1. Modeling the salvage value (resp., cost of disposal) of excess supply

In this e-companion we analyze the forward and spot market equilibria when we allow a unit cost of disposal or salvage value for excess supply. Let $c$ be this value. If $c > 0$ then the product has a positive salvage value. If $c < 0$ then the product has a cost of disposal. We assume further that $c \leq b$. That is, the salvage value does not exceed the marginal cost. The analysis in Section 3 of the main manuscript will be modified as follows. Producer $i$’s period 1 profit maximization problem can be written as

$$\max_{q_i \geq 0} \{ (P_s - b)q_i \}. \quad (54)$$

We can rewrite the problem as follows:

$$\max_{q_i \geq 0} \{ [a + \hat{\epsilon} - Q_f - q_i - q_s' - b]q_i \}. \quad (55)$$

Solving for the Nash equilibrium quantities and prices in this spot market Cournot game yields the result given in Proposition 8.

**Proposition 8** The Nash equilibrium of the Cournot game in period 1 is unique. The equilibrium quantities are as follows.

1. If $\hat{\epsilon} \leq Q_f - a + b$, then $q_1^*(\hat{\epsilon}, Q_f) = 0$; also, $P_s(\hat{\epsilon}, Q_f) = \max(c, a + \hat{\epsilon} - Q_f)$.

2. If $\hat{\epsilon} > Q_f - a + b$, then $q_1^*(\hat{\epsilon}, Q_f) = a + \hat{\epsilon} - Q_f + \frac{2b}{3}$; also, $P_s(\hat{\epsilon}, Q_f) = a + \hat{\epsilon} - Q_f + \frac{2b}{3}$.

Note that if demand turns out to be low (i.e., $\hat{\epsilon} \leq Q_f - a + b$), then producers will not sell in the spot market. The only sales will come from speculators which will clear their inventory at the market clearing price, $\max(c, a + \hat{\epsilon} - Q_f)$. This price can be negative whenever $c < 0$ and $\hat{\epsilon} < Q_f - a$—that is, the product is costly to dispose of and there is no demand for the product at a price greater or equal to zero. The expected spot market price will be:

$$E_\epsilon(P_s) = \int_{Q_f + b - a}^{\infty} \frac{a + \epsilon - Q_f + 2b}{3} dF(\epsilon) + \int_{-a + Q_f + c}^{Q_f - a + b} (a + \epsilon - Q_f) dF(\epsilon) + cF(-a + Q_f + c). \quad (56)$$

In a rational expectations equilibrium, the forward price will be an unbiased estimator of the spot price.

$$P_f = E_\epsilon(P_s). \quad (57)$$

Next, we solve the producers’ profit maximization problem and determine the optimal forward quantity to sell in period 0. The total expected profit of producer $i$ is

$$E_\epsilon(\Pi_i) = q_f^i (P_f - b) + E_\epsilon \left( q_s^i (P_s - b) \right). \quad (58)$$
Under the assumption of risk neutrality, the producer maximizes his expected profit:

$$\max_{q_i} E_e(\Pi_i).$$

(59)

We find that the forward market equilibrium is always symmetric and unique (i.e., $q^1_f = q^2_f$). Proposition 9 gives a formal statement of the equilibrium result.

**Proposition 9** There always exists a unique forward market Nash equilibrium. The forward market equilibrium is symmetric and is given by $q^1_f = q^2_f = Q/2$, where $Q \geq 0$ is a fixed point of $\hat{G}(\cdot)$:

$$\hat{G}(Q) = 2 \int_{Q-a+b}^{Q-a+c} F^c(e) \, de - 9 \int_{Q-a+c}^{Q-a+b} F(e) \, de \over 3 + 6F(Q - a + b) - 9F(Q - a + c).$$

(60)

**Proof of Proposition 9:** Producer $i$’s profit maximization problem can be written as

$$\max_{q^i_f} q^i_f \left( \int_{Q_f+b-a}^{Q_f+b} \frac{a + e - Q_f + 2b}{3} \, dF(e) + \int_{a+Q_f+c}^{Q_f+b-a} (a + e - Q_f) \, dF(e) + cF(-a + Q_f + c) - b \right)$$

$$+ \int_{Q_f+b-a}^{Q_f+b} \left( \frac{a + e - Q_f - b}{3} \right) \left( \frac{a + e - Q_f + 2b}{3} - b \right) \, dF(e)$$

(61)

for $i = 1, 2$ and $Q_f = q^1_f + q^2_f$. The first-order conditions (FOCs) and second-order conditions (SOCs) for maximizing seller $i$’s profit (59) are

$$q^i_f = \int_{Q_f}^\infty F^c(e) \, de - 9 \int_{Q_f}^w F(e) \, de \over 3 + 6F(z) - 9F(w), \quad i = 1, 2,$$

(62)

$$3q_f^i (3f(w) - 2f(z)) - 4 - 14F(z) + 18F(w) \leq 0, \quad i = 1, 2.$$

(63)

where $z = Q_f - a + b$, $w = Q_f - a + c$.

Equation (62) indicates that the equilibrium is always symmetric ($q^1_f = q^2_f$). After adding the reaction equations (62) for $q^1_f$ and $q^2_f$ we get that $Q_f$ is a fixed point of $\hat{G}(x)$ as given by

$$\hat{G}(x) = 2 \int_{x-a+b}^{x-a+c} F^c(x) \, de - 9 \int_{x-a+c}^{x-a+b} F(x) \, de \over 3 + 6F(x - a + b) - 9F(x - a + c).$$

One can show using identical steps as in the proof of Proposition 2 in the paper, that there exist a fixed point of $G(x)$ at which the second order condition is verified.

It is easy to see that the same insights from the comparative statics in Section 4 remain valid, with the only observation that the discussion is now in terms of $b - c$ rather than $b$. Specifically, if the fraction $(b - c)/a$ is below a threshold $\delta$, then the more uncertain the demand, the higher is the quantity sold in the forward market and the lower is the relative volume of additional spot sales. In contrast, if $(b - c)/a$ is above a threshold $\delta$, then the more uncertain the demand is, the lower the quantity sold in the forward market and the higher the expected additional spot sales. For intermediate values of $(b - c)/a$, the quantity sold in the forward market is U-shaped in the level of uncertainty.