements, option-based contracts) have been studied for resolving the double-sided moral hazard between two parties. Bhattacharyya and Lafontaine (1995) examine a joint production problem with risk-neutral parties and show that a linear contract is optimal; that is, linear contracts can do better or replicate performance of any other complicated contract, and attain the second-best outcome. In a related study, Kim and Wang (1998) show that with a risk-averse agent the linear contract structure is not optimal; moreover, the optimal nonlinear contract attains the second-best outcome.

In the operations management literature, there are a few studies that apply the double-sided moral hazard framework to contracting issues. Baiman et al. (2000) study joint efforts of a risk-neutral customer and a supplier to improve product quality and reduce external failures. They examine fixed-price contracts and show that the first-best outcome can be attained under two special conditions: when (i) decisions of the two parties can be directly contracted upon or (ii) the contract can be based on measures that allow for the separability of profit functions in the respective decisions. Such separation transforms a double-sided moral hazard problem into two single-sided moral hazard problems, which in turn can be resolved for risk-neutral parties. Corbett and DeCroix (2001) study the consumption of indirect materials in a supply chain with risk-neutral parties. They show that, under cost functions that are increasing and convex, the customer cannot attain the first-best outcome. In a related study, Corbett et al. (2004) show that if the first-best outcome requires positive efforts from at most one of the two parties, then the customer can attain the first-best outcome. More recently, Roels et al. (2010) have studied contracting issues in the outsourcing of collaborative services. They show that first-best decisions can be attained by incurring the costs of monitoring and verifying the other party’s efforts. However, the profits earned by the parties in this case are less than first best owing to the additional cost of monitoring. In the sequential decision setting, Bhattacharya et al. (2013b) study the contract design problem between a risk-neutral client and a risk-neutral customer support center. They show that the first-best outcome can be attained using a gain-share/cost-plus options mechanism. Bhattacharya et al. (2013a) study research collaboration in the healthcare industry. They show that the first-best outcome is guaranteed for a risk-neutral pharmaceutical firm when it outsources research to a risk-averse biotech firm with some bargaining power. Our work differs from these studies in that we study the design and performance of optimal contracts that not only provide sufficient incentives to resolve double-sided moral hazard but also ensure that a vendor’s exposure to financial distress is lower than her exogenous threshold. We show that, within the two classes of contracts studied in this paper, the client can attain first-best profits by customizing contractual terms to fit the vendor’s characteristics.

A relatively new area of research in operations management addresses how a firm’s contractual decisions affect the financial health of its suppliers and service providers. In a survey of automakers and their suppliers, Choi and Hartley (1996) find that the financial health of suppliers is an important selection criterion. Swinney and Netessine (2009) is among the first papers to study the relationship between a firm’s contractual choices and its endogenous...
effect on a supplier’s bankruptcy. They study, in a two-period model, a buyer’s choice between offering a short-term and a long-term contract to a supplier facing the risk of bankruptcy. The supplier declares bankruptcy if the random production costs for any period are higher than the contractual payments for that period. The authors show that a buyer’s preference for a long-term contract is increasing in his switching cost between suppliers. The long-term contract allows the buyer to make an up-front payment and thereby reduce the probability of supplier bankruptcy. In a related paper, Babich (2010) studies the role of subsidies with a cost-plus contract in managing a supplier’s performance. The supplier’s bankruptcy is a function of endogenous contractual payments and exogenous cash flows from the supplier’s other businesses. The buyer can influence the supplier’s bankruptcy risk by making up-front payments (subsidies) in addition to the supplier’s up-front cost requirement. Dong and Tomlin (2012) study the joint role played by business interruption insurance and operational measures in mitigating losses due to disruptions in the supply chain. We contribute to this literature in several ways. First, we model the financial health of the vendor as being purely endogenous to contractual payments. Second, we capture the vendor’s potential financial distress by modeling the stochastic evolution of her cumulative wealth. This approach allows us to capture the long-term, one-to-one relationship between client and vendor as well as the evolution of vendor wealth in terms of stochastic earnings (losses) across service calls. In this paper we demonstrate that the buyer can exploit contract features—other than up-front subsidies—that affect the supplier’s financial health.

3. Model Setup

We study a profit-maximizing client who outsources the repair and restoration services of his equipment to a vendor. We assume that such an outsourcing environment is exogenous to our problem setting, and we assume a long-term contractual relationship between client and vendor that continues throughout the equipment’s life cycle. To retain our central focus on the challenging issues of contracting and to achieve analytical tractability, we assume that the client has only one piece of equipment.

The equipment can be in only one of two states: operational or failed. We refer to the time period between two consecutive failures as uptime \((U)\); the time spent in restoring the failed equipment to its operational state is referred as downtime \((D)\). We also define the combination of a span of uptime and the subsequent downtime as the equipment’s operational cycle (OC); see Figure 1. In this paper we often refer to the period of realized downtime as a service call for clarity of exposition. We assume that the operational performance of the equipment is independent across operational cycles (i.e., that the realized uptimes and downtimes are independent across OCs). However, the realized uptime and downtime within a cycle may be correlated.

We assume that the equipment’s failure rate \(\lambda\) is determined by up-front static decisions. As discussed in §1, for a variety of equipment types the failure rate is determined mainly by decisions that concern the staffing quality of equipment operators, the expenditures on operator training, and the investment in operating environment. These decisions are often made with an eye on the long-term—either because they are irreversible in the short term (e.g., staffing decisions; see Hasija et al. 2008, Ren and Zhou 2008) or because, in many industries (e.g., data centers), low failure rate is a critical competitive edge and so such decisions are made at the strategic level. Similarly to the failure rate, the repair rate is determined mainly by decisions that concern the staffing quality of repair engineers and the expenditures on their training. In addition, a vendor’s capital investments in sophisticated diagnostic and repair equipment affects her ability to diagnose and rectify failures quickly. Taken together, these decisions are once again either irreversible in the short run or provide the vendor with an essential competitive edge; therefore, as with the failure rate, we assume that the repair rate \(\mu\) of the equipment is determined by up-front static decisions made by the vendor. Hereafter we abstract the cost of all such decisions made by the client and the vendor as investments in operating and repair technologies, respectively. In this problem setting we do not study opportunistic dynamic decisions—such as reducing the quality of repair during a particular repair cycle (cf. Murthy 1991), inducing frequent failures, or reducing the care in the usage of the equipment during an operating cycle—that may also influence system performance.

We define a measure of equipment reliability \(\tau\) that is equal to the mean time between failures \((1/\lambda)\). The client, through his decisions, determines the reliability of the equipment within a bounded range \([\tau, \bar{\tau}]\); the bounds reflect the reliability of current technologies used in the equipment. The vendor, through her decisions, determines the repair rate within a bounded range \([\mu, \bar{\mu}]\), which reflects the effectiveness of current repair methodologies. We assume that \(\bar{\tau} > 1/\mu\); this is consistent with the observed operational behavior of equipment in real-world settings, where the equipment’s average uptime \((1/\lambda \equiv \tau)\) is typically greater than the average downtime. We also assume that the failed equipment is restored to its original state (i.e., to reliability level \(\tau\)) after every repair. The up-front decisions by the customer and the vendor determine (respectively) the equipment’s failure and repair rates, which remain constant throughout its (presumably infinite) life cycle. Given these assumptions, we build our model following the conventional machine repairman model, which is commonly used to analyze repair-related problems.

We assume that the client generates revenue from his continued use of the equipment, which is subject to random breakdowns. The client earns revenue \(r\) (per unit of time) during uptime and incurs an opportunity cost \(o\) (also per unit of time) during downtime. The opportunity cost
Figure 1. Operational cycle of the equipment.

\[ \begin{align*}
& \text{F: failure epoch, R: repair epoch, OC: operational cycle, U: uptime, D: downtime} \\
& \end{align*} \]

captures the loss of customer goodwill, the cost of losing a customer to competition, and in some cases the institutional penalties levied on the client for disruption of service to customers.\(^7\) The vendor earns revenue from the contractual payments made by the client for her repair services.

We model the cost of up-front decisions in operating and repair technologies as the general cost functions \( C_0(\tau) \) and \( C_d(\mu) \), respectively. These cost functions are increasing and convex \((C_0'() > 0, C_0''(\tau) > 0, C_d(\mu) > 0, C_d''(\mu) > 0)\). We normalize the cost functions so that the investment required to achieve the equipment’s lowest reliability \( \bar{\tau} \) and lowest repair rate \( \bar{\mu} \) is zero. The upper bounds on the reliability \( \bar{\tau} \) and the repair rate \( \bar{\mu} \) represent the theoretical achievable levels, so we assume that the cost of achieving these theoretical levels is infinite. Thus, we have \( C_0(\tau) = 0, \lim_{\tau \to \bar{\tau}} C_0(\tau) = 0, \lim_{\tau \to \bar{\tau}} C_0'(\tau) = \infty, \lim_{\tau \to \bar{\tau}} C_0''(\bar{\tau}) = \infty, C_d(\mu) = 0, \lim_{\mu \to \bar{\mu}} C_d(\mu) = 0, \lim_{\mu \to \bar{\mu}} C_d'(\mu) = \infty, \text{ and } \lim_{\mu \to \bar{\mu}} C_d''(\mu) = \infty. \) These conditions on the cost structure imply that an interior point solution exists for a profit maximization problem.

The sequence of events in our focal setting, which we analyze in \( \S 4.2 \), is as follows. The client offers a take-it-or-leave-it contract to the vendor. Given the contractual terms, the client and the vendor make decisions in operating and repair technologies, respectively. We model these decisions as the outcome of a simultaneous investment game. In this investment game, the client’s aim is to maximize his profit rate \( \Pi_c \) and the vendor’s aim is to maximize her profit rate \( \Pi_v \) subject to her financial distress constraint. It is important to note that the client determines the optimal contract by using backward induction constrained by the outcome of this simultaneous investment game and the vendor’s individual rationality constraint. We formalize this setting as a contract design optimization problem in \( \S 4.2 \); see Equations (8)–(11). Figure 2 captures this two-step decision-making process, which is embedded in a typical double-sided moral hazard problem. The assumption of simultaneous nature of decisions allows us to model typical repair outsourcing settings, in which investments in repair and operating technology (e.g., staffing quality and training of repair engineers and equipment operators) are unobservable. It is, however, conceivable that in some settings these investments are observable and a sequential investment order may arise because of a differential power status between the two parties. We analyze such an alternative setting with sequential decisions in \( \S 5.1 \).

We use a model based on undiscounted cash flows to analyze the focal setting just described. For this purpose, we apply an exogenous opportunity cost of capital \( \alpha \) to the cost of up-front decisions \( C(\cdot) \) so that we may treat them as per-unit time cost \( \alpha C(\cdot) \) in the profit-rate functions (cf. Porteus 1986). We note that, since \( \alpha \) is an exogenous parameter, one can (without loss of generality) lump together \( \alpha \) and \( C(\cdot) \) and set \( \alpha C(\cdot) = c(\cdot) \). For the sake of brevity, from now on we use \( c_0(\tau) \) and \( c_d(\mu) \) in our discussion and analysis of the focal setting. One could alternatively use profit functions based on discounted cash flows that are computed using the discount rate \( \alpha \) (opportunity cost of capital). As we show in \( \S 5.3 \), we find equivalence between the two approaches.

3.1. Performance-Based Contract

We analyze the challenges of outsourcing repair and restoration services from the client’s perspective. The client designs and offers a performance-based contract to the vendor with the objective of maximizing his expected profit or, equivalently, the profit rate (i.e., profit earned per unit of time) over the contract period.

Without loss of generality, a performance-based contract can be split into a fixed component and a performance-linked variable component. In our setting, the fixed component \( w \) captures payments that are made per unit of time and the variable component \( T(D) \) captures the performance-linked compensation to the vendor, where \( D \) is the realized downtime (vendor’s performance) during a service call. It is intuitive that, in order to align the vendor’s incentives, the client will choose a structure for the optimal \( T(D) \) under which the vendor is penalized for longer downtimes. We restrict our analysis to the following two forms of \( T(D) \):

\[ T(D) = \begin{cases} 
 f - pD & \text{under a linear contract,} \\
 \bar{f} - \bar{p}I_{D > \bar{d}} & \text{under a tiered contract.} 
\end{cases} \]

Here \( f, \bar{f} \) are the fixed payments made during a service call and \( p, \bar{p} \) are the penalty parameters under the two contracts. The linear contract in our paper is similar to the performance-based contracts studied in Kim et al. (2011).
Figure 2. Two-step sequential decision making.

The distinction between linear and tiered contracts is the penalty structure levied against the vendor during a service call. Under a linear contract, the penalty levied in the \( i \)th service call is proportional to the realized performance \( D_i \). Under a tiered contract, a constant penalty is levied whenever the realized performance \( D_i \) is greater than the performance threshold \( d \).

The individual equilibrium profit rate functions of the client (\( \Pi_c \)) and the vendor (\( \Pi_v \)) in their respective efforts are as follows: \( \Pi_c = \lim_{t \to \infty} \frac{E[\sum_{i=1}^{N_t} rU_i - oD_i - T(D_i)]}{t} - c_u(\tau) - w; \) \( \Pi_v = \lim_{t \to \infty} \frac{E[\sum_{i=1}^{N_t} T(D_i)]}{t} + w - c_d(\mu). \)

Here \( N_t \) is a random variable that captures the number of operational cycles completed up to time \( t \), and \( U_i \) and \( D_i \) denote (respectively) the uptime and downtime realized during the \( i \)th operational cycle. Equations (1) and (2) imply that the profit rate functions \( \Pi_c \) and \( \Pi_v \) are determined by the joint efforts of client and vendor because the realization of \( N_t \) is contingent on both \( \tau \) and \( \mu \). This joint dependence creates a double-sided moral hazard that results in incentives for both the vendor and the client to underinvest (free-rider effects; see Holmstrom 1982).

### 3.2. Model for Induced Financial Distress

As discussed in §1, performance-based contracts expose a vendor to potential financial distress. In our setting, the stochastic nature of realized performance \( D \) implies that the interlinked performance-based compensation is also stochastic. This in turn may deplete the cumulative wealth of the vendor to undesirable levels, exposing her to financial distress. According to Hendricks and Singhal (2005, p. 36), the probability of financial distress is an important metric for the various stakeholders of a firm, including investors, managers, customers, etc. We incorporate these elements in our focal setting by adding a financial distress constraint to the vendor’s decision-making model. Thus the vendor, when making her profit-maximizing decision, ensures that her exposure to financial distress does not exceed her exogenous tolerance threshold. This modeling approach allows us to mimic a two-criteria decision-making framework that is often adopted by managers (cf. Eppen et al. 1989). Under such an approach, the manager makes a rational decision to maximize one criterion, such as expected profit rate, and a financial distress measure is developed as a second criterion; a threshold-type constraint on the magnitude of this measure is then appended to the manager’s decision-making model. Alternatively, managers at times may use a single-criterion approach whereby the cost of financial distress is added to the profit maximization objective. We analyze such an alternative approach in §5.2.

In this section, we first describe a stochastic wealth model that captures the dynamic wealth accumulation of the vendor under a given contract \((w, T(D))\). Then we use this stochastic model to compute the vendor’s exposure to financial distress that is induced by \((w, T(D))\). We assume that the vendor has initial wealth \( w_o \) that evolves stochastically over the contract period. During an operational cycle, the vendor earns revenue (cash inflow) in the form of contractual payments made by the client for her repair services; the vendor’s cost (cash outflow) consists of the investment rate \( c_d(\mu) \) and the performance-linked penalty, if any. We assume that the vendor does not reinvest her accumulated wealth and so earns no interest on it. Given these assumptions, we can write the process of the vendor’s wealth evolution under the offered contract \((w, T(D))\) as

\[
w_i = w_o + (w - c_d(\mu))t + \sum_{i=1}^{N_t} T(D_i),
\]

where \( w_i \) denotes the wealth of the vendor at time \( t \). Note that the realization of \( T(D_i) \) across operational cycles can be captured by a sequence of random variables that are independent and identically distributed. This implies that the wealth level \( w_i \) is a random sum \( (N_t) \) of random variables \( (T(D_i)) \). The distribution of these random variables is determined by (i) the contractual terms offered by the client to the vendor and (ii) the Nash equilibrium outcome of the simultaneous investment game.

Similar stochastic wealth models are employed extensively in the literature of actuarial science to study the wealth evolution of insurance firms (Asmussen and Albrecher 2000). An insurance firm receives a continuous cash inflow per unit time from the premium it charges to policyholders, and it is subject to stochastic cash outflows when the insured make claims. Our vendor’s wealth model, Equation (3), maps one-to-one to the wealth model of an...
In our model, the transfer payment $T(D)$ for a service call occurs at the completion of an operational cycle.

Our vendor’s wealth model, however, differs from wealth models of insurance firms in one important respect. Our model allows the distribution of $T(D)$ or “claims” to have positive support. The support of $T(D)$ is $(-\infty, f]$, where $T(D) > 0$ implies a cash inflow for the vendor during a service call. The vendor will see such instances when her realized performance is such that the corresponding penalty levied is less than the fixed payment. In contrast, for insurance firms the distribution of claims has only nonpositive support ($-\infty, 0$); in other words, all claims represent cash outflows. Following Iglehart (1969), we approximate the vendor’s wealth process as a Brownian motion with drift $\mu_{BM} = w - c_d(\mu) + E[T(\cdot)](\mu/(1 + \tau \mu))$ and variance $\sigma^2_{BM} = \text{Var}[T(\cdot)](\mu/(1 + \tau \mu))$.

We define the contract-induced financial distress level as the probability that the vendor’s wealth drops to zero before completion of the contract period (i.e., at some time $\hat{t} < \infty$). In other words, $\hat{t}$ denotes the time span between the start of a contractual agreement and the first instance of the vendor’s wealth falling to (or below) zero. In the actuarial science literature, this time span is referred to as the time to “ruin,” and the probability that $\hat{t} < \infty$ is used as the relevant measure of the insurance firm’s financial health (Asmussen and Albrecher 2000). We use the Brownian motion approximation and write the vendor’s induced financial distress level as

$$
\Pr(\tau, \mu, w, T(D)) = \Pr(\hat{t} < \infty) = \exp\left\{-2w \frac{(w-c_d(\mu))(1+\tau \mu)+E[T(\cdot)]\mu}{V[T(\cdot)]\mu}\right\}. \tag{4}
$$

We model the vendor’s problem as a profit maximization problem subject to the constraint that the induced financial distress level $\Pr(\tau, \mu, w, T(D))$ does not exceed a distress threshold $b$. We assume that the vendor’s distress threshold $b \in (0, 1)$ is determined exogenously and reflects the vendor’s financial state and/or business priorities.

4. Analysis

To benchmark the performance of the two contract structures, we first determine the optimal centralized decisions in the operating and repair technologies. In the rest of this paper, we use an asterisk (*) to denote the profit and decisions of the centralized entity.

4.1. Optimal Centralized Decisions

Given the model’s description in §3, we can write the centralized profit rate function $\Pi$ as

$$
\Pi = \lim_{t \to \infty} \frac{\text{E} \left[ \sum_{i=1}^{N} r U_i - o D_i \right]}{t} - c_d(\tau) - c_d(\mu). \tag{5}
$$

The following proposition characterizes the optimal centralized decisions in the operating and repair technology—in other words, the investments that maximize $\Pi$. Note that investments in operating and repair technology $\{c_d(\tau), c_d(\mu)\}$ uniquely map into the decisions $\{\tau, \mu\}$. Henceforth we formulate the decision spaces of the client and the vendor as $\tau \in [\underline{\tau}, \overline{\tau}]$ and $\mu \in [\underline{\mu}, \overline{\mu}]$, respectively.

**Proposition 1.** The optimal centralized decision $\{\tau^*, \mu^*\}$ is the unique solution to the following system of equations:

$$
\begin{align*}
(\tau + o)\mu & = c'_d(\tau), \tag{6} \\
(\tau + o)\tau & = c'_d(\mu). \tag{7}
\end{align*}
$$

**Proof.** All omitted proofs are given in the e-companion (available as supplemental material at http://dx.doi.org/10.1287/opre.2013.1210). \( \square \)

Proposition 1 characterizes the unique interior solution in the $\mathbb{R}^2_+$ space defined by $[\underline{\tau}, \overline{\tau}] \times [\underline{\mu}, \overline{\mu}]$. We denote the centralized optimal profit rate by $\Pi^*$, which is achieved by implementing the decision $\{\tau^*, \mu^*\}$. When the client outsources the repair services to the vendor, the implicit decentralized service supply chain can achieve a profit rate of no more than $\Pi^*$.

**Proposition 2.** Investments in operating technology or repair technology are substitutes for each other.

Proposition 2 implies that investments in operating and repair technologies are determined by the relative marginal costs of implementing them. More importantly, this proposition indicates that both the client and the vendor have incentives to underinvest in their respective decisions. Since the two investments are substitutable, each party has incentives to free ride on the other party’s effort, resulting in some loss of service supply chain profits. In the next section we analyze the decentralized setting.

4.2. The Client’s Problem Under Decentralized Decision Making

We formulate the client’s problem under decentralized decision making—in terms of a principal-agent model featuring double-sided moral hazard (Bhattacharyya and Lafontaine 1995, Roels et al. 2010)—as follows:

$$
\begin{align*}
\max_{T(\cdot), \mu} & \quad \Pi_c(\tau', \mu', w, T(\cdot)) \tag{8} \\
\text{s.t.} & \quad \tau' = \arg \max_{\tau \leq \tau'} \Pi_c(\tau, \mu', w, T(\cdot)), \tag{9} \\
& \quad \mu' = \arg \max_{\mu \leq \mu'} \Pi_c(\tau', \mu, w, T(\cdot)) \tag{10} \\
& \quad \text{s.t.} \quad \Pr(\tau', \mu, w, T(\cdot)) \leq b, \tag{11}
\end{align*}
$$

where $\tau'$ and $\mu'$ are the decision variables, $w$ is the stochastic variable representing the workload, and $b$ is the threshold for the probability of financial distress. The function $\Pi_c(\cdot)$ represents the client’s profit function, and $\Pi_c^*$ represents the optimal profit rate that the client can achieve by implementing the decision $\{\tau^*, \mu^*\}$.

In order to solve this optimization problem, we need to determine the optimal values of $\tau'$ and $\mu'$ that maximize the client’s profit function $\Pi_c(\tau', \mu', w, T(\cdot))$. This is a complex optimization problem that requires the use of advanced optimization techniques, such as dynamic programming or numerical methods. The solution to this problem will provide insights into the optimal decisions that the client should make in order to maximize its profit, given the constraints and objectives of the problem.

The optimal decentralized decisions $\{\tau^*, \mu^*\}$ are determined by solving the optimization problem (8) subject to the constraints (9), (10), and (11). The optimal decentralized decisions will be a function of the stochastic variable $w$ and the threshold $b$. The optimal decentralized decisions will provide insights into the optimal decisions that the client and the vendor should make in order to maximize their respective profits, given the constraints and objectives of the problem.
Here $\Pi_1$ and $\Pi_2$ are (respectively) the client’s and vendor’s profit rate functions as defined by Equations (1) and (2). To avoid trivial solutions, we assume that $\nu \in (0, \Pi_1^*)$. Also, we model the distribution of equipment uptime and downtime with exponential distributions in order to gain analytical tractability for computing the potential financial distress induced by offered contractual terms.

Equations (9) and (10) represent the simultaneous investment game that yields the Nash equilibrium outcome $(\nu^*, \mu^*)$. Observe that, within this investment game, the vendor makes her profit rate-maximizing decision $\mu^*$ subject to her financial distress constraint $Pr(\nu^*, \mu^*, w, T(\cdot)) \leq b$. Equation (8) represents the client’s search problem for the optimal contract $\{T(\cdot), w\}$ that will maximize his profit rate subject to investment game output $(\nu^*, \mu^*)$ and the vendor’s participation constraint (inequality (11)). Observe that the theoretical maximum profit rate attainable by the client in the decentralized case is via a contract that (i) induces the client and the vendor to make investments equal to the optimal centralized decisions (thus rendering moot the free-rider issue) and (ii) makes the participation constraint of the vendor “tight.” A $\{T(\cdot), w\}$ that satisfies these two conditions is called the first-best contract. Although the set of feasible $\{T(\cdot), w\}$ in the client’s problem is infinite, as we show below, he can attain the first-best outcome within the class of linear and tiered contract structures for all feasible values of exogenous parameters. More specifically, the client can design a first-best contract by limiting his search space to $\{T(\cdot), w\} \in \mathcal{L} \cup \mathcal{T}$, where $\mathcal{L}$ is the class of linear contracts, $\mathcal{T}$ is the class of tiered contracts, and $\mathbb{R}^+$ is the set of positive real numbers. Next we shall analyze each class of contracts in turn.

4.2.1. Linear Contractual Structure. Recall that the variable component of a linear contract is

$$T(D) = f - pD, \quad (12)$$

where $f$ is the fixed payment made during the service call and $p$ is the penalty rate. At the end of the service call, the client levies a penalty ($pD_i$) that is proportional to the vendor’s realized performance $D_i$ for that service call.

**Lemma 1.** A linear contract $\{w, f, p\}$ attains the first-best outcome for the client only if

(a) $p = f \cdot \mu^*$;
(b) $w = c_d(\mu^*) + \nu$.

Lemma 1 describes the necessary conditions for designing a linear contract that can attain the first-best outcome for the client. By applying Lemma 1, we can reduce the client’s problem of searching for a first-best contract (within the class of linear contracts) into a single decision $\{p\} \in \mathbb{R}^+$.

We find that the client can attain the first-best outcome with a linear contract if the vendor’s exogenous parameters $\{\nu, b\}$ are within a particular range. In the next proposition we characterize the set of parameters for which the linear contract attains the first-best outcome.

**Proposition 3.** (a) For a vendor with reservation value $\nu \in [(r + o)\tau^* \mu^*]/((1 + \tau^* \mu^*)(1 + 2\tau^* \mu^*), \Pi_1^*)$, the client (i) can attain the first-best outcome by designing a unique linear contract

$$\{w, f_1, p_1\} = \left[ c_d(\mu^*) + \nu, \frac{(r + o)\tau^* \mu^*}{1 + \tau^* \mu^*} \right]$$

if $b \in \{\exp[-(2w_0\nu(1 + \tau^* \mu^*)]/((r + o)^2\tau^2 \mu^*)] \}$.

(ii) can attain the first-best outcome by designing a unique linear contract

$$\{w, f_2, p_2\} = \left[ c_d(\mu^*) + \nu, \frac{2w_0\nu(1 + \tau^* \mu^*)}{\log(1/b) \mu^*} \right]^{1/2}$$

if $b \in \{0, \exp[-(2w_0\nu(1 + \tau^* \mu^*)]/((r + o)^2\tau^2 \mu^*)] \}$.

(b) For vendors with the reservation value $\nu \in (0, (r + o)\tau^* \mu^*/((1 + \tau^* \mu^*)(1 + 2\tau^* \mu^*))$, the client (i) can attain the first-best outcome with a unique linear contract $\{w, f_1, p_1\} = \{c_d(\mu^*) + \nu, [(r + o)\tau^* \mu^*/((1 + \tau^* \mu^*)(1 + 2\tau^* \mu^*))] \}$ if $b \in \{\exp[-(2w_0\nu(1 + \tau^* \mu^*)]/((r + o)^2\tau^2 \mu^*)] \}$.

(ii) can attain the first-best outcome with a unique linear contract

$$\{w, f_2, p_2\} = \left[ c_d(\mu^*) + \nu, \frac{2w_0\nu(1 + \tau^* \mu^*)}{\log(1/b) \mu^*} \right]^{1/2}$$

if $b \in \{0, \exp[-(2w_0\nu(1 + \tau^* \mu^*)]/((r + o)^2\tau^2 \mu^*)] \}$.

(iii) cannot attain the first-best outcome under a linear contract structure if $b \in \{0, \exp[-(2w_0\nu(1 + \tau^* \mu^*)]/((r + o)^2\tau^2 \mu^*)] \}$. Proposition 3 broadly classifies vendors into two groups based on their reservation values $\nu$ and distress thresholds $b$. A vendor belongs to the first group (region A of Figure 3) if she has a high reservation value and/or a high distress threshold. With such a vendor, the client can attain the first-best outcome by offering a customized linear contract in which contractual terms are appropriately adjusted according to the vendor’s characteristics $\{\nu, b\}$. In contrast, a vendor belongs to the second group (region B of Figure 3) if she has a low reservation value and/or a low distress threshold. With such a vendor, the client cannot attain the first-best outcome by offering a linear contract.

Proposition 3 shows that linear contracts can attain the first-best outcome only for a subset of parameter values. This result complements the extant literature on double-sided moral hazard. In economics, Bhattacharyya and Lafontaine (1995) and Kim and Wang (1998) show...
that attaining the first-best outcome in a double-sided moral
dehazard environment is not feasible under general condi-
tions. Bhattacharyya and Lafontaine find that, in their setting, linear contracts are optimal but can attain only the second-best outcome. In other words, although linear contracts do equal or better than any other contract type, they fail to attain the first-best outcome in their setting. Furthermore, Kim and Wang find that linear contracts lose their optimality if the agent is risk averse. In the operations management literature, it has been shown that the first-best outcome can be attained under three special conditions: (i) when the decisions of both parties are verifiable, which allows the contract to be based directly upon those decisions (Baiman et al. 2000); (ii) when there exists a contractible performance measure that can reduce the profit functions of the two parties to endogenous functions of their respective decisions (Cooper and Ross 1985, Baiman et al. 2000); or (iii) when the first-best outcome require positive effort from at most one of the parties (Corbett et al. 2004).

None of these three special conditions applies in our setting. Condition (i) does not apply because neither the client’s decision in operating technology nor the vendor’s decision in repair technology is verifiable. Condition (ii) does not apply because it requires a performance measure that could reduce the profit rate functions of client and vendor to their respective decisions (i.e., \( \Pi_x = g_x(\tau) \) and \( \Pi_y = g_y(\mu) \)). Such reduction would convert the double-sided moral hazard problem to a single-sided moral hazard problem that could be completely resolved, but Equations (1) and (2) imply that such a reduction is not feasible in our setting. Finally, condition (iii) does not apply because in our setting the optimal decision is an interior point in the joint decision space of client and vendor; hence both parties must exert positive efforts in order to attain the first-best outcome. Given that our setting does not fit into these three conditions reported in Baiman et al. (2000), Cooper and Ross (1985), and Corbett et al. (2004), the results of Proposition 3 cannot be inferred directly from these papers and hence extend their findings.

To understand what enables a linear contract to attain the first-best outcome one should understand the interaction between the “fixed fee per service call” component \( f \) and the penalty rate \( p \). The term \( f \) plays the dual role of carrot and stick for vendor and client, respectively. The fixed component per unit time \( w \) covers the investment cost \( c_i(\mu^*) \) and the reservation value \( v \), so \( f \) provides an opportunity for the vendor to gain additional profits. At the same time, if the client underinvests in \( \tau \) then the frequency of failure will increase, and with each failure the client must pay \( f \) in addition to the per-unit-time payment \( w \). The term \( p \) serves as a stick in that it discourages the vendor from underinvesting.

The limited ability of linear contracts to attain the first-best outcome is primarily driven by the vendor’s trade-off between maximizing her profit rate and keeping financial distress below her tolerance threshold. Intuitively, if the vendor has a high reservation value (or, equivalently, a large share in the supply chain profit) then she has strong incentives to make the optimal decision. However, if the vendor has a low reservation value then a linear penalty exposes her to higher level of financial distress, which in turn may incentivize her to make suboptimal decisions. This implies that vendors with low reservation value must trade off between investment decisions and the associated potential financial distress levels. Therefore, unless vendors with low reservation value have a strong appetite for potential financial distress, a linear contract will not attain the first-best outcome.

Next we show that the client can circumvent the financial distress concerns of a vendor in region B, and thus incentivize her to implement optimal centralized decision \( \mu^* \), by using linear contracts that pay the vendor a surplus over her reservation value \( v \). Contracts that are so devised to attain system-optimal decisions are known as coordinating contracts. Of course, even though such contracts attain the optimal system performance, they are not first best for the client because he must now pay the vendor a surplus. In other words, the client has to pay a cost for exposing the vendor to financial distress level that is higher than her tolerance threshold. Corollary 1 presents the analytical expression for the profit rate earned by a region-B vendor, \( \Pi_y(> v) \), under the coordinating linear contract. By paying a profit rate \( \Pi_y \) that is higher than the vendor’s reservation value \( v \), the client reduces the vendor’s trade-off between

**Figure 3.** First-best region for linear and tiered contract structures.

![Diagram](https://example.com/diagram)
making system-optimal investment decisions and limiting her exposure to financial distress.

**Corollary 1.** The client can attain first-best decisions with a vendor \( \{v_B, b_B\} \) in region B by paying her

\[
\Pi_v = \left( -1 + \tau^* \mu^* \right) \sqrt{2w_v \mu^*} + \sqrt{2w_v (1 + \tau^* \mu^*)} \mu^* + 4 \log(1/b_B) (2 \tau^* \mu^* + 1) (\tau^* + \tau^* \mu^*) \right) \cdot \left( 2 (\tau^* \mu^* + 1) \sqrt{(1 + \tau^* \mu^*) \log(1/b_B)} - 1 \right)^2 > v_B.
\]

The results of Proposition 3 have important implications. Linear contracts are intuitive, yet they are seldom observed in practice. One possible explanation for this is that most vendors may actually belong to region B; in other words, vendors typically have a low reservation value and/or a low financial distress threshold. Since linear contracts do not attain the first-best outcome in region B, there may exist a class of contracts that outperform linear contracts in this region. As we show in the following section, tiered contracts outperform linear contracts in region B. In fact, we find that the tiered contract structure attains the first-best outcome for the client in region B and in region A.

### 4.2.2. Tiered Contractual Structure

Contracts in repair and restoration settings are seldom linear; instead, they levy a constant penalty on the vendor if her performance deteriorates beyond a threshold level (see Table 1). Often such contracts have multiple performance brackets (tiers), each with a corresponding fixed penalty that is levied when the vendor’s realized performance falls within that bracket. As described in §3.1, this paper analyzes a one-tier contract structure under which the client proposes to levy a constant penalty \( \tilde{p} \) only if the vendor’s realized performance \( D \) deteriorates beyond a threshold level \( \tilde{d} \) during a service call:

\[
T(D) = \tilde{f} - \tilde{p}I_{D > \tilde{d}}.
\]

In this equation, \( I_{D > \tilde{d}} \) is a indicator function on \( \tilde{d} \) (i.e., \( I_{D > \tilde{d}} \) is set equal to 1 if \( D \geq \tilde{d} \) and to 0 otherwise), \( \tilde{f} \) is the fixed fee paid per service call, and \( \tilde{p} \) is the constant finite penalty levied after service call if \( D \geq \tilde{d} \).

**Proposition 4.** The client can attain the first-best outcome with a tiered contractual structure for all \( v \in (0, \Pi^* \) and for all \( b \in (0, 1 \). Specifically, the client

(a) can attain the first-best outcome with a tiered contract

\[
[w, \tilde{f}, \tilde{p}, \tilde{d}]
\]

\[
= \left\{ c_d(\mu^*) + v, \frac{c_d(\mu^*)(1 + \tau^* \mu^*)}{\bar{d} \mu^*}, \frac{c_d(\mu^*)(1 + \tau^* \mu^*)}{\mu^*}, \frac{2}{\bar{d} \mu^*} \right\}
\]

if \( b \in \left[ \exp\left\{ -\frac{8w_v \mu^*}{c_d(\mu^*)^2 (1 + \tau^* \mu^*)(\mu/e^2 - 1) \mu^*} \right\}, 1 \right] \); or

(b) can attain the first-best outcome with a tiered contract

\[
[w, \tilde{f}, \tilde{p}, \tilde{d}]
\]

\[
= \left\{ c_d(\mu^*) + v, \frac{2w_v v(1 + \tau^* \mu^*)}{\log(1/b)} \mu^* (e^{\mu^* d} - 1)^{1/2}, \frac{2w_v v(1 + \tau^* \mu^*)}{\log(1/b)} \mu^* (e^{-\mu^* d} - e^{-2\mu^* d})^{1/2} , \tilde{d} \right\}
\]

if \( b \in \left( 0, \exp\left\{ -\frac{8w_v \mu^*}{c_d(\mu^*)^2 (1 + \tau^* \mu^*)(e^{\mu^*/2} - 1) \mu^*} \right\}\right) \); and \( \tilde{d} \geq 2/\mu \) and satisfies

\[
d \left( 1 - \frac{1}{\mu^*} \right) \geq \frac{\mu^*}{v (1 + \tau^* \mu^*)} \left( c_d(\mu^*) + v - c_d(\mu^*) \right) \frac{1}{\mu^*}
\]

\[
+ c_d(\mu^*) \left( \frac{1}{\mu^*} + \tau^* \right) \forall \mu \in [\mu^*, \mu^*].
\]

Proposition 4 shows that tiered contracts not only outperform linear contracts but also enable the client to attain the first-best outcome. This adds to the literature on double-sided moral hazard by showing that the first-best outcome can be attained for all possible parameter values.

The results in Proposition 4 are particularly interesting because their intuition is not straightforward. Intuitively, the limited penalty structure of tiered contracts might have a countervailing effect: while reducing the vendor’s exposure to financial distress, it may also reduce her incentives to invest optimally in the repair technology. In particular, under a tiered contract, no penalty is levied unless the vendor’s performance deteriorates beyond a threshold level \( d \). This contract term reduces the vendor’s probability of earning negative profits in consecutive operational cycles. At the same time, it also limits the vendor’s downside to underinvesting in repair technology and so may not eliminate the free-rider aspect of the double-sided moral hazard. Yet it is interesting that we find the client is able to choose a financial reward and penalty structure such that he can incentivize the vendor aptly to invest in the first-best decision while satisfying her financial distress concerns.

Our results for tiered contracts have important managerial implications. First, we show that such contracts can attain the client’s first-best outcome irrespective of the vendor characteristics \( \{v, b\} \). This result provides theoretical support for the observed prevalence of tiered contracts in practice. Second, our results show how choosing an appropriate contract structure can help the client attain the first-best outcome—by incentivizing the vendor to make the first-best decision while satisfying her financial distress concerns—without paying the vendor any surplus in excess of her reservation value. Therefore, we show that using appropriate (tiered) contracts eliminates the possibility of losses due to agency issues in our context.
5. Contract Performance Under Alternative Settings

In this section we analyze the comparative performance of tiered and linear contracts under three alternative repair outsourcing settings. In §5.1 we analyze settings in which investment decisions in repair and operating technology are made sequentially. Section 5.2 examines settings in which the vendor incorporates a cost of induced financial distress into her decision making. Finally, in §5.3 we analyze our focal setting using a discounted cash flow model.

5.1. Sequential Decisions

So far we have focused on repair outsourcing settings wherein the investment decisions in operating and repair technology are made simultaneously by the client and the vendor, respectively (see Figure 2). The simultaneous nature of investment decisions allows us to model repair outsourcing settings, in which investments in repair and operating technology are unobservable. In practice, however, it is conceivable that in some settings these investments are observable and a sequential investment order may arise because of a differential power status of the two parties. For example, the client may exercise his higher power status to enforce a sequential investment setting and to become the leader in such a setting. Serving as a leader may be preferred by the client as it provides him an opportunity to do better than the simultaneous investment setting. Similarly, there may be scenarios where the vendor acts as the leader in the investment game. To cover all such possibilities, we analyze the comparative performance of linear and tiered contracts under two scenarios: sequential investment games in which the client (scenario 1, S1) or the vendor (S2) acts as the leader. Our findings are summarized in Figure 4.

We find that structural limitation of the linear contractual structure persists—regardless of whether the client or the vendor acts as leader. In contrast, the tiered contractual structure continues to attain the first-best outcome in every case. In particular, when the client acts as the leader (S1, panel (a) in the Figure 4) he does at least as well under each contract type as in the simultaneous setting. This follows directly from the fact that being a first mover cannot hurt the client. Further, we find that region of no first-best for the linear contractual structure (region B in Figure 3) is now split into two subregions: B_1 and B_2. In region B_1, we find that there exists no linear contract that can satisfy the necessary conditions for attaining the first-best outcome. However, in region B_2 (the shaded region in Figure 4(a)), we find that there do exist linear contracts that satisfy those necessary conditions but because of analytical intractability we are unable to verify whether the sufficient conditions are also satisfied in this region. We characterize these regions in Proposition 5.

**Proposition 5.** For the sequential investment game in which the client acts as the leader, the following statements hold.

(a) There exists a linear contract that satisfies the necessary conditions for attaining the first-best outcome if \( v \in [v_2, v_1] \) and distress threshold \( b \in [b_2, b_1] \), where
\[
v_1 = ((r + o)\tau \mu^*)/((1 + \tau \mu^*)(1 + 2\tau \mu^*)), \quad v_2 = ((r + o)\tau \mu^*)/((r + o)^2\tau \mu^*), \quad b_1 = \exp\left(-(2\omega v(1 + \tau \mu^*)^3)/((r + o)^2\tau \mu^*))\right), \quad b_2 = \exp\left(-(2\omega v(1 + \tau \mu^*)^3)/(r + o)\right).
\]

(b) No linear contract satisfies the necessary conditions for attaining the first-best outcome if \( v \in (0, v_2) \) and \( b \in (0, b_2) \).

When the vendor acts as the leader (S2, panel (b) in the Figure 4), we find that linear contracts may attain the first-best outcome in region D but not in region C (see Proposition 6 for characterization of these regions). In comparison with the simultaneous setting, the linear contractual form may do either better or worse depending on the vendor’s characteristics \( v \) and \( b \). In particular, linear contracts
do worse if \( v/\log(1/b) \leq (r + \sigma^2/\mu^* - 1)^2/(2w_0(1 + \sigma^2/\mu^*)) \). Intuitively, if \( v \) and/or \( b \) is low, then the penalty required is so low that it may provide incentives for the vendor to deviate from the first-best decision \( \mu^* \). In contrast, we find that tiered contracts once again have the ability to eliminate any such deviation while providing appropriate incentives for making the first-best decision \( \mu^* \). In Proposition 7, we provide tiered contractual terms that can help the client to attain the first-best outcome under \( S_2 \).

**Proposition 6.** For the sequential investment game in which the vendor acts as the leader, the client cannot attain the first-best outcome using a linear contract with a vendor who has reservation value \( v \in (0, ((r + \sigma^2/\mu^* K_2)/(1 + \sigma^2/\mu^* K_1))) \) and distress threshold \( b \in (0, (e^{v_1 + c_1(\mu^*)} + \mu^*(1 + \tau^2/\mu^*)^2)/(1 + \tau^2/\mu^*)^2)] \).

\[ \begin{align*}
K_1 &= (3c_1(\tau^*)\mu^* + c_1(\tau^*)(1 + 2\tau^2/\mu^*)) > 0 \quad \text{and} \quad K_2 = 2(r + \mu^2/\mu^*) > 0,
\end{align*} \]

where \( K_1 = (3c_1(\tau^*)\mu^* + c_1(\tau^*)(1 + 2\tau^2/\mu^*)) > 0 \) and \( K_2 = 2(r + \mu^2/\mu^*) > 0 \).

**Proposition 7.** For the sequential investment game in which the vendor acts as the leader, the client can always attain the first-best outcome by using a tiered contractual structure. In particular, for a vendor with \( v \in (0, \Pi^*) \), the client can attain the first-best outcome with a tiered contract

\[ \begin{align*}
(a & \{w_5, f_5, p_5, d_5\} = \left\{ \frac{c_1(\mu^*) + v, c_1(\mu^*) + \mu^*(1 + \tau^2/\mu^*)}{\mu^*}, \right.
\left. c_1(\mu^*) + \mu^*(1 + \tau^2/\mu^*) d_5 \right\} \right) \\
& \text{if } b \in (\hat{b}, 1), \text{ where} \hat{b} = \exp\left(-\frac{8w_0v\mu^*}{c_1(\mu^*)^2(1 + \tau^2/\mu^*)(1 + v_1 + c_1(\mu^*) - 1\mu^*)}\right);
\end{align*} \]

(b) \( \{w_6, f_6, p_6, d_6\} \)

\[ \begin{align*}
& = \left\{ v + c_1(\mu^*), \left( \frac{2w_0v(1 + \tau\mu)}{\log(1/b)\mu^*(e^{v_1 + c_1(\mu^*)} - 1)} \right)^{1/2}, \right. \\
& \left. \frac{2w_0v(1 + \tau\mu)}{\log(1/b)\mu^*(e^{v_1 + c_1(\mu^*)} - 1)} \right)^{0.5}, \bar{d} \right\} \\
& \text{if } b \in (0, \hat{b}), \text{ where } \bar{d} \geq 2/\mu \text{ and satisfies} \\
& \bar{d}(1 - \frac{1}{(e^{v_1 + c_1(\mu^*)} - 1)}) \\
& \geq \frac{\mu^*(v + c_1(\mu^*) - c_1(\mu))}{v(1 + \tau^2/\mu^*)} \left( \frac{1}{\mu^*} + \tau \right) + \frac{v + c_1(\mu^*)}{\mu^*} + \frac{\mu^*(v + c_1(\mu^*) - c_1(\mu))\tau(r + o)}{v(1 + \tau^2/\mu^*)2c_1(\tau)(1 + \tau^2/\mu^*)^2\mu + c_2(\tau)} \\
& \text{for all } \mu \in [\mu, \mu^*] \text{ and } \tau \in [\tau, \tau^*]. \quad (15)
\end{align*} \]

These results imply that tiered contracts outperform linear contracts in attaining the first-best outcome for the client irrespective of whether the investment decisions by client and the vendor are made simultaneously or sequentially. As discussed before, in practice both scenarios of simultaneous and sequential decision making are plausible. Therefore, it is interesting to find that dominance of tiered contracts over linear contracts is robust to the order of investment decisions made by the client and the vendor.

### 5.2. Cost of Financial Distress

In this paper we study the impact of PBC-induced financial distress on vendors’ rational decision making by using a two-criteria approach that is often practiced by managers (see the discussion in §3.2). Alternatively, at times, managers may prefer to adopt a single-criterion approach in which they attach an exogenous cost \( K \) to the financial distress level. To analyze the comparative performance of the two contracts under such an approach, we formulate the client’s optimization problem as

\[ \begin{align*}
\max_{\tau, \mu, \phi} & \quad \Pi_c(\tau, \mu, w, T(\cdot)) \\
\text{s.t.} & \quad \tau^* = \arg\max_{\tau \in \hat{\tau}} \Pi_c(\tau, \mu, w, T(\cdot)), \\
& \quad \mu^* = \arg\max_{\mu \in \hat{\mu}} \Pi_c(\tau, \mu, w, T(\cdot)) \quad (\hat{\tau}, \hat{\mu}) - K(B(\tau, \mu, w, T(\cdot)) \geq \phi,
\end{align*} \]

where \( B(\cdot, \cdot, \cdot, \cdot) \) is a metric of financial distress and \( \phi \) denotes the vendor’s exogenous outside option value. We study two different measures for \( B(\cdot, \cdot, \cdot, \cdot) \): (i) a probability-based induced financial distress level, as we use in our focal setting, and (ii) a measure based on cash flow volatility. It is important to note here that under this approach the first-best outcome can only be attained if \( B(\cdot, \cdot, \cdot, \cdot) \) equals zero. In other words, the client can earn first-best profits only if the level of financial distress, induced by a performance-based contract, can be reduced to zero. Given the inherent stochasticity in the system, this cannot be achieved with both the linear and tiered contractual structures. Therefore, under this approach, we compare the efficacy of these contractual structures by analyzing the optimal profits earned by the client under each contractual structure.

The probability-based measure has the critical advantage of incorporating the cumulative effect of transfer payments. With this measure, however, studying the Nash equilibrium outcome of the investment game ((17) and (18)) becomes analytically intractable. To circumvent this difficulty, we analyze the comparative performance of tiered and linear contracts via an extensive numerical study. The details of this numerical study are provided in the e-companion. In particular, we study an efficacy measure \( \bar{d} \), which is defined as the relative increase in the client’s optimal profit rate \( \bar{\Pi} \).
when using tiered contracts as compared with the case of using linear contracts; thus, \( \delta = (\Pi_{1}^L - \Pi_{1}^T)/\Pi_{1}^L \), where the superscripts \( L \) and \( T \) denote (respectively) the linear and tiered contract type. We find that tiered contracts perform strictly better than optimal linear contracts with \( \delta \geq 3.75\% \) and that, for half of the input parameter vectors in our sample,\(^{12} \) the gain was no less than 16.98\% (see Figure 5).

For the second measure—cash flow volatility—we use variance in cash flows earned during an operational cycle. This variance is endogenously determined by the offered contract and the investment decisions of both vendor and client. Using this measure, we find that any linear contract is strictly dominated by a tiered contract. The following definition and proposition summarize our results under this formulation.

**Definition 1.** From the client’s perspective, a contract \( T_i(\cdot) \) is strictly dominated by another contract \( T_j(\cdot) \) if \( \Pi_i^j > \Pi_i \) and \( \Phi_i^j > \Phi_i \), where \( \Pi_i^j \) and \( \Phi_i^j \) denote (respectively) the client’s profit and the value of the vendor’s objective function under contract \( i \in \{1, 2\} \).

**Proposition 8.** With explicit costs of financial distress in the vendor’s profit function, linear contracts are strictly dominated by tiered contracts. That is, for every linear contract, there exists at least one tiered contract that strictly dominates the linear contract.

To summarize, similar to the two-criteria approach used in our study of the focal setting, we find that under a single-criterion approach the client can better coordinate vendors’ decisions using tiered contracts than using linear contracts.

We also analyze an extension of this model ((16)–(19)) to study repair outsourcing settings in which additional explicit costs are incurred by the client as a result of the vendor’s financial distress. To gain insights for such settings, we analyze an extension in which an explicit cost term, \( K_c \mathbb{M}(\tau, \mu', w, T(\cdot)) \), is appended to the client’s profit function:

\[
\max_{T(\cdot), w} \Pi_c(\tau', \mu', w, T(\cdot)) - K_c \mathbb{M}(\tau', \mu', w, T(\cdot))
\]

s.t. \( \tau = \arg\max_{\tau \in \mathcal{T}} \Pi_c(\tau', \mu', w, T(\cdot)) \),

\[
\mu' = \arg\max_{\mu \leq \mu \leq \mu} \Pi_c(\tau', \mu', w, T(\cdot))
\]

\[
\Pi_c(\tau', \mu', w, T(\cdot)) - K_c \mathbb{M}(\tau', \mu', w, T(\cdot)) \geq \phi.
\]

Here \( K_c \) denotes an exogenous cost parameter and \( \mathbb{M}(\tau, \mu', w, T(\cdot)) \) is the metric of the vendor’s distress. Again, for both the probability based and the cash flow volatility based measure, we find that tiered contracts outperform linear contracts. For the probability-based measure, we find numerically that optimal tiered contracts perform strictly better than optimal linear contracts with \( \delta \geq 4.44\% \); for half of the parameter vectors in our sample,\(^{15} \) the gain was no less than 21.48\% (see Figure 6). For the cash flow volatility-based measure, we find that for every linear contract, there exists at least one tiered contract that strictly dominates the linear contract.\(^{14} \)

### 5.3. Discounted Cash Flows

In line with the classical machine repairman model, we have modeled cash flows in the repair outsourcing setting as a renewal-reward process. This implies that the client’s and the vendor’s cash flow streams, which are endogenously determined by the contractual form and payment terms, emulate compound Poisson processes. Given the up-front nature of decisions, in some settings managers may prefer to use the discounted present value of these cash flow streams when making their decisions. We compare
the performance of linear and tiered contracts under such settings by reformulating the client's optimization problem ((8)–(11)) by using objective functions—for client and vendor both—that are based on net present value (NPV). To construct NPV-based objective functions, following Iglehart (1969) we approximate the cash flow streams as Brownian motions and then apply continuous discounting to compute the present value of contract-induced cash flow streams. Finally, we subtract the cost of up-front decisions $C(\cdot)$ to construct the NPV-based objective functions. With this approach, we find equivalence between the two client optimization problems that are formulated using profit functions based on discounted and undiscounted cash flows. This equivalence implies that the comparative performance of the linear and tiered contracts is not sensitive to how the cash flows are formulated. Instead, the better performance of tiered contracts is driven by their ability to aptly incentivize the vendor to make the first-best decision while satisfying her financial distress constraint. The details of this analysis are given in the e-companion.

6. Conclusions and Future Research

In this paper we study the contracting issues that arise when a firm outsources the repair and restoration of its equipment to an external service provider. The design of an optimal contract for such outsourcing of repair services is complicated by two primary challenges. First, the contract must resolve the double-sided moral hazard problem that naturally occurs in such a setting. Second, a performance-based contract exposes the service provider to financial distress due to the inherent stochasticity in the system that in turn creates disincentives for the vendor to make system-optimal decisions.

We find that if the vendor has a high reservation value and/or a high financial distress threshold, then the client can attain the first-best outcome with either a linear or a tiered contract structure. However, if the vendor has a low reservation value and/or a low distress threshold, then only the tiered contract structure allows the client to attain the first-best outcome in all cases. The dominance of the tiered over the linear contract—regardless of vendor characteristics—may explain the preference for tiered contracts over linear contracts that is observed in practice.

One can conceive of additional advantages, not studied in this paper, of using tiered contracts in repair outsourcing settings. For example, the client may prefer the tiered contractual structure as it requires monitoring of the vendor's performance only up to a threshold. Also, the vendor may prefer the tiered contractual structure as it allows her to set simple target-based performance goals for her repair engineers or allows her to avoid incurring penalties for small delays that naturally occur, and may not be under her control, with each breakdown. One example is the pre-repair setup of the machine area, which is typically done with assistance of the client's staff.

This work contributes to the operations literature in a number of ways. First, the paper establishes a model that captures important challenges in the repair and restoration service outsourcing industry by combining elements of three different fields of research: the machine repairman model from the operations literature, the double-sided moral hazard framework from the economics literature, and the model of vendor’s potential financial distress from the actuarial science literature. Under this model, we find that the client’s problem of designing an optimal contract can be reduced to a search space of the two classes of contracts that we study. Second, results based on our model show that tiered contracts can attain the client’s first-best outcome for all parameter values. Third, past studies have shown that, in a double-sided moral hazard environment, no contract can attain the first-best outcome under general settings. In some exceptional settings in which certain special conditions apply, a few studies have shown that the first-best outcome can be attained. Even though none of those special conditions apply in our setting, we show how a firm can design contracts that achieve the first-best outcome.

Fourth, our analysis also contributes to the relatively recent body of literature that studies contracting issues and their implications on the service provider’s (or supplier’s) financial health. Complementing this literature—which focuses mostly on the role of up-front payments or subsidies in balancing the supplier’s trade-off between system-optimal investment decisions and her financial concerns—we show that contractual features can be leveraged to attain the first-best outcome without paying any surplus to the vendor.

Our choice of performance-based contracts that penalize the vendor for each occurrence of downtime are best suited for an environment in which the frequency of transfer payments between client and vendor are of the same order as the failure rate. In some settings, however, the frequency of transfer payments may be much lower than the failure rate (e.g., when the contract penalizes the vendor based on average downtime over a prespecified (finite) period of time). Although we do not explicitly consider such settings, the insights of our paper extend directly to them. Tiered contracts will continue to attain the first-best outcome under all conditions, and there will be a range of parameter values for which linear contracts cannot attain the first-best outcome.

In this paper, we focus on a one-to-one contractual relationship between the firm and the service provider. Further, we focus on settings where the failure and repair rates are primarily determined by the respective parties’ up-front decisions. There are other facets of repair outsourcing settings that are not addressed in this paper but deserve thorough consideration. For example, some environments may feature many-to-one relationships between firms and a service provider. In such a setting, the vendor's exposure to financial distress is naturally reduced because of diversification. However, the vendor may also seek to exploit
“economies of scale” advantages, which could create incentives for opportunist decision making. Another alternative setting is one where failure and repair investment decisions could be reversible in the short run, which might provide incentives for dynamic decision making. It is also conceivable that in some settings both the failure and repair rate are determined by co-efforts of client and vendor. We believe that each of these settings involves interesting trade-offs, especially in light of the financial uncertainties embedded in performance-based contracting. Given the importance of repair outsourcing in today’s world, we consider each of these settings to be an important topic for future research.

Supplemental Material
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Endnotes
3. Raymond Corporation offers a range of products and after-sale services, including field service support for repair of forklift trucks.
6. Ruin theory provides well-established models that study the financial risks of insurance firms in settings of stochastic wealth evolution.
7. In 2010, DBS Bank was assessed a fine of 230 million Singapore dollars by the Central Bank of Singapore for disruption of the bank’s ATM, online, and mobile services for more than seven hours.
8. Often such contracts have multiple performance brackets (tiers), but in this paper we use a single-tiered structure for tractability. However, the insights of our analysis are robust to a more general tiered contract structure.
9. Following the literature on service contracting, we use the equilibrium profit rate for our analysis (cf. Hasija et al. 2008, Ren and Zhou 2008).
10. The wealth evolution model of an insurance firm is \( X(t) = x + at - \sum_{i=1}^{M_t} x_i \), where \( a \) is the initial wealth of the firm, \( a \) is the per-unit time premium earned by the firm, \( x_i \) is the stochastic cash outflow due to the \( i \)th claim, and \( M_t \) is the stochastic number of claims up to time \( t \).
11. To satisfy the vendor’s financial distress constraint while attaining the first-best decisions (\( \tau^*, \mu^* \)) and ensuring that the vendor’s participation constraint is satisfied.
12. We generated an initial sample of 1,500 input parameter vectors. Out of this sample, we excluded 97 vectors (less than 6.5%) for which MATLAB numerical output was below the tolerance threshold for optimal search. We also excluded 110 vectors for which 200 vectors for which \( \Pi^* \) < 0. Thus, we report findings based on using the 1,293 remaining input parameter vectors.
13. We generated an initial sample of 1,500 input parameter vectors. Out of this sample, we excluded 102 vectors (less than 6.9%) for which MATLAB numerical output was below the tolerance threshold for optimal search. We also excluded 200 vectors for which 200 vectors for which \( \Pi^* \) < 0. Thus, here we report findings based on using the 1,198 remaining input parameter vectors.
14. The formal proof of this finding is omitted as it directly follows from the workings presented in the proof of Proposition 8.

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